

$$1) a) f(x) = \ln x \quad \left\{ \begin{array}{l} f'(x) = \frac{1}{x} \end{array} \right.$$

$$b) f(x) = \log x \quad \left\{ \begin{array}{l} f'(x) = \frac{1}{x \ln 10} \end{array} \right.$$

$$2) y = \ln(x^3 + \cos x) \quad \left\{ \begin{array}{l} y' = \frac{3x^2 - \sin x}{x^3 + \cos x} \end{array} \right.$$

$$3) y(t) = \tan t + \ln(\tan t)$$

$$y'(t) = \sec^2 t + \ln(\tan t) + \tan t \cdot \frac{1}{\tan t} \cdot \sec^2 t$$

$$y'(t) = \sec^2 t (\ln(\tan t) + 1)$$

$$4) y = (2x+1)^{\sin x}$$

$$\ln y = \sin x \ln(2x+1)$$

$$\frac{y'}{y} = \cos x \cdot \ln(2x+1) + \sin x \cdot \frac{1}{2x+1} \cdot 2$$

$$y' = y \left(\cos x \ln(2x+1) + \frac{2 \sin x}{2x+1} \right)$$

$$y' = (2x+1)^{\sin x} \left(\cos x \ln(2x+1) + \frac{2 \sin x}{2x+1} \right)$$

$$5) s(t) = 2t^3 - 105t^2 + 1500t$$

$$a) v(t) = 6t^2 - 210t + 1500$$

$$b) v(5) = 600 \text{ ft/s}$$

$$v(15) = -300 \text{ ft/s}$$

$$v(35) = 1500 \text{ ft/s}$$

$$0 = 6t^2 - 210t + 1500$$

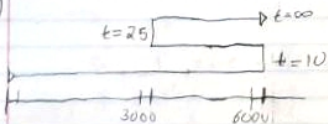
$$0 = 6(t^2 - 35t + 250)$$

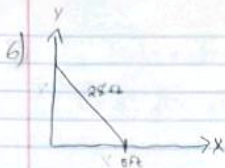
$$0 = 6(t-10)(t-25)$$

c) Particles at rest at $t = 10, 25$

d) $[0, 10)$ increasing, $(10, 25)$ decreasing, $(25, \infty)$ increasing

e)





$$\frac{dx}{dt} = 1.2 \frac{ft}{s}, \quad \frac{dy}{dt} = ?$$

$$x = 5, \quad y = ?$$

$$5^2 + y^2 = 784$$

$$x^2 + y^2 = 784$$

$$y = \sqrt{759} \approx 27.55 \text{ ft}$$

$$2xy' + 2yy' = 0$$

$$2(5)(1.2) + 2(\sqrt{759})y' = 0$$

$$2\sqrt{759}y' = -12$$

$$y' = \frac{-12}{2\sqrt{759}} \left\{ \frac{dy}{dt} = -\frac{6}{\sqrt{759}} \approx -0.22 \frac{ft}{s} \right\}$$

7) a) $y = 3x^2 - 4x \left\{ \frac{dy}{dx} = 6x - 4 \right\}$

b) $y = \ln(t^3 + 1) \left\{ \frac{dy}{dt} = \frac{3t^2}{t^3 + 1} \right\}$

8) $y = \sqrt{x}, \quad x = 4, \quad \Delta x = 0.1, \quad dy = ?, \quad \Delta y = ?$

$$\Delta y = f(4 + 0.1) - f(4) \left\{ \begin{array}{l} f(4.1) = \sqrt{4.1} \\ f(4) = \sqrt{4} \end{array} \right.$$

$$\Delta y = \sqrt{4.1} - \sqrt{4} \left\{ \Delta y \approx 0.025 \right\}$$

$$y' = \frac{1}{2\sqrt{x}} \quad dy = f'(x) dx$$

$$dy = \frac{1}{2\sqrt{4}} \cdot 0.1 \left\{ dy \approx 0.025 \right\}$$

9) Linear Approximation: $(3.01)^3$ } x^3 at $x=3.01$

$$f(a) = 3.01^3$$

$$f(a) + f'(a)(x-a)$$

$$f(a) = 27.2709$$

$$f'(a) = 3x^2$$

$$f'(3.01) = 27.1803$$

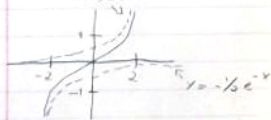
$$27.2709 + 27.1803(x - 3.01)$$

$$27.2709 + 27.1803x - 81.8127$$

$$f(3.01) = 27.1803(3.01) - 54.5418$$

$$f(3.01) \approx 27.27$$

10) a) $y = \sinh x$ } $x = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$
 $y = \frac{1}{2}e^x$

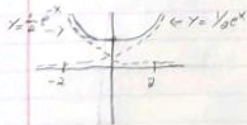


Asymptotes:

$$y = \frac{1}{2}e^x$$

$$y = -\frac{1}{2}e^{-x}$$

b) $y = \cosh x$ } $x = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$



Asymptotes:

$$y = \frac{1}{2}e^x$$

$$y = \frac{1}{2}e^{-x}$$

11) $\cosh x - \sinh x = e^{-x}$: prove

$$\frac{e^x + e^{-x}}{2} - \frac{e^x - e^{-x}}{2} = e^{-x}$$

$$\frac{e^x + e^{-x} - e^x + e^{-x}}{2} = e^{-x}$$

$$\frac{2e^{-x}}{2} = e^{-x} \quad \left\{ e^{-x} = e^{-x} \right\}$$

12) $\cosh(2x) = \cosh^2 x + \sinh^2 x$

$$\frac{e^{2x} + e^{-2x}}{2} = \left(\frac{e^x + e^{-x}}{2} \right)^2 + \left(\frac{e^x - e^{-x}}{2} \right)^2$$

$$\frac{e^{2x} + e^{-2x}}{2} = \frac{(e^x)^2 + 2e^x e^{-x} + (e^{-x})^2}{4} + \frac{(e^x)^2 - 2e^x e^{-x} + (e^{-x})^2}{4}$$

$$\frac{e^{2x} + e^{-2x}}{2} = \frac{2e^{2x} + 2e^{-2x}}{4} \quad \left\{ \frac{e^{2x} + e^{-2x}}{2} = \frac{e^{2x} + e^{-2x}}{2} \right\}$$

13) a) $y = \cosh x \quad \left\{ \cosh x = \frac{e^x + e^{-x}}{2} \right\} \quad \left\{ \cosh' x = \frac{e^x - e^{-x}}{2} = \sinh x \right\}$

b) $y = \sinh x \quad \left\{ \sinh x = \frac{e^x - e^{-x}}{2} \right\} \quad \left\{ \sinh' x = \frac{e^x + e^{-x}}{2} = \cosh x \right\}$

14) $y = \cosh(x^3) \quad \frac{dy}{dx}(\cosh(x^3)) \quad \frac{dy}{dx}(x^3)$

$$y' = \sinh(x^3) 3x^2$$