Binary Number Project

acknowledgements to Kyle Gillette for this assignment

Welcome to the Binary Number project. In this challenging but interesting exercise we will be exploring how decimal numbers are represented in binary form, and how mathematical operations are performed using binary numbers. We will also be working more in depth with String operations.

This is "Pair Programming" Assignment

While you are working on this project, you will use the "Pair Programming" approach. In pair programming, two programmers share one computer. One student is the "driver," who controls the keyboard and mouse. The other is the "navigator," who observes, asks questions, suggests solutions, and thinks about slightly longer-term strategies. I would suggest that the 'navigator' have the directions open on their screen, and the 'driver' have the code on their screen. The two programmers should switch roles about every 20 minutes. Working in pairs should make you much better at programming than would be by working alone. The resulting work of pair programming nearly always outshines that of the solitary programmer, with pairs producing better code in less time. For this assignment each partner will receive the same grade.

When you have selected your partner and you are sitting next to each other, you may begin work on this assignment. Remember that you both need to be working together as a unified pair in order to successfully complete this assignment. No 'going rogue' on your own. Good luck and have fun!

***Be sure that both of you have duplicate saved copies of your work each day**. You need to have something to work with tomorrow in the event your partner is absent.*

Download, extract, and save the Binary Number project to your computer. Open up the project by clicking on the BlueJ package file icon. Notice the (incomplete) `BinaryNumber` class and the `BinaryNumberTest` class (which should be listed as 'no source'). You may run `BinaryNumberTest` at any time to see how much of the assignment you have completed.

Your assignment is to complete the `BinaryNumber` class that represents (guess what?) binary numbers. The `BinaryNumber` class will represent a binary number as a `String` entirely composed of 1s and 0s. (For example "10011001".) While the length (number of digits, or bits) of a `BinaryNumber` is arbitrary, you should assume that all `BinaryNumbers` being used for calculations will all have the same number of bits (digits).

Here are the instructions for you to complete the `BinaryNumber` class:

1. Write a constructor that accepts a `String` as a parameter, and uses it to set the instance variable.
2. Complete the `getNumber` method that returns the number (which is a `String`).
3. Now the assignment gets a bit (ha!) more challenging. Do you recall how the `substring` method of the `String` class works? There are two versions of it (it is an *overloaded* method), namely `subtring(beginIndex)` and `substring(beginIndex, endIndex)`. A useful way to remember how either version of substring works is to think of a loaf of sliced bread (see the diagram on the next page).

0	1	2	3	4	5	6	7	8	9	0	1
S	l	i	c	e	d		B	r	e	a	d

0 1 2 3 4 5 6 7 8 9 0 1 2

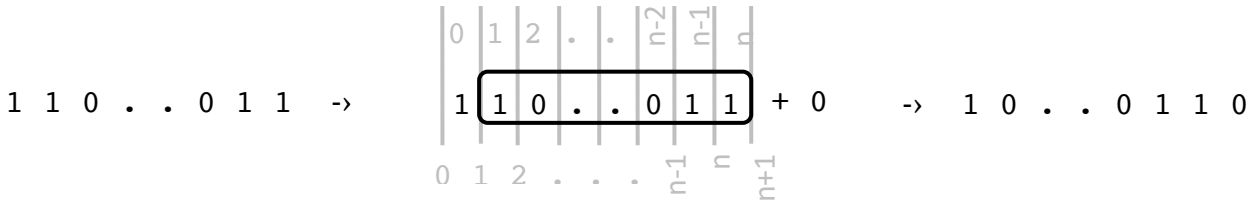
Think of the slices (as numbered on the bottom) as isolating a specific piece of bread (as numbered on the top).

For example (given the diagram above), if you were looking for everything except the *first* letter, you would start at slice 1, and take the rest of the loaf. In the same way `substring(1)` would yield (in this case) the `String "liced Bread"` (yuk! That's disgusting!).

For another example (given the diagram above), if you were looking for everything except the *last* letter, you would start at slice 0, and go through slice 11 (or $n - 1$ for a string that is n characters in length). In the same way `substring(0,11)` would yield (in this case) the `String "Sliced Brea"` (now that sounds better!).

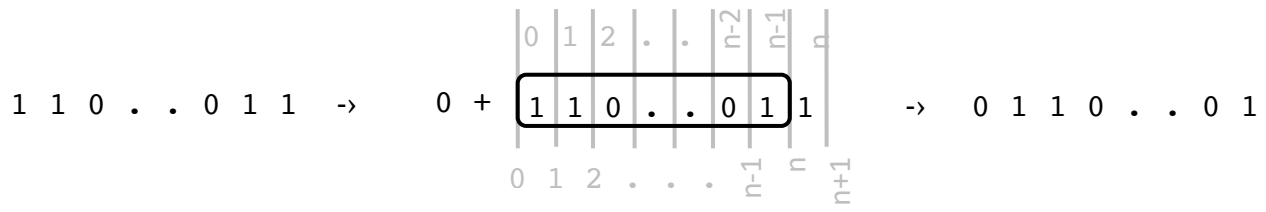
As a third example (given the diagram above), if you were looking for the letter in position 3 (the `"c"` in `"sliced"`), you would start with slice number 3 and end with slice number 4. In the same way, a call to `substring(3, 4)` would yield the single character `String "c"`.

Now, we will put that knowledge to use in writing the next three methods. First, complete the `shiftLeft()` method that returns a new `BinaryNumber`, where each bit is moved one position to the left, while the left-most bit is eliminated, and the right-most bit automatically becomes a zero. Note that the number of digits in the new and old binary number remains the same. You may find it helpful to remember that `substring(beginIndex)` returns the `String` starting at `beginIndex` through the rest of the `String`. Find the proper substring, concatenate a 0 to the end of it, and use it to `return new BinaryNumber(String);`.



Notice that because each bit moves over one position to the left, this method creates a new `BinaryNumber` that is twice the value of the original (hence this method performs multiplication by 2). Can you see the inherent problem with possible misevaluation because the left-most digit is discarded each time?

4. Next complete the `shiftRight()` method that returns a new `BinaryNumber` where each bit is moved one position to the right, while the right-most bit is eliminated, and the left-most bit automatically becomes a zero. Note that the number of digits in the new and old binary number remains the same. You may find it helpful to remember that `substring (beginIndex, endIndex)` returns the `String` starting at `beginIndex` and ending at `endIndex - 1`.



Notice that because each bit moves over one position to the right, this method creates a new `BinaryNumber` that is half the value of the original (hence this method performs division by 2). Can you see the inherent problem with loss of precision because the right-most digit is discarded each time? This is the root of the issue with integer division as the 'one's digit' is always ignored, so odd numbers always divide evenly. For example, when 0111 (which is 7) is halved (shifted right), it becomes 0011 (which is 3).

5. Your next task is to complete the `getBit` method that looks at the bit in the given position (starting from the right-hand side at position 0) and returns `true` if the bit is a 1, or `false` if the bit is a zero. For example, if the binary number was `"00001000"`, a call to `getBit(3)` would return `true`, while a call to `getBit(4)` would return `false`. You'll have to write a routine that will look up the single character at the given location and then return either true or false based on what it is. There are several ways to do this, but probably the most obvious is to use the `String` class's `substring(beginIndex, endIndex)` method.

   Just remember that for this method, however, the `String` positions start at 0 on the right-hand side, and go up to the left. You will have to do some subtraction to figure out the appropriate opposite position. It might help you to remember the `String` class's `length` method, which returns the number of characters in the entire string (in other words, it's length). Perhaps you have already realized that the calculation to find the proper start position is:

   ```
   number.length() - position - 1.
   ```

   Another important consideration is the proper way to compare two objects for equality, in particular, two `String` objects. **Never test for `String` equality like this: `oneString == anotherString`**. What the 'double equals sign' (==) does is compare if the two `String` objects have *the same address in memory*, not if they have the same characters. You should use the `String` class's built-in `equals` method instead, which is specifically designed to compare the characters in each string to see if they are the same or not. The proper way to see if the two `Strings` are equivalent is to call `oneString.equals(anotherString)`. We'll learn later on that properly written objects that are to be compared to one another need to include an `equals` method that defines how equality is to be measured.

   Make sure the `getBit(int position)` *method is working correctly before you go any further.* All of the other remaining methods are dependent on this one working properly.

6. Write a `not()` method that returns a new `BinaryNumber` that has exactly the opposite values as the original `BinaryNumber`. In other words, any 1s become 0s and any 0s become 1s. The number of digits remains the same.

   You will probably need to loop through each bit, one-at-a-time (using `getBit`), and then build a new `String` that has the opposite values. Be careful to think about how you are looping through the original binary number string, and then consider the order of the digits as you are building the new binary number string. It's easy to make a mistake by accidentally building the new string in reverse order.

7. Complete the `and` method that takes another `BinaryNumber` as a parameter, and returns a new `BinaryNumber`. The two `BinaryNumbers` that are being evaluated to form a third new one are the existing `BinaryNumber (this`, the one you are in right now) and the other `BinaryNumber (other)` that is being passed as a parameter. Loop through each of those two `BinaryNumbers` looking at the same bit position in each of them. Since both binary numbers have the same number of digits, you can use a single loop, and then use the loop index to get the

```
    111000
and 100110
    100000
```

appropriate bit from both numbers. The rules for the `and` method come directly from Boolean logic; if both bits are `true` then that same bit in the new (third) binary number is set to `true`. If either of the two bits is a `false`, then that bit in the new binary number is `false`. You get the idea.

8. Complete the `or` method that takes another `BinaryNumber` as a parameter, and returns a new `BinaryNumber`. The `or` method is very similar to the `and` method you just wrote, except that if *either* of the two bits being examined are `true`, the corresponding bit in the new binary number will be set to `true`. The only time the new bit will be `false` is if both bits being compared are `false` as well.

```
    111000
or  100110
    111110
```

9. Complete the `xor` (known as an *exclusive or*) method that takes another `BinaryNumber` as a parameter, and returns a new `BinaryNumber`. The `xor` method is very similar to the `or` method you just wrote, except that only if <u>one</u> of the two bits being examined are `true`, the corresponding bit in the new binary number will be set to `true`, otherwise it will be set to `false`. In other words, if *both* bits are `true`, or if *both* bits are `false`, then the corresponding bit in the new binary number will be `false`.

```
    111000
xor 100110
    011110
```

10. Complete the `add` method that takes another `BinaryNumber` as a parameter, and returns a new `BinaryNumber`. While this method uses the same looping procedure that you did for the previous 3 methods, this one requires some careful consideration. Just like adding two decimal numbers, if the sum of two digits is more than 9, you have to keep track of the 'carry' number that spills over into the next digit's calculation. The same thing holds true for binary numbers (which, of course can only be 1s or 0s). In other words, when adding two binary bits together, you also have to take into account the amount (if any) that was 'carried over' from the calculation of previous digits. So there are essentially three bit values that have to be added together for any given digit calculation, one bit from `this` binary number, one bit from the `other` binary number, and the 'carry' bit (if any). The possible sum of those three bit values is either 0, 1, 2, or 3. A digit total of 2 or 3 means that 2 should be subtracted from the total and the 'carry' bit should be set to 1. If you have an integer variable called `count` that stores the given digit total, and an integer variable named `carry` that stores the digit carry value, and a `String newNumber` that is used to create the new `BinaryNumber`, then as you loop through each digit position starting at 0, the following routine might prove helpful to you:

```
     1 1 1 1
    011110
add 000111
    100101
```

```
int count = 0;
if(this.getBit(i))
    count ++;
if(other.getBit(i))
    count ++;
if(carry > 0)
{
    count++;
    carry--;
}
newNumber = (count % 2) + newNumber;
carry += count / 2;
```

11. And now for the most intriguing part of this assignment, let us specifically consider in detail the mathematical numbering concept known as *two's compliment*. A two's-complement number encoding system allows for binary numbers to represent both positive and negative values. Specifically, two's complement is defined as the additive inverse of a binary number. In other words, when a binary number and it's "two's complement" are added together, the sum is always zero. For example, given the equation `x + y = 0`, solving for `x` yields `x = -y`. This is how calculating two's complement can be used to represent a negative value using binary numbers.

    So how do we calculate a two's complement binary value? Actually it is way easier than it sounds. Simply take the inverse (using the `not` method) of the binary number, and then add 1 to it.

    ```
                  011110
    add (not)  100001
                  111111

                1 1 1 1 1 1
                  111111
    add (one)  000001
                 1000000
    ```

    Note that simply adding a binary number to it's opposite (not) does not equal zero (actually this is the *one's complement*). Rather the sum will be a binary number consisting of all 1s, which in a normal binary numbering scheme represents the highest numerical value that a binary number of a given length can store. Ah, but if you add 1 to that total, all of the 1s flip to 0s, *except* for an extra last leading overflow 1s digit that comes from the carry forward from all the previous digits. But remember, the number of digits of a binary number does not ever change (the number of bits is constant), so that extra overflow leading digit is simply thrown away, and we wind up with all 0s instead.

    ```
                1 1 1 1 1 1
                  011110
    add (2's)  100010
                 1000000
    ```

    Complete the `twosComplement` method returns a new `BinaryNumber`. Use the `not` method to create a new `BinaryNumber` that is the opposite of the original, and then add 1 to it. Note that to properly add 1 to any binary number, you need to first figure out how many digits are in the binary number you are adding it to, and then fill in the appropriate amount of leading zeros of your binary number 1. Since our `BinaryNumber` class stores the number value as a `String`, can you think of a method that could tell you what the length of the `String` might be?

12. Write the `binary2decimal` method that returns the decimal (base 10) integer value of the binary number. For example the binary number 00110011 is equivalent to decimal number 51. How do you convert a binary number to a decimal number, you ask? How did you do it by hand back in August? Start at bit position 0 and if the bit is 1 (true) add $2^{(position)}$ to the total.

    As described above, our `BinaryNumbers` class will be following the two's complement encoding scheme that is widely used (including by Java) for representing negative binary numbers. Two's complement uses the most significant bit (the last one on the left) as a 'sign bit'. By using a 'sign bit' we get a whole new range of numbers available to us – specifically negative numbers. Therefore if the last bit (the one on the far left) is true, then the number is negative. Check the last bit once more (after you have summed up all the values), and if it is `true`, then subtract $2^{(number\ of\ digits)}$ from the total. For example, normally the 6 bit binary number 111111 would be equivalent to 63, but using a two's complement encoding scheme it would 63 – 2^6 (or 64) which equals -1. In fact, a handy check is that *any two's complement binary number that is entirely composed of 1s is always equivalent to -1*.

    So think about it for a minute; if we have an 8 bit number (one byte) that would mean we can normally represent the number 0 – 255 (which represent 2^8 numbers). Does that mean that by using the most significant bit (the one on the far left representing 2^7) as a 'sign bit' that we now can represent all the values between negative 255 and positive 255? Unfortunately no, because binary and decimal numbers have a 1-to-1 relationship, and if 8 bits can only represent 256 different binary numbers, it also can only represent 256 different decimals. Because that last (and most significant) bit is used to determine the sign, the largest positive number that can be

represented in 8 bits is 01111111 (which is 127), while the smallest negative number is 10000000 (which is 128 – 2^8 [or 256] which equals -128). Thus we get the range from -128 to 127, which (including 0 of course) is 512 different numbers.

Then why don't we just keep a separate bit just dedicated to the number sign, and use the rest for number values, you ask? (By the way, this is called a *sign and magnitude* encoding scheme.) Using this scheme the 8 bit binary number range would go from 0 to 127 (based on the first 7 bits) and either positive or negative (based on the last bit), which would include both a positive and a negative zero (both of which are literally nonsensical). One's complement (which is simply flipping all the 1s to 0s and 0s to 1s) does the same thing (there are two values for zero). Using two's complement, on the other hand, gives only one zero representation and an extra number value to boot. That's why it is so widely used.

Say we have the maximum positive value for a 10 bit two's complement binary number, which would be 0111111111, or 511 in decimal form. What happens when we add 1 to that number? In decimal form then answer is obviously 512, but it is not so apparent in a two's complement (or any form of) binary number representation.

```
decimal              binary      2's Comp

                  1 1 1 1 1 1 1 1 1
  511               0111111111     (511)
+   1             +  0000000001   +  (1)
  512               1000000000     (-512)
```

This resulting miscalculation is called (can you guess?) arithmetic overflow. It happens when we try to represent a number outside the range of maximum (or minimum) values. We don't come across this problem with decimal values *because we simply add another leading significant digit* if needed. You can't do that with binary numbers because their number of bits is fixed from the beginning, so their range is automatically limited. Why don't we just make them with a really large number of bits to start with? Because every binary number would have that large number of bits, which would use up a great amount of computer resources (both memory and speed). In Java, a primitive `int` is represented by 4 bytes (32 bits), and uses a two's complement encoding scheme. That means that an `int` can represent from -2,147,483,648 to 2,147,483,647 (from -2^{16} to $2^{16} - 1$), and if you try to add 1 to the maximum integer value, you will 'wrap around' and get the minimum integer value. This is the source of arithmetic overflow errors.

If you are interested, there is a simple test when using a two's complement encoding scheme for detecting when arithmetic overflow has occurred when adding two numbers. You can be sure that arithmetic overflow has occurred if two positive numbers added together yield a negative number, and conversely, if two negative numbers added together yield a positive number.

13. Lastly, complete the `toString` method that includes both the binary and decimal representations of the number. For example, your `toString` might return a `String` that contains something like "`Binary: 00011  Decimal: 3`". The exact format (letters, spacing, etc.) is not important, just so long as it contains both the binary and decimal values of the number.

Good luck. Show me the results of your successful test when you are finished. Feel free to see me if you have any questions or difficulties in the meantime.