PSTAT174_Lab03

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4/16/2021

1. Consider the AR(2) process below:

$X_t = 0.8X_{t-1}$-0.12X_{t-2}+Z_t with Z_t in N (0, 1).

(a) Express the processes in terms of the back shift operator, B.

(1 - 0.8B + 0.12B^2)$X_t = Z_t$

(b) Determine whether each process is causal and/or invertible. (Hint: use polyroot()).

```
polyroot(c(1,-0.8,0.12))
```

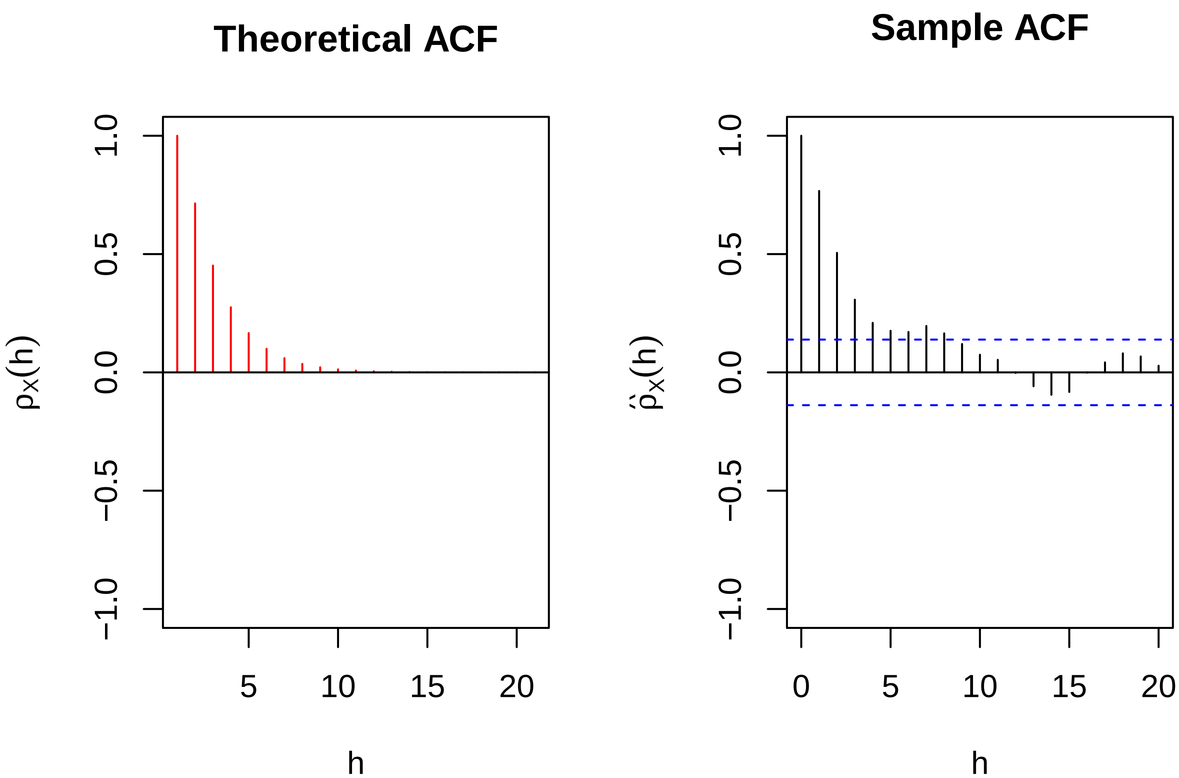
```
## [1] 1.666667+0i 5.000000+0i
```

In this case it is known to be autoregressive therefore we can conclude that it is invertible. And since the zeros fall outside of the unit circle we can therefore conclude the at AR(2) is casual.

```
set.seed(1234)
ar2 <- arima.sim(model = list(ar = c(0.8,-0.12),sd = 1),n = 200)

theo_acf <- ARMAacf(ar = c(0.8,-0.12),lag.max = 20, pacf = FALSE)
op <- par(mfrow = c(1,2))
# Theoretical ACF
plot(theo_acf,type = "h",ylim = c(-1,1),
main = "Theoretical ACF",
col = "red",
ylab = expression(rho[X](h)), xlab = "h")
abline(h = 0) # Add horizontal line


# Sample ACF
acf(ar2,lag.max = 20,
main = "Sample ACF",
ylim = c(-1,1),
xlab = "h",
ylab = expression(hat(rho)[X](h)))
```




```
par(op)
```

(d) Use the above simulation to manually construct the Yule-Walker estimates ϕ^1, ϕ^2 and σ_Z^2. Also, use the pre-installed function ar.yw() for estimation.

```
# Estimation with Yul-Walker eqns
acv_ar <- acf(ar2,type = "covariance",main = "Sample ACF",plot = F)

Rho <- toeplitz(acv_ar$acf[c(1,2)]/acv_ar$acf[1])
rho <- acv_ar$acf[c(2,3)]/acv_ar$acf[1]

phi_hat <- solve(Rho) %*% rho
phi_hat
```

```
##             [,1]
## [1,]  0.9210879
## [2,] -0.2011451
```

```
 # Estimate of noise variance
sigma_z <- acv_ar$acf[1]*(1-t(rho)%*%solve(Rho)%*%rho)
sigma_z
```

```
##          [,1]
## [1,] 1.024407
```

```
#estimation
yw <- ar.yw(ar2,order = 2)
yw$x.mean # mean estimate
```

```
## [1] -0.01351863
```

```r
yw$ar # Parameter estimates
```

```
## [1]  0.9210879 -0.2011451
```

```r
yw$var.pred # Error variance
```

```
## [1] 1.040007
```