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PSTAT 174 HW#1 04/07/21

1. The correct option for this instance would be Statement (II) Deterministic trends are better suited to extrapolation than sochastic trends. Because for Statement (I) Stochastic are pertained to be not explainable, therefore we can conclude statement (I) is not correct. Then for statement (III) some trends are known to be true but some are not.

2. $X_1 = Z_1$ $X_t = X_{t-1} + Z_t$ $t = 2, 3, \ldots$

$Z \sim WN(\mu_z, \sigma_z^2)$

$E(Z_t) = \mu_z$ $Var(Z_t) = \sigma_t^2$

$Cov(Z_t, Z_s) = 0$ $t \neq s$

I) If $\mu_z \neq 0$ then random walk is nonstationary with mean

$E(X_1) = E(Z_1) = \mu_z$

$E(X_2) = E(X_1 + Z_2)$
$= \mu_z + \mu_z$
$= 2\mu_z$

$E(X_3) = E(X_2 + X_3)$
$= E(X_2) + E(X_3)$
$= 2\mu_z + \mu_z$
$3\mu_z$

$E(X_t) = t\mu_z$ random walk is known to be independent from time.
Therefore $\mu_z \neq 0$ b/c random walk is non-stationary if mean is true.

II) $\sigma_z^2 = 0$ then random walk is non-stationary in variance

$Var(X_1) = Var(Z_1)$
$= \sigma_z^2$

$Var(X_2) = Var(X_1 + Z_2)$
$= Var(X_1) + Var(Z_2) + 2Cov(X_1, Z_2)$
$= \sigma_z^2 + \sigma_z^2 + 2 \times 0$
$= 2\sigma_z^2$

$Var(X_3) = Var(X_2 + Z_3)$
$= Var(X_2) + Var(Z_3) + 2Cov(X_2, Z_3)$
$2\sigma_z^2 + \sigma_z^2 + 0 =$
$= 3\sigma_z^2$

$V(X_t) = t\sigma_z^2 \qquad t = 1, 2, \ldots$

Therefore, we know variance of random walk is nonstationary in variance to be not true.

III) If $\sigma_z^2 > 0$, then random walk is nonstationary in the variance

In section (II) $V(X_t) = t\sigma_z^2$ therefore be known to be dependent upon t. The Known case of section (III) is known to be nonstationary if variance true.

3. $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ $\hat{\rho}_4 = \frac{\hat{\gamma}(4)}{\hat{\gamma}(0)} = \frac{\sum_{i=1}^{n-4}(x_i - \bar{x})(x_{t+4} - \bar{x})}{\sum_{i=1}^{n}(x_t - \bar{x})^2}$

Quarter	Rainfall	$(x_t - \bar{x})$	$(x_t - \bar{x})^2$	$(x_{t+4} - \bar{x})$	$(x_t - \bar{x})(x_{t+4} - \bar{x})$
z_1	$x_{t1} = 25$	$25-24=1$	1		$(1)(2) = 2$
z_2	$x_{t2} = 19$	$19-24=-5$	25		$(-5)(14) = -70$
z_3	$x_{t3} = 10$	$10-24=-14$	196		$(-14)(-2) = 28$
z_4	$x_{t4} = 32$	$32-24=8$	64		$(8)(-4) = -32$
z_5	$x_{t5} = 26$	$26-24=2$	4	$26-24=2$	
z_6	$x_{t6} = 38$	$38-24=14$	196	$38-24=14$	
z_7	$x_{t7} = 22$	$22-24=-2$	4	$22-24=-2$	
z_8	$x_{t8} = 20$	$20-24=-4$	16	$20-24=-4$	
	$\sum x_t = 192$		506		-72

$\bar{x} = \frac{192}{8} = 24$ $\bar{x} = 24$

$\hat{\rho}_4 = \frac{-72}{506} = -.142$

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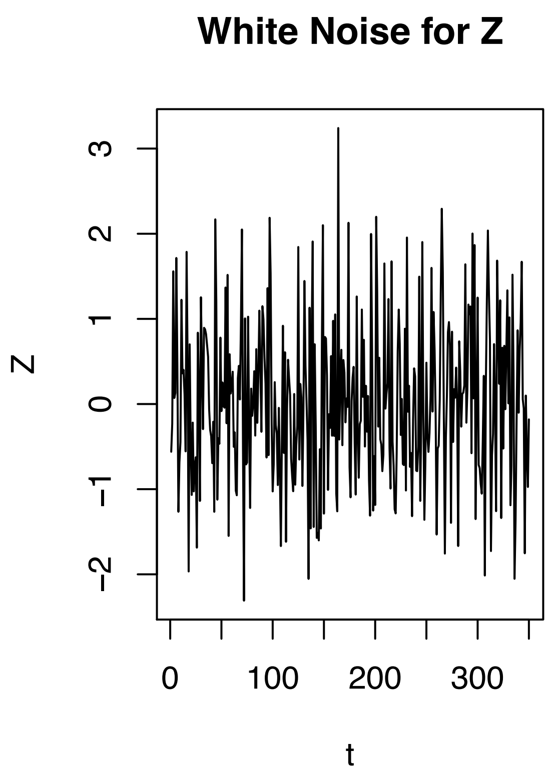
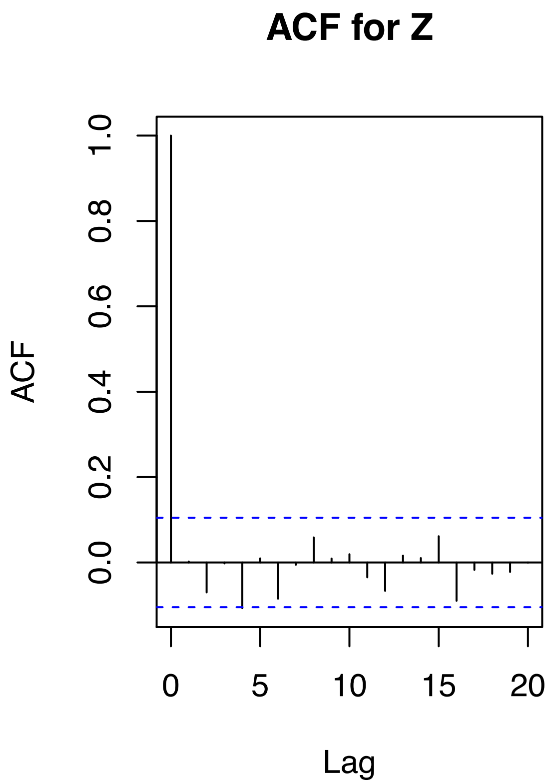
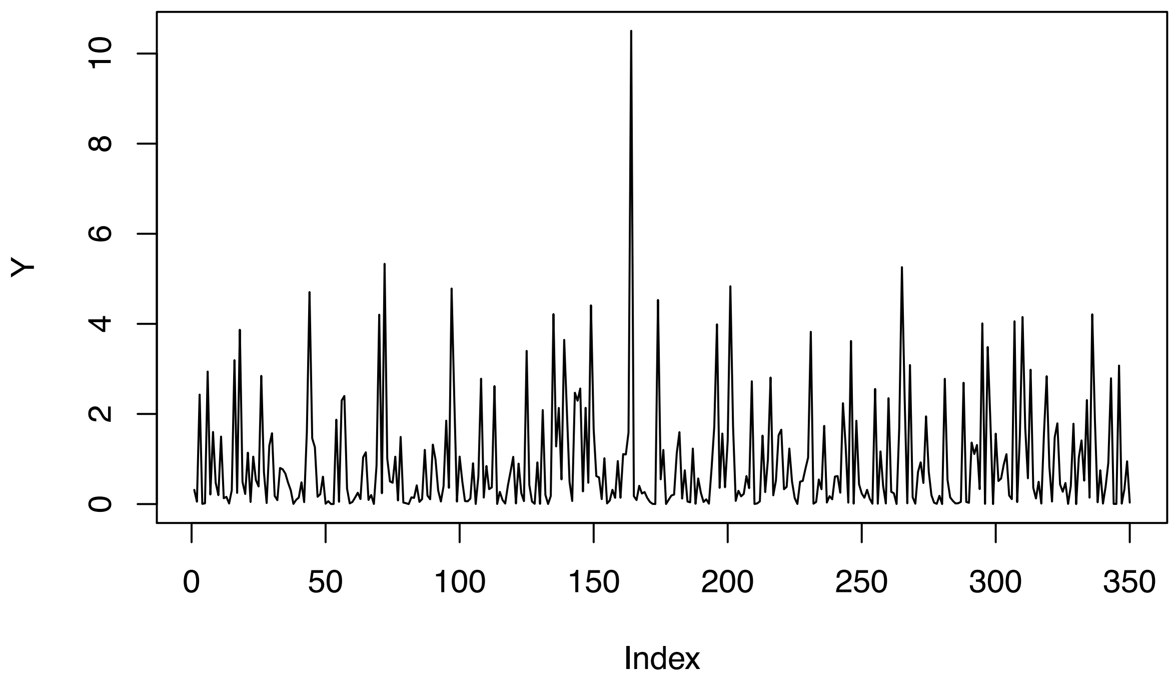
```
#Part 5 A
 set.seed(123)

Z <- rnorm(n = 350, mean = 0, sd = 1)

par(mfrow = c(1, 2))

plot.ts(Z, xlab="t", ylab="Z",main="White Noise for Z")

par(mfrow = c(1, 2))
```



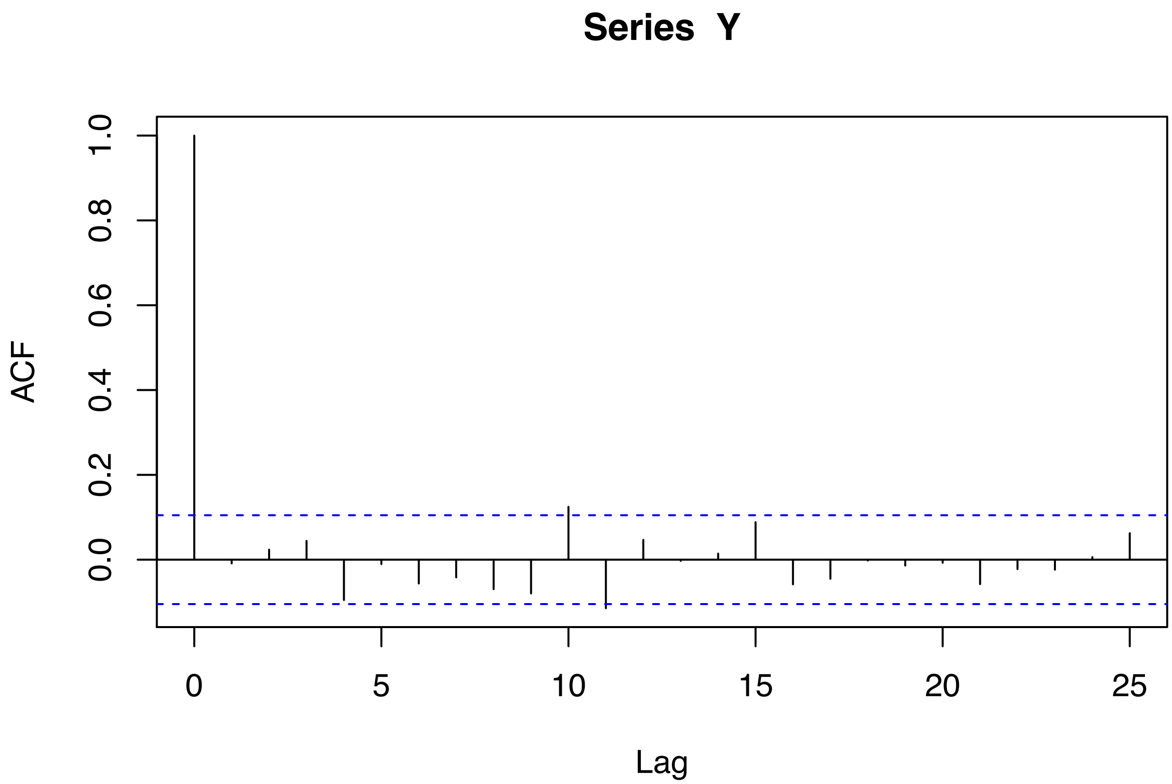

```
acf(Z, main = "ACF for Z", lag.max = 20)
```




```r
#Part 5 B
Y = Z^2

plot(Y, type = "l")
```




```r
acf(Y)
```


Series Y
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
Lag

4. $f(z) = 1 - .4z$

$z = 1/.4$

$= 2.5$

$g(z) = 1 + z - 6z^2$

$a = -6 \quad b = 1 \quad c = 1$

$$x_z = \frac{-1 \pm \sqrt{1+24}}{-12}$$

$$= \frac{-1 \pm 5}{-12}$$

$z = .5$ or $z = -.333$

polyroot(c(1,-.4)) [1] 2.5+0i

polyroot (c(1,1,-6)) [1] .5000000-0i -0.3333333+0i

5(c) Analyze graphs from (a) & (b)

> As shown in the R file the graphs given in the time series for $Y = Z^2$ has known positive values on y-axis. Where Z has positive and negative values that have symmetry for mean zero.

> Both are stationary considering no trend, no knowledge of seasonality, no change in varability, & no change of behavior.

> There no difference in the acf. function. The acf is known to be 1 at lag 0. When lag is greater than 0 since ACF is approx 0 they will be within interval

> ∵ $Y = Z^2$ and Z is Gaussian, $Y \sim \chi_1^2$ dist.

5(d) $Y_t = Z_t^2 \sim \chi^2_1$

$\mu_Y(t) = E[Y_t] = 1$
$\sigma^2_Y(t) = Var(Y_t) = 2(1) = 2$

$\rho_Y(t, t+h) = Cov(Y_t, Y_{t+h})$

$$\begin{cases} h=0 & Cov(Y_t, Y_{t+h}) = 1 \\ h \neq 0 & Cov(Y_t, Y_{t+h}) = \dfrac{Cov(Y_t, Y_{t+h})}{\sqrt{Var(Y_t)}\sqrt{Var(Y_{t+h})}} = 0 \end{cases}$$

Therefore,

$$\rho_Y(t, t+h) \begin{cases} h=0 & ,1 \\ h\neq 0 & ,0 \end{cases}$$