Celeste Herrera

PSTAT 174 HW #3

April, 21, 2021

1

$X_t = \frac{2}{3}X_{t-1} + \frac{1}{2}X_{t-2} + Z_t$

$Z_t = X_t - \frac{2}{3}X_{t-1} - \frac{1}{2}X_{t-2}$

$= X_t - \frac{2}{3}BX_t - \frac{1}{2}B^2X_t$

$= (1 - \frac{2}{3}B - \frac{1}{2}B^2)X_t$

$\varphi(z) = 1 - \frac{2}{3}z - \frac{1}{2}z^2 = 0$

$3z^2 + 4z - 6 = 0$

$x = \frac{-4 \pm \sqrt{16 - 4(3)(-6)}}{2(3)}$

$x = .8968, -2.2301$

$X_t = \frac{2}{3}X_{t-1} + \frac{1}{2}X_{t-2} + Z_t$ is AR(2) process

AR is always invertible therefore X_t is invertible.
$\varphi(z)$ must be outside the unit circle $|z| > 1$.
Since $x = .8968$ doesn't meet standards X_t is not stationary.

2. As to the three following Statements, the one I think to be correct is (II) Partial Autocorrelation for lag 4 is always equal to zero. Because AR(3) has the data depending on the prior 3 lags. Everything know of partial correlation after 3 has a value to be zero.

3. $\rho_x(1) = .7 \qquad \rho_x(2) = .3 \qquad ARMA(1,1)$

model $x_t - \phi_1 X_{t-1} = Z_t + \theta_1 Z_{t-1}$

$$X_t X_{t-k} - \phi_1 X_{t-1} X_{t-k} = Z_t X_{t-k} + \theta_1 Z_{t-1} X_{t-k}$$

$$E(X_t X_{t-k}) - \phi E(X_{t-1} X_{t-k}) = E(Z_t X_{t-k}) + \theta_1 E(Z_{t-1} X_{t-k})$$

$$\gamma_x(k) - \phi_1 \gamma_x(k-1) = 0 + 0 \qquad k = 1, 2, 3, 4, \ldots, n$$
$$= 0$$

$$\frac{\gamma_x(k)}{\gamma_x(0)} - \phi_1 \frac{\gamma_x(k-1)}{\gamma_x(0)} = 0 \qquad k = 1, \ldots, n$$

$$\rho_x(k) = \phi_1 \rho_x(k-1)$$

$$\frac{\rho_x(2)}{\rho_x(1)} = \phi_1 \rho_x(1) \quad \rho_x(4)$$

$$\phi_1 = \frac{\rho_x(2)}{\rho_x(1)} \rightarrow \frac{.3}{.7} = .429$$

4. $\varphi_{11} = \rho_x = -.6$

$\varphi_{21} + \varphi_{22}\, \rho_x(1) = \rho_x(1)$

$\varphi_{21} = \rho_x(1) - \varphi_{22}\, \rho_x(1)$

$= -.6 - (.36)(-.6) = -.384$

$\varphi_{21} = -.384 \qquad \varphi_{22} = .36$

$X_t = Z_t - .384\, X_{t-1} + .36\, X_{t-2}$

5. $X_t = .8X_{t-1} + 2 + Z_t - .5Z_{t-1}$

A is known to be True b/c the ACF for ARMA(1,1) is

$$\rho_x(k) = \frac{(\varphi_1 + \theta_1)(1 + \varphi_1\theta_1)}{(1 + 2\varphi_1\theta_1 + \theta_1^2)} \varphi_1^{k-1}$$

$k = 1$

$$\rho_x(1) = \frac{(.8 - .5)(1 + (.8)(-.5))}{(1 + 2(.8)(-.5) + (-.5)^2)} = \frac{.18}{.45}(1) = .4$$

(B) is known to be true.

$$\rho_x(k) = \frac{(\varphi_1 + \theta_1)(1 + \varphi_1\theta_1)}{(1 + 2\varphi_1\theta_1 + \theta_2^2)} \varphi_1^{k-1} = \rho_x(1)\varphi_1^{k-1}$$

$\rho_x(k)$ multiple of $\rho_x(1)$ & φ_1^{k-1}

since we have the knowledge of $\varphi_1 = .8$ only iff $k \geq 2$ then φ_1^{k-1} will be smaller than 1.

Therefore $\rho_x(k) < \rho_x(1)$ for $k \geq 2$

(C) known to be true. ARMA(1) as:

$X_t = \varphi_1 X_{t-1} = Z_t + \theta_1 Z_{t-1}$ $\quad Z_t \sim WN(0,1)$

can be later rewritten as $\Downarrow$

$X_t - .8X_{t-1} = 2 + Z_t - .5Z_{t-1}$

(D) Is known to be true.

ARMA(p,q) be stationary iff. $\varphi(z)$ outside unit circle

$\underbrace{X_t - .8X_{t-1}}_{(1 - .8B)X_t} = 2 + Z_t - .5Z_{t-1}$

$\varphi(z) = 1 - .8z = 0$

$\frac{1}{.8} = \frac{.8z}{.8}$ $\quad z = \frac{1}{.8} \rightarrow 1.25$

$|1.25| > 1$ so stationary.

(e) is known to be false

$E[X_t] = E[.8X_{t-1} + 2 + Z_t - .5Z_{t-1}]$

$= .8E[X_{t-1}] + E[2] + E[Z_t] - .5E[Z_{t-1}]$

Knowing $E[X_t] = E[X_{t-1}] = \mu_x$

$\mu_x = .8\mu_x + 2 + 0 + .5(0)$

$\mu_x - .8\mu_x = 2$

$.2\mu_x = 2$

$\mu_x = 10$

E is false $\mu_x \neq 2$

6. parameter redundancy pertain to AR & MA characteristic polynomial $\phi(z)$ $\theta(z)$

I) $X_t = \frac{1}{2}X_{t-1} + Z_t - \frac{1}{2}Z_{t-1}$

$X_t - \frac{1}{2}BX_t = Z_t - \frac{1}{2}BZ_t$

$\Rightarrow X_t(1-\frac{1}{2}B) = Z_t(1-\frac{1}{2}B)$

$X_t = Z_t$

* parameters are redundant

II) $X_t = \frac{1}{2}X_{t-1} + Z_t - \frac{1}{9}Z_{t-2}$

$X_t - \frac{1}{2}X_{t-1} = Z_t - \frac{1}{9}Z_{t-1}$

$X_t - \frac{1}{2}BX_t = Z_t - \frac{1}{9}BZ_t$

$X_t(1-\frac{1}{2}B) = Z_t(1-\frac{1}{9}B)$

* parameters are not redundant

III) $X_t = -\frac{5}{6}X_{t-1} - \frac{1}{6}X_{t-2} + Z_t + \frac{8}{12}Z_{t-1} + \frac{1}{12}Z_{t-2}$

$X_t + \frac{5}{6}X_{t-1} + \frac{1}{6}X_{t-2} = Z_t + \frac{8}{12}Z_{t-1} + \frac{1}{12}Z_{t-2}$

$X_t + \frac{5}{6}BX_t + \frac{1}{6}B^2X_t = Z_t + \frac{8}{12}BZ_t + \frac{1}{12}B^2Z_t$

$X_t(1+\frac{5}{6}B+\frac{1}{6}B^2) = Z_t(1+\frac{8}{12}B+\frac{1}{12}B^2)$

$X_t(1+B/3)(1+B/2) = Z_t(1+B/6)(1+B/2)$

$X_t(1+B/3) = Z_t(1+B/6)$

* Parameters are redundant