Deliverable - Updated Objective and Constraints

Group 17
F. Feng, S. Xiong, Z. Fan, E. Qu

February 2025

1 Simplify Approach Description

To streamline the mathematical formulation while preserving its core functionality, several simplifications were applied. The original formulation included highly detailed constraints for different types of classification operations and routing decisions. These were condensed into generalized constraints that maintain operational feasibility without complexity.

The key simplifications include: First, we reduce redundant constraints that specify individual yard conditions separately. Instead, we now use a unified set of constraints that apply to all classification yards. Second, we merge classification yard constraints by distinguishing only between normal and shifted classifications rather than considering multiple levels of routing restrictions. Third, we eliminate unnecessary integer constraints where binary indicators were sufficient to enforce logical conditions. Fourth, we add penalty terms for shifted cars instead of modelling complex yard congestion effects explicitly, simplifying computations while still discouraging inefficient routing.
These changes make the model more computationally efficient while still capturing the essential trade-offs involved in freight train service planning.

2 Updated Objective Function

We aim to minimize the total operational cost of freight train services while ensuring routing and yard management. There exist three cost components: train operation cost, which minimizes unnecessary train services; distance-driven shipment cost, which encourages shipments to follow the shortest possible routes; and yard handling cost, which ensures that classification yards are not overloaded. Additionally, we introduce a penalty for shifted service cars to discourage excessive use of inefficient routing.

$$\min \sum_i \sum_j c_{ij} y_{ij} + \sum_s \sum_i \sum_j d_{ij} x_{ij}^s + \sum_j C_{\text{yard},j} f_{ij} + \sum_k P_{\text{shifted},k} S_k \tag{1}$$

where:

- c_{ij} is the train operation cost between yard i and yard j.
- y_{ij} is a binary variable: 1 if a train service operates between yard i and j, otherwise 0.
- d_{ij} represents the distance-driven shipment cost between yard i and yard j.
- x_{ij}^s is a binary variable: 1 if shipment s is assigned to a train service from yard i to yard j, otherwise 0.
- $C_{\text{yard},j}$ represents the yard handling cost at yard j.
- f_{ij} is the number of cars flowing from yard i to yard j.
- $P_{\text{shifted},k}$ is a penalty for shifted cars at yard k.
- S_k represents the number of shifted service cars in double-hump yard k.

3 Updated Constraints

3.1 Flow Conservation

Each shipment follows exactly one train service route from its start to destination:

$$\sum_j x_{ij}^s = 1, \quad \forall s \in S, i, j \in N \tag{2}$$

At any intermediate yard j, the number of incoming cars equals the number of outgoing cars:

$$\sum_j x_{ij}^s = \sum_k x_{jk}^s, \quad \forall j, s \tag{3}$$

3.2 Train Capacity

Shipments are only assigned to existing train services:

$$x_{ij}^s \leq y_{ij}, \quad \forall i, j, s \tag{4}$$

The total number of cars assigned to a train does not exceed its maximum capacity:

$$\sum_s x_{ij}^s n_s \leq L_{\text{Max}}, \quad \forall i, j \tag{5}$$

3.3 Yard Capacity with Shifted and Normal Classification

Each classification yard has a maximum capacity, distinguishing between normal and shifted classification operations:

$$\sum_{j} f_{ij} \leq C_{\text{Yard},i}, \quad \forall i \tag{6}$$

$$\sum_{j} x_{ij}^{s} \leq C_{\text{Sort},i}, \quad \forall i \tag{7}$$

For double-hump yards k, we define separate classification limits:

$$\sum_{j} x_{ij}^{s} h_{ijk}^{0} \leq C_{\text{Normal},k}, \quad \forall k \tag{8}$$

$$\sum_{j} x_{ij}^{s} h_{ijk}^{1} \leq C_{\text{Shifted},k}, \quad \forall k \tag{9}$$

where:

- h_{ijk}^{0} is a binary variable: 1 if shipment s is classified normally at yard k, otherwise 0.
- h_{ijk}^{1} is a binary variable: 1 if shipment s is shifted in a double-hump yard k, otherwise 0.

The number of shifted cars at yard k is:

$$S_k = \sum_{i} \sum_{j} \sum_{s} x_{ij}^{s} h_{ijk}^{1}, \quad \forall k \tag{10}$$

3.4 Routing Constraints with Shifted Cars

To ensure that a shipment is classified only once per yard:

$$h_{ijk}^{0} + h_{ijk}^{1} \leq 1, \quad \forall i, j, k \tag{11}$$

To ensure shifted cars only exist if the corresponding train service exists:

$$h_{ijk}^{1} \leq y_{ij}, \quad \forall i, j, k \tag{12}$$