Sha-Categories & Quasi-isomorphisms

Thomas Wilskow Thorbjørnsen

November 17, 2021

Abstract

Fill inn abstract

Sammendrag

Fyll inn sammendraget

Acknowledgements

Thank the people in your life who has made this journey easier :D

Contents

Chapter 1

Bar and Cobar Construction

[../../thesis.tex]subfiles

1.1 Algebras, Coalgebras and Twisting Morphisms

In this section we will look at a result of associative algebras over a field $\mathbb{K}$. Given a coassociative conilpotent coalgebra C and an associative algebra A, we say that a linear transformation $\alpha : C \to A$ is twisting if it satisfies the Maurar-Cartan equation:

$$\partial\alpha + \alpha \star \alpha = 0.$$

Let $Tw(C, A)$ be the set of twisting morphisms, then considering it as a functor $Tw : Coalg^{op} \times Alg \to Ab$ we want to show that it is represented in both arguments. Moreover, this representation give rise to an adjoint pair of functors, called the Bar and Cobar construction.

$$\begin{tikzcd} Alg \arrow[r, bend left, "B"] \arrow[r, phantom, "\top"] & \substack{Conil\\Coalg} \arrow[l, bend left, "\Omega"] \end{tikzcd}$$

To obtain this result we need to define a twisting morphism. Thus this section will define algebras, coalgebras and convolution algebras before we state the result of the Bar and Cobar construction.

1.1.1 Algebras

Let $\mathbb{K}$ be a field. A unital associative algebra is a $\mathbb{K}$-module with structure morphisms called multiplication and unit:

$$(\cdot_A) : A \otimes_{\mathbb{K}} A \to A$$
$$1_A : \mathbb{K} \to A$$

These structure morphisms should satisfy associativiy and identity laws, i.e. $(A, (\cdot_A), 1_A)$ is a monoid.

$$\begin{aligned} (associativity) \quad & (a \cdot_A b) \cdot_A c = a \cdot_A (b \cdot_A c) \\ (unitality) \quad & 1_A(1) \cdot_A a = a = a \cdot_A 1_A(1) \end{aligned}$$

Alternatively, we may represent the axioms with commutative diagrams:

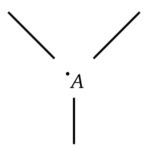
(associativity)

$$\begin{tikzcd} A \otimes_{\mathbb{K}} A \otimes_{\mathbb{K}} A \arrow[r, "(\cdot_A)\otimes id_{\mathbb{K}}"] \arrow[d, "id_{\mathbb{K}}\otimes(\cdot_A)"] & A \otimes_{\mathbb{K}} A \arrow[d, "(\cdot_A)"] \\ A \otimes_{\mathbb{K}} A \arrow[r, "(\cdot_A)"] & A \end{tikzcd}$$

(unitality)

$$\begin{tikzcd} A \otimes_{\mathbb{K}} \mathbb{K} \arrow[r, "id_A\otimes 1_A"] \arrow[dr, "\simeq"'] & A \otimes_{\mathbb{K}} A \arrow[d, "(\cdot_A)"] & \mathbb{K} \otimes_{\mathbb{K}} A \arrow[l, "1_A\otimes id_A"'] \arrow[dl, "\simeq"] \\ & A & \end{tikzcd}$$

Another approach to write down the axioms of an alegbra is by the use of electric circuits.



1.1.2 Coalgebras

1.1.3 Derivations, Coderivations and Convolution Algebras

1.1.4 Twisting Morphisms

1.2 Strongly Homotopy Associative Algebras, Coalgebras and Twisting Morphisms

1.2.1 Sha Algebras

1.2.2 Sha Coalgebras

1.2.3 Twisting Sha Morphisms