Q1

i) Espérance

X	1	2	3	4	5
$P(X=x)$	$1/5$	$1/5$	$1/5$	$1/5$	$1/5$

$$E(x) = \sum x \cdot P(X=x)$$
$$= 1 \cdot 1/5 + 2 \cdot 1/5 + 3 \cdot 1/5 + 4 \cdot 1/5 + 5 \cdot 1/5$$
$$= 3$$

ii) Variance

$$Var(X) = E(X^2) - [E(x)]^2$$
$$= (1^2 \cdot 1/5 + 2^2 \; 1/5 + 3^2 \cdot 1/5 + 4^2 \cdot 1/5 + 5^2 \cdot 1/5) - 3^2$$
$$= 11 - 3^2$$
$$= 2$$

iii) Écart-type

$$\sigma = \sqrt{var}$$
$$= \sqrt{2}$$

Q2

i) Norme euclidienne de u

$$\|\vec{u}\| = \sqrt{u_1^2 + \dots + u_d^2} \qquad \|\vec{u}\| = \sqrt{\sum_{i=1}^{d} u_i^2}$$

ii) Produit scalaire euclidien $u \cdot v$

$$\vec{u} \cdot \vec{v}^T = (u_1 \ \cdots \ u_d) \begin{pmatrix} v_1 \\ \vdots \\ v_d \end{pmatrix}$$

$$= u_1 v_1 + \dots + u_d v_d$$

$$= \sum_{i=1}^{d} u_i v_i$$

iii) Produit matrice - vecteur Au

$$A^{n \times d} = \begin{pmatrix} a_{11} & \cdots & a_{1d} \\ a_{21} & \cdots & a_{2d} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nd} \end{pmatrix} \qquad u^{1 \times d} = (u_1 \ \cdots \ u_d)$$

$$\underbrace{n \times d \cdot 1 \times d}_{\times}$$

Transposé

$$\hookrightarrow \underbrace{n \times d \cdot d \times 1}_{\checkmark}$$

$$A u^T = \begin{pmatrix} a_{11} & \cdots & a_{1d} \\ a_{21} & \cdots & a_{2d} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nd} \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_d \end{pmatrix}$$

$$= \begin{matrix} a_{11} \cdot u_1 + a_{12} u_2 + \dots + a_{1d} u_d \\ a_{21} u_2 + \cdots + \dots + a_{2d} u_d \\ \vdots \quad \vdots + \cdots + \dots + \vdots \quad \vdots \\ a_{n1} u_n + \cdots + \dots + a_{nd} u_d \end{matrix}$$

$$= \left(\sum_{j=1}^{d} a_{ij} u_j \right)_{i=1}^{n}$$

Q3

ALGO2(n) est plus rapide puisqu'elle fait un calcul direct tandis que ALGO1(n) doit itérer à travers n entrées

ALGO1(n)

```
result = 0
for i = 1...n
  result = result + i
return result
```

ALGO2(n)

```
return (n+1) * n/2
```

somme → $O(n)$

somme → $O(1)$

Q4

i) $\frac{df}{dx} = ?,\quad \text{where } f(x,\beta) = x^2 \exp(-\underset{\substack{\uparrow \\ \text{cst}}}{\beta} x)$

$u' \cdot v + u\ v'$

$u = x^2 \quad u' = 2x$

$v = e^{-\beta x} \quad v' = -\beta e^{-\beta x}$

$$\frac{df}{dx} = 2x\ \exp(-\beta x) - x^2 \beta \exp(-\beta x)$$

$$= \exp(-\beta x)(2x - \beta x^2)$$

ii) $\frac{df}{d\beta} = ?,\quad \text{where } f(x,\beta) = \underset{\substack{\uparrow \\ \text{cst}}}{x} \exp(-\beta x)$

$$\frac{df}{d\beta} = x \ \frac{d}{d\beta} \exp(-\beta x)$$

$$= x \cdot (-x \exp(-\beta x))$$

$$= -x^2 \exp(-\beta x)$$

iii) $\frac{df}{dx} = ?,\quad \text{where } f(x) = \sin(\exp(x^2))$

$g(x) = \exp(x^2)$

$h(x) = \sin(g(x))$

$$\frac{df}{dx} = \frac{d}{dx}(\sin(g(x)))$$

$$= \cos(g(x)) \cdot \frac{d}{dx}(g(x))$$

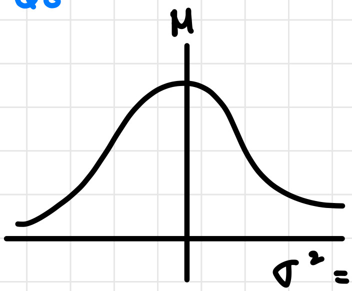
$$\frac{d}{dx}\exp(x^2) = \exp(x^2)\ \frac{d}{dx} x^2$$

$$= \exp(x^2) \cdot 2x$$

$$\frac{df}{dx} = \cos(\exp(x^2)) \cdot \exp(x^2) \cdot 2x$$

$$= 2x \cos(\exp(x^2))(\exp(x^2))$$

Q5



$\sigma^2 = 1$

$$Var(X) = \underbrace{E(X^2)}_{?} - \underbrace{[E(X)]^2}_{\mu^2}$$

$$\begin{aligned} E(X^2) &= Var(X) + \mu^2 \\ &= \sigma^2 + \mu^2 \\ &= 1 + \mu^2 \end{aligned}$$