Statistics 452: Statistical Learning and Prediction

Chapter 7, Part 1: Simple Extensions of the Linear Model

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Polynomial Regression

- We have used polynomial regression before.
- This was the standard way to extend the linear model.
- The text recommends against a degree of more than 4.
- To avoid collinearity we can use the covariates returned by the `poly()` function (more on these functions in the next set of notes).

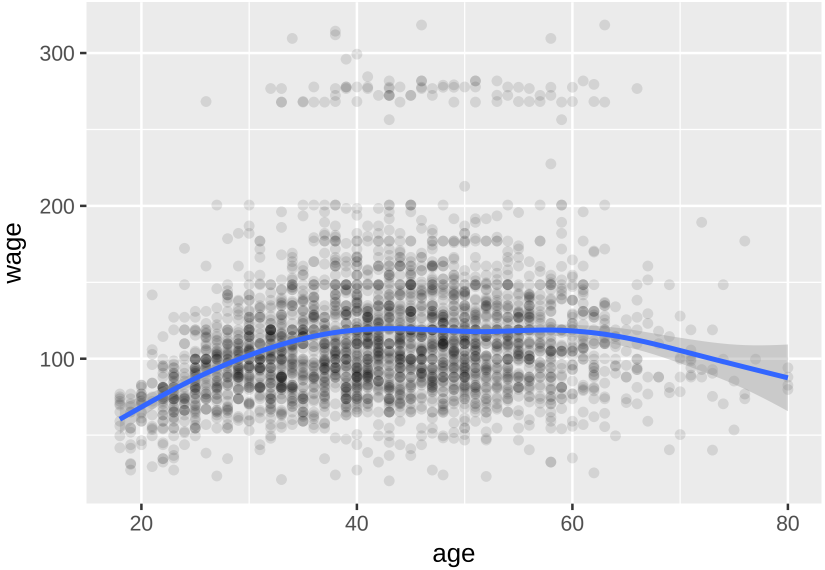
Example: Wage Data

- Predict wages as a function of age.

```r
library(ISLR)
data(Wage)
head(Wage,n=3)
```

```
##        year age            maritl     race       education
## 231655 2006  18 1. Never Married 1. White    1. < HS Grad
## 86582  2004  24 1. Never Married 1. White 4. College Grad
## 161300 2003  45        2. Married 1. White 3. Some College
##                    region       jobclass         health health_ins
## 231655 2. Middle Atlantic  1. Industrial      1. <=Good      2. No
## 86582  2. Middle Atlantic 2. Information 2. >=Very Good      2. No
## 161300 2. Middle Atlantic  1. Industrial      1. <=Good     1. Yes
##         logwage      wage
## 231655 4.318063  75.04315
## 86582  4.255273  70.47602
## 161300 4.875061 130.98218
```

```r
library(ggplot2)
ggplot(Wage,aes(x=age,y=wage)) + geom_point(alpha=0.1) +
  geom_smooth(formula=formula(wage ~ poly(age,4)))
```



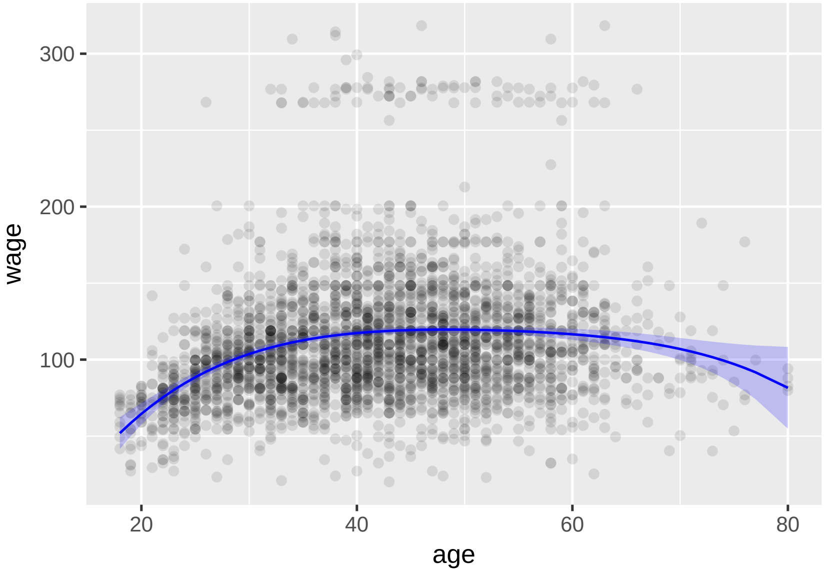
DIY Confidence Bands

- Use `predict()` to get the predictions and plot CI bands with `geom_ribbon()`
- First the predictions and point-wise CIs.

```
wfit <- lm(wage ~ poly(age,4),data=Wage)
newWage <- data.frame(age = sort(unique(Wage$age)))
wpred <- data.frame(newWage,predict(wfit,newdata=newWage,interval="confidence"))
head(wpred,n=3)
```

```
##   age      fit      lwr      upr
## 1  18 51.93145 41.54284 62.32006
## 2  19 58.49674 49.92674 67.06674
## 3  20 64.57188 57.52864 71.61511
```

```r
ggplot(Wage,aes(x=age,y=wage)) + geom_point(alpha=0.1) +
  geom_ribbon(aes(x=age,y=fit,ymin=lwr,ymax=upr),
              data=wpred,fill="blue",alpha=.2) +
  geom_line(aes(y=fit),data=wpred,color="blue")
```

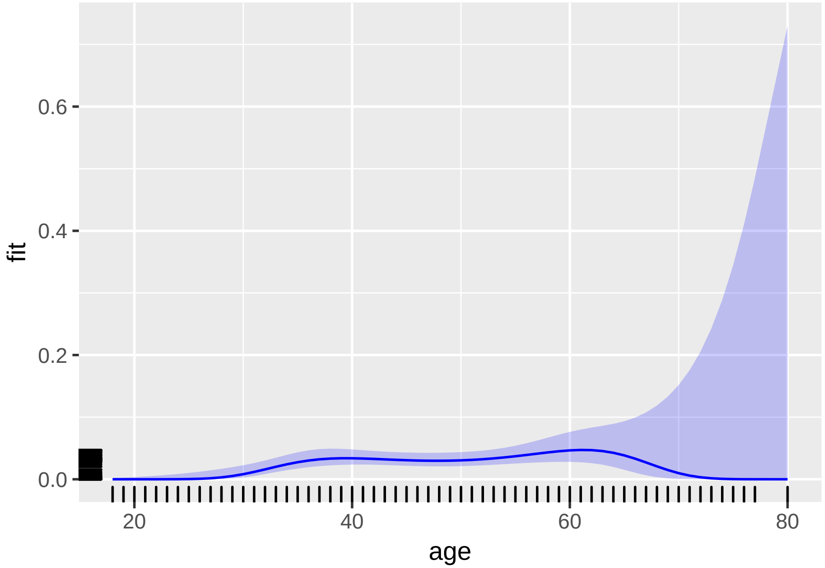


Classification Example: Wage > 250K

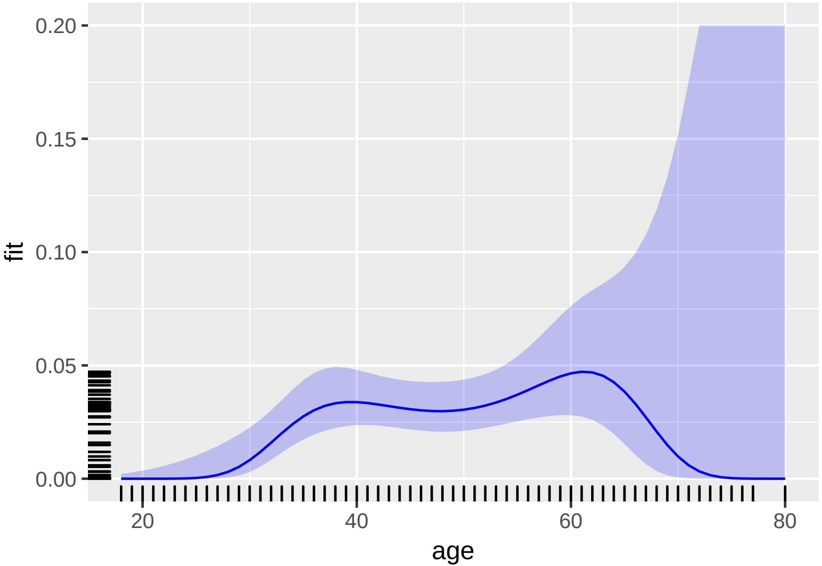
```
library(dplyr)
Wage <- mutate(Wage,wage250 = as.numeric(wage>250))
wfit <- glm(wage250 ~ poly(age,4),data=Wage,family=binomial())
wpred <- data.frame(newWage,
          predict(wfit,newdata=newWage,se.fit=TRUE)) # predicted logits
expit <- function(x) { exp(x)/(1+exp(x)) }
wpred <- mutate(wpred,
                lwr=expit(fit-2*se.fit),upr=expit(fit+2*se.fit),
                fit = expit(fit))
# Don't go below 0 or above 1
head(wpred,n=3)
```

```
##   age          fit   se.fit residual.scale          lwr         upr
## 1  18 9.826427e-09 6.140231              1 4.560982e-14 0.002112586
## 2  19 7.577844e-08 5.254561              1 2.067720e-12 0.002769461
## 3  20 4.746699e-07 4.464911              1 6.283757e-11 0.003572811
```

```r
ggplot(wpred,aes(x=age,y=fit)) + geom_rug() +
  geom_ribbon(aes(ymin=lwr,ymax=upr),fill="blue",alpha=.2,limits=c(0,.2)) +
  geom_line(aes(y=fit),data=wpred,color="blue")
```



```
wpred <- mutate(wpred,upr = pmin(upr,0.2))
ggplot(wpred,aes(x=age,y=fit)) + geom_rug() +
  geom_ribbon(aes(ymin=lwr,ymax=upr),fill="blue",alpha=.2,limits=c(0,.2)) +
  geom_line(aes(y=fit),data=wpred,color="blue")
```



Step Functions

- In Epidemiology it is common to discretize age and treat as a categorical variable.
  - Categorical variables are coded as dummy variables for regression.
- The regression will fit a separate mean for each category, which is more flexible than, say, linear in age.

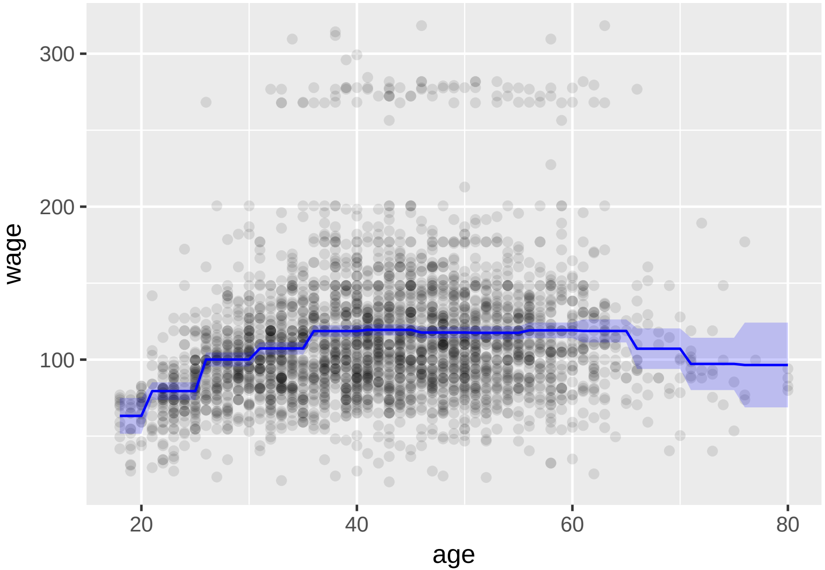
```
with(Wage,range(age))
```

```
## [1] 18 80
```

```
agebreaks <- c(15,20,25,30,35,40,45,50,55,60,65,70,75,80)
wfit <- lm(wage ~ cut(age,breaks=agebreaks),data=Wage)
newWage <- data.frame(age = sort(unique(Wage$age)))
wpred <- data.frame(newWage,predict(wfit,newdata=newWage,interval="confidence"))
head(wpred,n=3)
```

```
##   age     fit      lwr      upr
## 1  18 63.2435 51.55813 74.92886
## 2  19 63.2435 51.55813 74.92886
## 3  20 63.2435 51.55813 74.92886
```

```r
ggplot(Wage,aes(x=age,y=wage)) + geom_point(alpha=0.1) +
  geom_ribbon(aes(x=age,y=fit,ymin=lwr,ymax=upr),
              data=wpred,fill="blue",alpha=.2) +
  geom_line(aes(y=fit),data=wpred,color="blue")
```



Classification Example: Wage > 250K

```r
library(dplyr)
wfit <- glm(wage250 ~ cut(age,agebreaks),data=Wage,family=binomial())
wpred <- data.frame(newWage,
             predict(wfit,newdata=newWage,se.fit=TRUE)) # predicted logits
expit <- function(x) { exp(x)/(1+exp(x)) }
wpred <- mutate(wpred,
                lwr=expit(fit-2*se.fit),upr=expit(fit+2*se.fit),
                fit = expit(fit))
# Don't go below 0 or above 1
head(wpred,n=3)
```

```
##   age          fit   se.fit residual.scale lwr upr
## 1  18 3.181005e-09 1603.114              1   0 NaN
## 2  19 3.181005e-09 1603.114              1   0 NaN
## 3  20 3.181005e-09 1603.114              1   0 NaN
```

```r
ggplot(wpred,aes(x=age,y=fit)) + geom_rug() +
  geom_ribbon(aes(ymin=lwr,ymax=upr),fill="blue",alpha=.2,limits=c(0,.2)) +
  geom_line(aes(y=fit),data=wpred,color="blue")
```