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A Simple Method to Obtain Consistent and Clinically Meaningful Pelvic Angles From Euler Angles During Gait Analysis

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Clinical gait analysis usually describes joint kinematics using Euler angles, which depend on the sequence of rotation. Studies have shown that pelvic obliquity angles from the traditional tilt-obliquity-rotation (TOR) Euler angle sequence can deviate considerably from clinical expectations and have suggested that a rotation-obliquity-tilt (ROT) Euler angle sequence be used instead. We propose a simple alternate approach in which clinical joint angles are defined and exactly calculated in terms of Euler angles from any rotation sequence. Equations were derived to calculate clinical pelvic elevation, progression, and lean angles from TOR and ROT Euler angles. For the ROT Euler angles, obliquity was exactly the same as the clinical elevation angle, rotation was similar to the clinical progression angle, and tilt was similar to the clinical lean angle. Greater differences were observed for TOR. These results support previous findings that ROT is preferable to TOR for calculating pelvic Euler angles for clinical interpretation. However, we suggest that exact clinical angles can and should be obtained through a few extra calculations as demonstrated in this technical note.

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The model most commonly used in clinical gait analysis describes orientation using Euler angles, which are also referred to as *Cardan* or *Bryant* and are equivalent to Grood and Suntay (1983) angles (Davis et al., 1991; Kadaba et al., 1990). Euler angles are mathematically useful because the orientation of a rigid body (e.g., the pelvis) in a reference frame (e.g., the gait laboratory) is uniquely determined by three angles and three associated axes around which a sequence of rotations is performed. In clinical applications, Euler angles are generally used instead of more computationally robust methods such as helical/screw angles (Woltring, 1994) or quaternions (Haug, 1989) because they are considered to be more conducive to clinical interpretation. However, Euler angles are not necessarily physically significant or clinically meaningful. In addition, it is well known that the order of rotation is critically important in determining the Euler angles reported (Cheng, 2000; Dumas et al., 2004).

Previous studies have shown that the sequence of rotations most frequently used in clinical gait analysis, tilt-obliquity-rotation (TOR), can yield pelvic obliquity angles that deviate considerably from clinical expectations (Baker, 2001; Foti et al., 2001). Similar problems occur with another sequence, rotation-tilt-obliquity (RTO), which is also used in clinical gait analysis (Baker, 2001).

A suggested solution has been to use the rotation-obliquity-tilt (ROT) sequence instead because this sequence gives pelvic obliquity angles in better agreement with clinical expectations (Baker, 2001; Foti et al., 2001). However, a limitation of this approach is that angles of clinical interest may not exactly correspond with any Euler angles.

This article proposes a simple alternative approach in which clinical joint angles are defined and then calculated in terms of the Euler angles. The advantages of this approach are that it produces the exact angles of clinical interest regardless of the underlying Euler rotation sequence and it is easy to implement in current gait analysis systems. The current study applies this approach to the calculation of pelvic angles. Our specific aims were (1) to precisely define clinically relevant pelvic angles, (2) to derive equations for accurately calculating these clinical angles from Euler angles using TOR and ROT, and (3) to quantify the differences between the clinical and Euler angles.

Methods

In the conventional gait model, the pelvis (rigid body B) is defined by the pelvic markers R_{ASIS} (right anterior superior iliac spine), L_{ASIS} (left anterior superior iliac spine), and SAC (sacral) (Figure 1) (Davis et al., 1991; Kadaba et al., 1990). To measure the orientation of B in the gait laboratory (A), right-handed mutually perpendicular unit vectors $\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$ and $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$ are fixed in A and B, respectively,

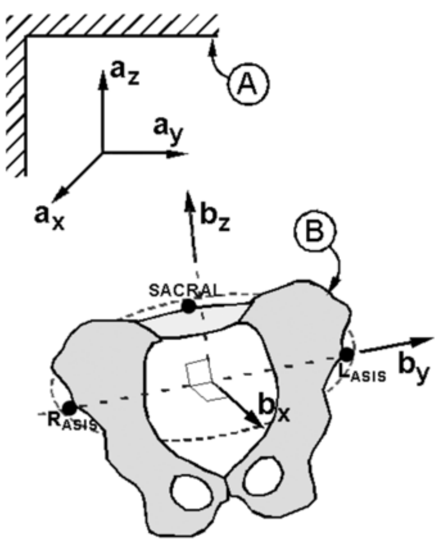

Figure 1 — Coordinate systems for the room and pelvis.

following a commonly used convention (Tebbutt et al., 2002) with

- $\mathbf{a}_x$ pointing forward along the walkway
- $\mathbf{a}_z$ pointing vertically upward
- $\mathbf{a}_y$ pointing to the subject's left, that is, $\mathbf{a}_y = \mathbf{a}_z \times \mathbf{a}_x$
- $\mathbf{b}_y$ directed from R_{ASIS} to L_{ASIS}
- $\mathbf{b}_z$ perpendicular to the plane containing R_{ASIS}, L_{ASIS}, and SAC
- $\mathbf{b}_x$ pointing from posterior to anterior, that is, $\mathbf{b}_x = \mathbf{b}_y \times \mathbf{b}_z$

Three angles used to describe the orientation of B in A in the conventional gait model are tilt, obliquity, and rotation. The clinical and mathematical definitions of these angles are identical when the orientation of B in A can be described by a *simple rotation* around either $\mathbf{b}_x$ or $\mathbf{b}_y$ or $\mathbf{b}_z$. For example, in a simple rotation around $\mathbf{b}_x$, the patient elevates the left hip higher than the right hip by an "obliquity" angle denoted θ_o. In a simple rotation around $\mathbf{b}_y$, the patient leans forward by a "tilt" angle denoted θ_t. In a simple rotation around $\mathbf{b}_z$, the patient rotates to the left by a "rotation" angle denoted θ_r.

Alternately, when there are *successive rotations* about two or more of $\mathbf{b}_x$ and $\mathbf{b}_y$ and $\mathbf{b}_z$, the clinical and mathematical meanings of the various angles may differ. Mathematically, the orientation of B in A is described by Euler angles and depends on the order of the successive rotations. Clinically, angles are measured between physically identifiable lines (e.g., the angle between the local vertical and the line joining R_{ASIS} to L_{ASIS}), and the clinical meanings of obliquity, tilt, and rotation are associated with, but not necessarily the same as, θ_o, θ_t, and θ_r.

One way to *mathematically* describe the orientation of B in A is to align $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$ with $\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$ and then subject B to right-handed successive rotations characterized by $\theta_t\,\mathbf{b}_y$, $\theta_o\,\mathbf{b}_x$, and $\theta_r\,\mathbf{b}_z$ in the TOR rotation sequence commonly used in the conventional gait model. Alternately, B can be subjected to right-handed successive rotations characterized by $\theta_r\,\mathbf{b}_z$, $\theta_o\,\mathbf{b}_x$, and $\theta_t\,\mathbf{b}_y$ in the ROT rotation sequence. Direction cosine matrices were calculated to describe the mathematical relationships between $\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$ and $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$ for TOR and ROT.

Additional angles were defined to describe parameters of clinical importance. These angles are similar to those described by Baker (2001). A pelvis elevation angle ϕ was defined as the angle between $\mathbf{b}_y$ and the horizontal plane in the room (the plane perpendicular to $\mathbf{a}_z$). This angle answers the question "how much is L_{ASIS} elevated above R_{ASIS}" and is regarded as positive when L_{ASIS} is higher than R_{ASIS} and negative when L_{ASIS} is lower than R_{ASIS}. The pelvis elevation angle is most closely related to obliquity as $\phi = \theta_o$ when $\theta_t = \theta_r = 0°$.

A pelvic progression angle ψ was defined as the angle between $\mathbf{b}_y$ and the vertical plane perpendicular to $\mathbf{a}_x$. This angle answers the question "how far forward is R_{ASIS} relative to L_{ASIS}" and is regarded as positive when R_{ASIS} is forward of L_{ASIS} and negative when R_{ASIS} is behind L_{ASIS}. The pelvis progression angle is most closely related to rotation as $\psi = \theta_r$ when $\theta_o = \theta_t = 0°$.

Finally, a pelvis lean angle γ was defined as the angle between $\mathbf{b}_x$ and the horizontal plane in the room. This angle is regarded as positive when $\mathbf{b}_x$ points below the horizontal (in the $-\mathbf{a}_z$ direction) and negative when $\mathbf{b}_x$ points above the horizontal (in the $+\mathbf{a}_z$ direction). The pelvis lean angle is most closely related to tilt as $\gamma = \theta_t$ when $\theta_o = \theta_r = 0°$.

Equations were derived to express the clinical angles ϕ, ψ, and γ in terms of the Euler angles θ_r, θ_t, and θ_o for TOR and ROT. This was accomplished using the TOR and ROT rotation matrices, which relate $\mathbf{b}_x$, $\mathbf{b}_y$, $\mathbf{b}_z$ to $\mathbf{a}_x$, $\mathbf{a}_y$, $\mathbf{a}_z$. The element in the $\mathbf{b}_j$ row and $\mathbf{a}_i$ column of the rotation matrix is, by definition, the cosine of the angle between $\mathbf{b}_j$ and $\mathbf{a}_i$. Since each clinical angle is clearly defined in terms of the angle between two vectors (e.g., $\mathbf{b}_y$ and $\mathbf{a}_z$ for the pelvis elevation angle), the appropriate element of the rotation matrix can be used directly to determine the clinical angle.

The differences between θ_o, θ_r, θ_t and ϕ, ψ, γ were expressed as obliquity, rotation, and tilt difference terms $\varepsilon_o = \theta_o - \phi$, $\varepsilon_r = \theta_r - \psi$, and $\varepsilon_t = \theta_t - \gamma$, respectively. These differences are graphed as a function of θ_r, θ_t, and θ_o for TOR and ROT. A case example is also presented for a patient who walked with substantial pelvic obliquity and rotation.

Results

The equations for calculating the clinical angles from the Euler angles are as follows.

Pelvis elevation angle ϕ

$$\text{TOR successive rotations: } \phi = 90° - \operatorname{acos}(\sin\theta_r \sin\theta_t + \sin\theta_o \cos\theta_r \cos\theta_t)$$

$$\text{ROT successive rotations: } \phi = 90° - \operatorname{acos}(\sin\theta_o)$$

$$= \theta_o \text{ when } -90° \le \theta_o \le 90°$$

Pelvis progression angle ψ

$$\text{TOR successive rotations: } \psi = 90° - \operatorname{acos}(\sin\theta_r \cos\theta_t - \sin\theta_o \sin\theta_t \cos\theta_r)$$

$$\text{ROT successive rotations: } \psi = 90° - \operatorname{acos}(\sin\theta_r \cos\theta_o)$$

Pelvis lean angle γ

$$\text{TOR successive rotations: } \gamma = 90° - \operatorname{acos}(\sin\theta_t \cos\theta_r - \sin\theta_o \sin\theta_r \cos\theta_t)$$

$$\text{ROT successive rotations: } \gamma = 90° - \operatorname{acos}(\sin\theta_t \cos\theta_o)$$

Obliquity from the ROT sequence is exactly the same as the pelvis elevation angle ϕ for the range of angles that occur clinically. For all other angles, the clinical and Euler angles differ when the Euler angles are large (Figure 2). The differences for TOR are generally greater than the differences for ROT.

Results from the case example illustrate these differences for a patient who walked with substantial pelvic obliquity and rotation (Figure 3). The angles from TOR differ greatly from the clinical angles (maximum errors of 8.1°, 6.1°, and 9.4° for obliquity, rotation, and tilt, respectively). The angles from ROT are close to the clinical angles, but still differ for rotation and tilt (maximum errors of 1.3° and 1.2°, respectively).

Discussion

This article demonstrates that angles of clinical interest can be calculated exactly and easily from the Euler angles typically used by current gait analysis systems. By incorporating a few extra calculations in gait analysis software, we can obtain clinically meaningful angles that do not depend on the underlying sequence of Euler

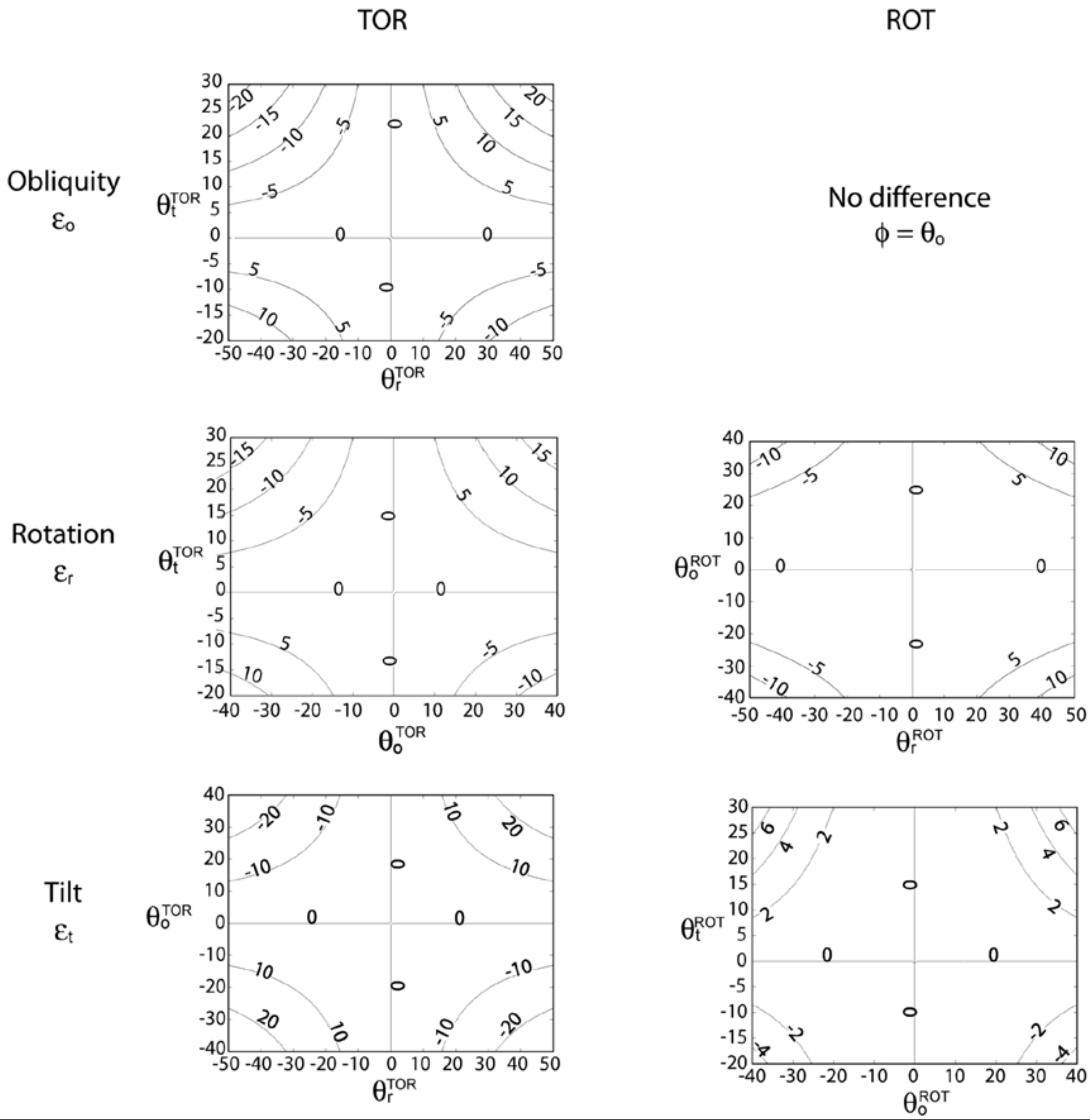

Figure 2 — Contour plots showing differences between the Euler and clinical angles in degrees for (a) obliquity/elevation, (b) rotation/progression, and (c) tilt/lean. Differences in TOR obliquity, rotation, and tilt are shown for $\theta_o = 0°$, $\theta_r = 0°$, and $\theta_t = 0°$, respectively. Differences increase for larger angles. Differences in ROT rotation and tilt are independent of θ_t and θ_r, respectively.

rotations. Previous studies have recommended the use of Euler angles from the rotation sequence that best approximates angles of clinical interest (Baker, 2001). Our results confirm the previous finding that ROT is a more appropriate sequence than TOR for calculating clinical pelvic angles (Baker, 2001; Foti et al., 2001). The difference between the ROT Euler angles and clinical pelvic angles is probably not clinically significant. However, we propose that it is preferable to calculate clinical angles exactly given the ease with which this can be accomplished.

Other computational methods exist that also avoid the sequence dependence of Euler angles (Cheng, 2004; Dumas et al., 2004; Haug, 1989; Kinzel et al., 1972; Kuipers, 2002; Woltring, 1994). Some of these methods, which include quaternions and helical/screw angles, also eliminate the problem of gimbal lock, which can occur when certain Euler angles are multiples of 90° (Kuipers, 2002; Woltring, 1994). Despite the computational advantages of these other approaches, clinical gait analysis has relied on Euler angles because they usually correspond with angles of clinical interest. The approach proposed in this study builds on this established foundation of Euler angles in gait analysis and would therefore be easy to implement in clinical gait laboratories.

The clinical angles defined in this technical note are essentially projections of anatomical vectors onto convenient reference planes. These angles answer

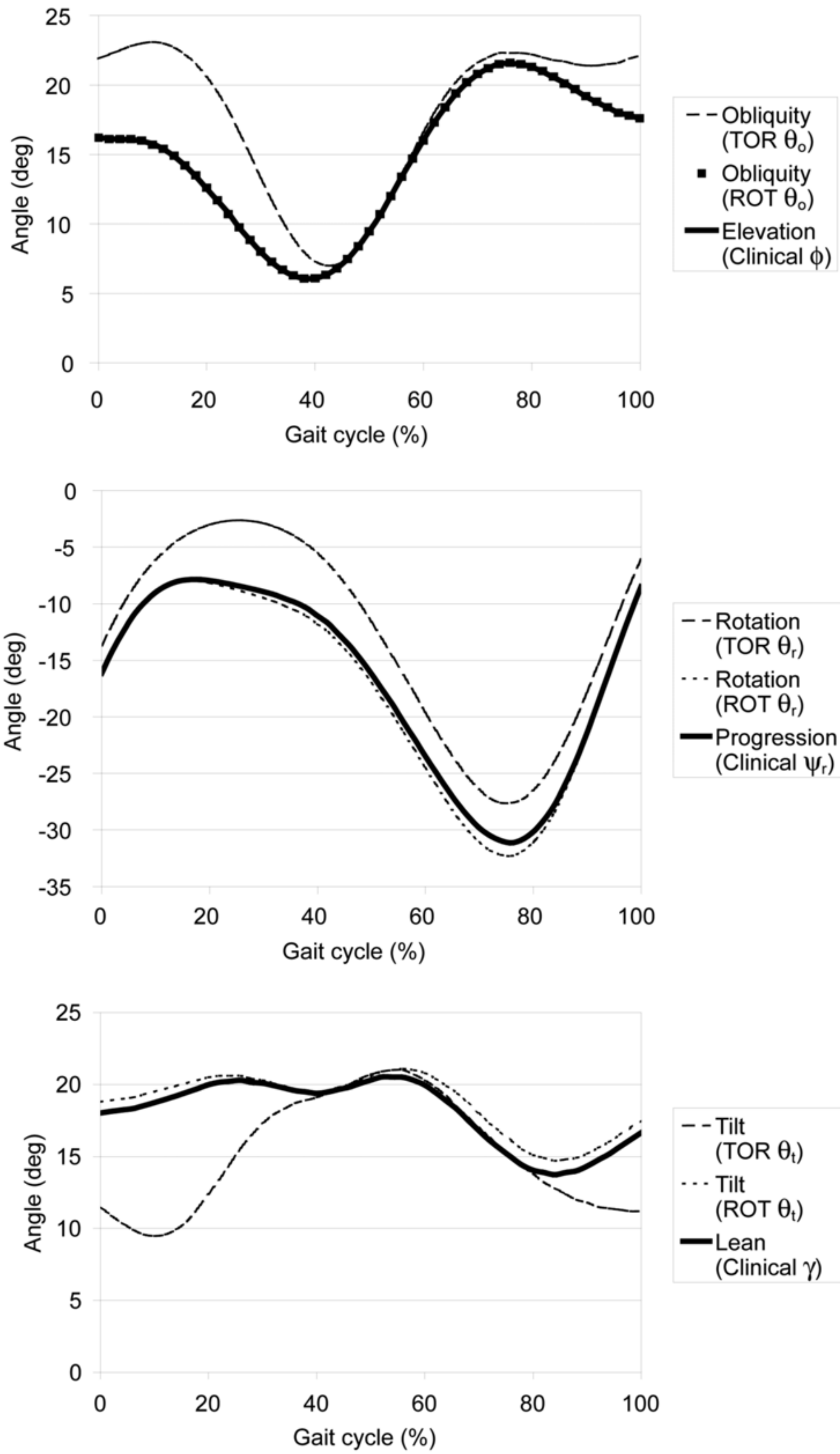

Figure 3 — Case example for patient with substantial obliquity and rotation. TOR results differ significantly from the clinical angles. ROT results are close to, but still different from, the clinical angles.

physical questions such as "how much higher is one hip than the other?" Other angles may also be of interest, and more than three clinical angles could be reported. It should also be noted that the equations presented here are based on a specific laboratory orientation. The process described could be used to derive equations to calculate additional angles or to account for gait laboratory configurations using other rotation sequences or axis conventions.

Like distances, angles are inherently positive quantities. However, angles may be regarded as negative when one associates a sense with their value. In this study, we defined pelvis elevation as positive when L_{ASIS} is higher than R_{ASIS} and negative when L_{ASIS} is lower than R_{ASIS}. This sense is consistent with what might be reported clinically as left pelvis elevation. If separate left and right elevation results are desired, the signs would need to be reversed when considering right pelvis elevation. Similar sign changes would be needed to report separate left and right pelvis progression angles.

A computational advantage of Euler angles over clinical angles is that Euler angles can fully determine the orientation of one rigid body relative to another (e.g., the pelvis relative to the room). Three or more clinical angles may be insufficient for uniquely determining a segment's orientation. Because Euler angles uniquely determine orientation, they are more appropriate for tracking the orientations of multiple rigid-body segments, as is done in gait analysis. Euler angles are also convenient for standardizing the reporting of results. Once the sequence of Euler angles is known, angles of clinical interest can be easily calculated, as demonstrated herein. We propose that Euler angles continue to be used for tracking segment orientations, but that clinical angles be used to present the results for clinical interpretation.

Our proposed method takes advantage of the Euler angles already used in clinical gait analysis systems. The discrepancy between Euler angles and clinical angles is greatest for patients whose joint angles differ most from typical gait kinematics. These are precisely the patients for whom clinically meaningful data are most important. We have presented a simple method for obtaining clinically meaningful angles from the Euler angles currently used in clinical gait analysis. This method will produce the same angles regardless of the rotation sequence used and can be readily implemented in current clinical gait analysis systems.

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