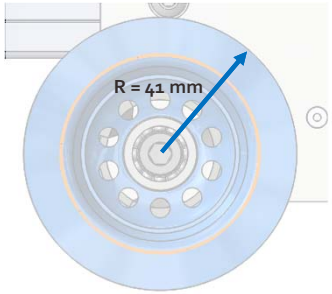
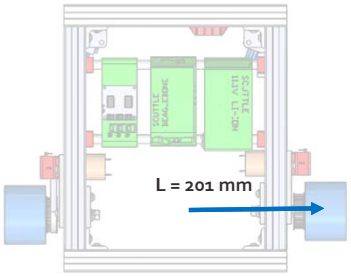
SCUTTLE Kinematics

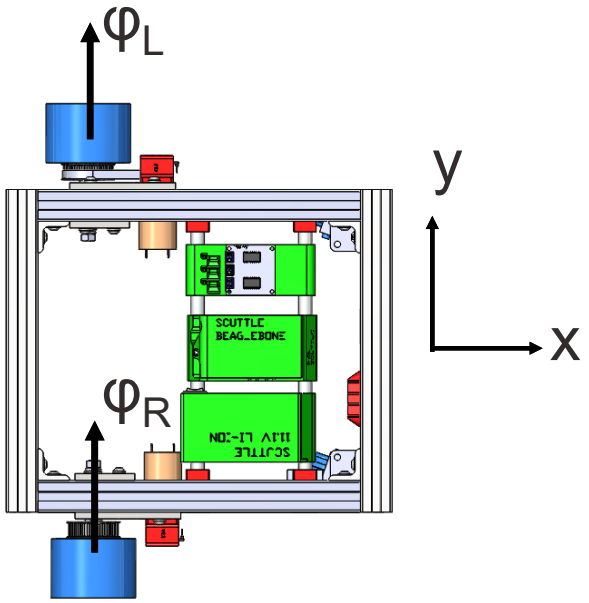
- This section covers:
  - Robot geometry, r, and L
  - Important variables: phi, x, y, and theta
  - Kinematic equation: convert wheel speeds to chassis speeds
  - The time-derivatives of the wheel and chassis displacements
  - Rotation matrix to convert body-fixed coordinates to global coordinates

SCUTTLE Kinematics

- The **Chassis Geometry** determines the equations for kinematics.
- The **radius**, r, is the radius of the driven wheel
- The **half-wheelbase**, L, is the space from wheel edge to edge divided in two







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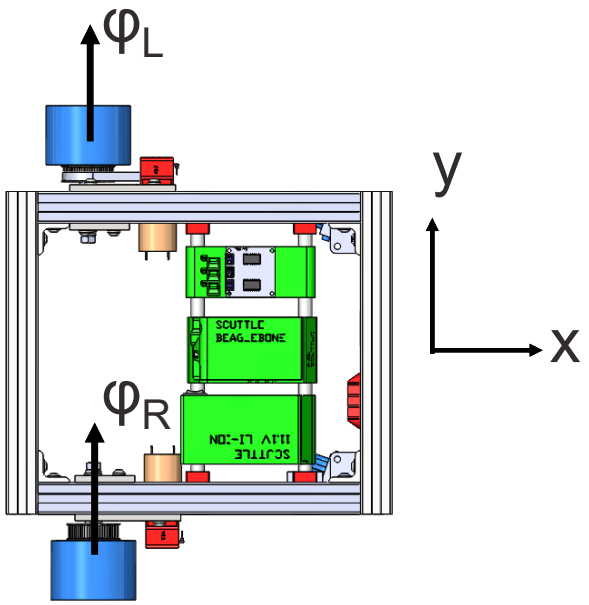
- **Phi dot** is the derivative of phi with respect to time.

  $\dot{\varphi}_L$ = pdl, as in phi_dot_l

  $\dot{\varphi}_R$ = pdr, as in phi_dot_r




```
phiDots = np.array( [ pdl, pdr ] ) # python syntax
```

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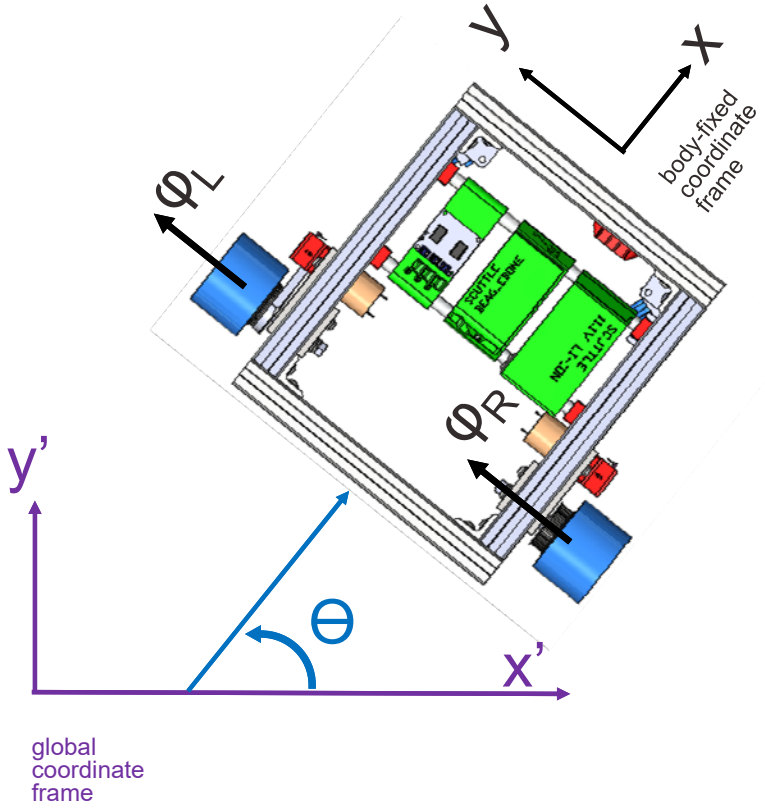
- **Theta** describes the difference between the body-fixed frame and the global frame.
- The **rotation matrix** converts body-fixed coordinates to the global coordinates

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x_{bf} \\ y_{bf} \end{bmatrix}$$



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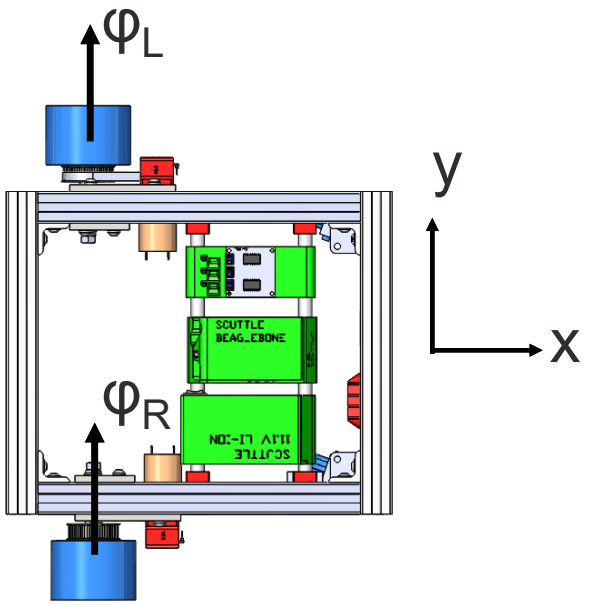
- Phi is the angle of the wheel.
  - It is used to define incremental changes in wheel position and to calculate wheel speeds
- The x,y coordinate system has x pointing forward on the bot.
  - Positive movement of both phi's result in positive movement of the robot along the x-direction
- The Kinematic Equation generates chassis motion information.
  - input the wheel speeds and output the (translational and rotational) chassis speeds

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} R/2 & R/2 \\ -R/2L & R/2L \end{bmatrix} \begin{bmatrix} \dot{\varphi}_L \\ \dot{\varphi}_R \end{bmatrix}$$

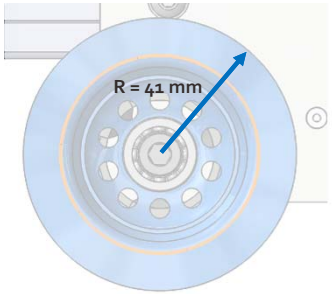


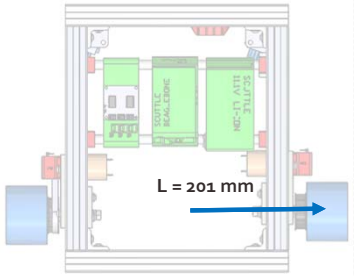
SCUTTLE Kinematics

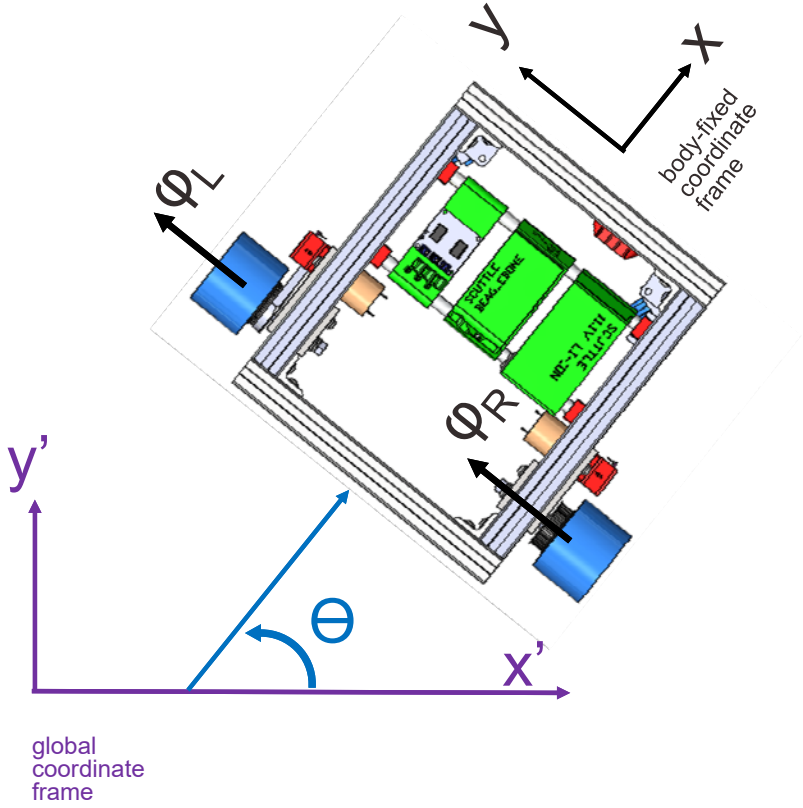
- Inverse Kinematic equation:
  - Input the desired speed and angular speed, and output the left and right wheel speeds.
  - These equations are written in the robot-fixed frame

$$\begin{bmatrix} \dot{\varphi}_L \\ \dot{\varphi}_R \end{bmatrix} = \begin{bmatrix} 1/R & -L/R \\ 1/R & L/R \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix}$$

matrix multiplication: $[C] = [A][B]$







SCUTTLE Kinematics

- Comparing forward and inverse kinematics:

"Kinematics"
Use the **wheel** speeds to obtain the **chassis** speeds

$$\begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} R/2 & R/2 \\ -R/2L & R/2L \end{bmatrix} \begin{bmatrix} \dot{\varphi}_L \\ \dot{\varphi}_R \end{bmatrix}$$

"Inverse Kinematics"
Use the **chassis** speeds to obtain the **wheel** speeds

$$\begin{bmatrix} \dot{\varphi}_L \\ \dot{\varphi}_R \end{bmatrix} = \begin{bmatrix} 1/R & -L/R \\ 1/R & L/R \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix}$$