Question 18.14

x	1	2	3	5	7	8
$f(x)$	3	6	19	99	291	444

as the equation 18.37 applied

→ interior knot selected as x_1

$$(2-1)f''(1) + 2(3-1)f''(2) + (3-2)f''(3) = \frac{6}{3-2}[f(3)-f(2)] + \frac{6}{2-1}[f(3)-f(2)]$$

$$f''(1) + 4f''(2) + f''(3) = 6.(19-6) + 6.(19-6)$$

$$\boxed{f''(1) + 4f''(2) + f''(3) = 156}$$

→ interior knot selected as x_2 :

$$(3-2)f''(2) + 2(5-2)f''(3) + (5-3)f''(5) = \frac{6}{5-3}[f(5)-f(3)] + \frac{6}{3-2}[f(5)-f(3)]$$

$$f''(2) + 6f''(3) + 2f''(5) = 3(99-19) + 6(99-19)$$

$$\boxed{f''(2) + 6f''(3) + 2f''(5) = 720}$$

→ interior knot selected as x_3 :

$$(5-3)f''(3) + 2(7-3)f''(5) + (7-5)f''(7) = \frac{6}{7-5}[f(7)-f(5)] + \frac{6}{5-3}[f(7)-f(5)]$$

$$2f''(3) + 8f''(5) + 2f''(7) = 3(291-99) + 3(291-99)$$

$$\boxed{2f''(3) + 8f''(5) + 2f''(7) = 1152}$$

→ interior knot selected as x_4 :

$$(7-5) f''(5) + 2(8-5) f''(7) + (8-7) f''(8) = \frac{6}{8-7}[f(8)-f(7)] + \frac{6}{7-5}[f(8)-f(7)]$$

$$2f''(5) + 6f''(7) + f''(8) = 6(444-291) + 3(444-291)$$

$$\boxed{2f''(5) + 6f''(7) + f''(8) = 1377}$$

→ Additionally $f''(1) = f''(8) = 0$ as the rules declared.

→ Found equations can be stored in a system of matrices.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 6 & 2 & 0 & 0 \\ 0 & 0 & 2 & 8 & 2 & 0 \\ 0 & 0 & 0 & 2 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f''(1) \\ f''(2) \\ f''(3) \\ f''(5) \\ f''(7) \\ f''(8) \end{bmatrix} = \begin{bmatrix} 0 \\ 156 \\ 720 \\ 1152 \\ 1377 \\ 0 \end{bmatrix}$$

$f''(1) = 0$

$f''(2) = 15.367$

$f''(3) = 94.533$

$f''(5) = 68.718$

$f''(7) = 206.594$

$f''(8) = 0$

cubic

→ Use equation at 18.36 to define each interval:

$\boxed{x \in [1,2]}$

$$f_1(x) = \underbrace{\frac{f''(1)}{6(2-1)}(2-x)^3}_{} + \frac{f''(2)}{6(2-1)}(x-1)^3 + \left[\frac{f(1)}{2-1} - \cancel{\frac{f''(1)(2-1)}{6}}^{\,0}\right](2-x) + \left[\frac{f(2)}{2-1} - \frac{f''(2)(2-1)}{6}\right](x-1)$$

$$f_1(x) = 0 + 2.5611(x-1)^3 + 3(2-x) + (6 - 2.5611)(x-1)$$

$$f_1(x) = 2.5611(x-1)^3 + 3(2-x) + 3.43883(x-1)$$

$\boxed{x \in [2,3]}$

$$f_2(x) = \frac{f''(2)}{6(3-2)}(3-x)^3 + \frac{f''(3)}{6(3-2)}(x-2)^3 + \left[\frac{f(2)}{3-2} - \frac{f''(2)(3-2)}{6}\right](3-x) + \left[\frac{f(3)}{3-2} - \frac{f''(3)(3-2)}{6}\right](x-2)$$

$$f_2(x) = 2.5611(3-x)^3 + 15.7555(x-2)^3 + 3.43883(3-x) + 3.245(x-2)$$

$\boxed{x \in [3,5]}$

$$f_3(x) = \frac{f''(3)}{6(5-3)}(5-x)^3 + \frac{f''(5)}{6(5-3)}(x-3)^3 + \left[\frac{f(3)}{5-3} - \frac{f''(3)(5-3)}{6}\right](5-x) + \left[\frac{f(5)}{5-3} - \frac{f''(5)(5-3)}{6}\right](x-3)$$

$$f_3(x) = 7.878(5-x)^3 + 2.863(x-3)^3 - 22.011(5-x) + 26.594(x-3)$$

$\boxed{x \in [5,7]}$

$$f_4(x) = \frac{f''(5)}{6(7-5)}(7-x)^3 + \frac{f''(7)}{6(7-5)}(x-5)^3 + \left[\frac{f(5)}{7-5} - \frac{f''(5)(7-5)}{6}\right](7-x) + \left[\frac{f(7)}{7-5} - \frac{f''(7)(7-5)}{6}\right](x-5)$$

$$f_4(x) = 5.727(7-x)^3 + 17.216(x-5)^3 + 26.594(7-x) + 76.635(x-5)$$

$\boxed{x \in [7,8]}$

$$f_5(x) = \frac{f''(7)}{6(8-7)}(8-x)^3 + \frac{f''(8)}{6(8-7)}(x-7)^3 + \left[\frac{f(7)}{8-7} - \frac{f''(7)(8-7)}{6}\right](8-x) + \left[\frac{f(8)}{8-7} - \frac{f''(8)(8-7)}{6}\right]$$

$$* (x-7)$$

$$f_5(x) = 34.432(8-x)^3 + 256.57(8-x) + 444(x-7)$$

ⓐ

→ $f(4)$ is in the range of $[3,5]$, so we put 4 into $f_3(x)$

$$f_3(4) = 7.878(5-4)^3 + 2.863(4-3)^3 - 22.011(5-4) + 26.594(4-3)$$

$$\boxed{f_3(4) = 15.325}$$

→ $f(2.5)$ is in the range of $[2,3]$, so we put it into $f_2(x)$

$$f_2(2.5) = 2.5611(3-2.5)^3 + 15.7555(2.5-2)^3 + 3.43883(3-2.5) + 3.245(2.5-2)$$

$$\boxed{f_2(2.5) = 5.632}$$

ⓑ $$f_2(3) = 2.5611(3-3)^3 + 15.7555(3-2)^3 + 3.43883(3-3) + 3.245(3-2)$$

$$\boxed{f_2(3) = 19,0005 \approx 19}$$

$$f_3(3) = 7.878(5-3)^3 + 2.863(3-3)^3 - 22.011(5-3) + 26.594(3-3)$$

$$\boxed{f_3(3) = 19.002 \approx 19}$$