Question 18.5

$$f[x_i, x_j] = \frac{f(x_i) - f(x_j)}{x_i - x_j}$$

$$f[x_i, x_j, x_k] = \frac{f[x_i, x_j] - f[x_j, x_k]}{x_i - x_k}$$

x_i	$f(x_i)$	First-order	Second-order	Third-order
2	$f(2) = 8$	$f[2.5, 2]$	$f[3.2, 2.5, 2]$	$f[4, 3.2, 2.5, 2]$
2.5	$f(2.5) = 14$	$f[3.2, 2.5]$	$f[4, 3.2, 2.5]$	
3.2	$f(3.2) = 15$	$f[4, 3.2]$		
4	$f(4) = 8$			

$$f[2.5, 2] = \frac{f(2.5) - f(2)}{2.5 - 2} = \frac{14 - 8}{0.5} = 12$$

$$f[3.2, 2.5] = \frac{f(3.2) - f(2.5)}{3.2 - 2.5} = \frac{15 - 14}{0.7} = 1.42857$$

$$f[4, 3.2] = \frac{f(4) - f(3.2)}{4 - 3.2} = \frac{8 - 15}{0.8} = -8.75$$

$$f[3.2, 2.5, 2] = \frac{f[3.2, 2.5] - f[2.5, 2]}{3.2 - 2}$$

$$= \frac{1.42857 - 12}{1.2}$$

$$= -8.809525$$

$$f[4, 3.2, 2.5] = \frac{f[4, 3.2] - f[3.2 - 2.5]}{4 - 2.5}$$

$$= \frac{-8.75 - 1.42857}{1.5}$$

$$= -6.785713$$

$$\underbrace{f[4, 3.2, 2.5, 2]}$$

$$\frac{f[4, 3.2, 2.5] - f[3.2, 2.5, 2]}{4 - 2}$$

$$= \frac{-6.785713 + 8.809525}{2}$$

$$= 1.011906$$

$$f_1(x) = b_0 + b_1(x - x_0)$$

$$= 8 + 12(2.8 - 2)$$

$$\boxed{f_1(x) = 17.6}$$

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$= 17.6 - 2.114286$$

$$\boxed{f_2(x) = 15.485714}$$

$$f_3(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

$$f_3(2.8) = 8 + 12(2.8-2) - 8.809525(2.8-2)(2.8-2.5)$$
$$+ 1.011906(2.8-2)(2.8-2.5)(2.8-3.2)$$

$$= 17.6 - 2.114286 - 0.097142976$$

$$\boxed{f_3(2.8) = 15.388571024}$$

(b)

$$R_1 = f[x_2, x_1, x_0](x-x_0)(x-x_1)$$
$$= f[3.2, 2.5, 2](2.8-2)(2.8-2.5)$$
$$= -8.809525.(0.8)(0.3)$$
$$\boxed{R_1 = -2.114286}$$

$$R_2 = f[x_3, x_2, x_1, x_0](x-x_0)(x-x_1)(x-x_2)$$
$$= 1.011906(2.8-2)(2.8-2.5)(2.8-3.2)$$
$$\boxed{R_2 = -0.097142976}$$

$$R_3 = f[x_4, x_3, x_2, x_1, x_0](x-x_0)(x-x_1)(x-x_2)(x-x_3)$$
$$= \frac{f[x_4, x_3, x_2, x_1] - f[x_3, x_2, x_1, x_0]}{x_4 - x_0}(x-x_0)(x-x_1)(x-x_2)(x-x_3)$$

1.8477

$$\boxed{R_3 = 0.21286}$$