Question 18.8

chosen to be closest to 3.5

x_i	$f(x_i)$	First-order	Second-order
1.8	16.415	$f[5, 1.8]$	$f[6, 5, 1.8]$
5	5.375	$f[6, 5]$	
6	3.5		

$$f[5, 1.8] = \frac{f(5) - f(1.8)}{5 - 1.8} = \frac{5.375 - 16.415}{3.2} = -3.45$$

$$f_1(x) = b_0 + b_1(x - x_0)$$

$$f_1(3.5) = 16.415 - 3.45(3.5 - 1.8)$$

$$\boxed{f_1(3.5) = 10.55}$$

→ For trying out x_2 is chosen to be 6 since it is the next thing closest to 3.5

$$f[6,5] = \frac{f(6) - f(5)}{6 - 5} = -1.875$$

$$f_2(x) = \underbrace{b_0 + b_1(x - x_0)} + b_2(x - x_0)(x - x_1)$$

$$= 10.55 + f[6, 5, 1.8](3.5 - 1.8)(3.5 - 5)$$

$$= 10.55 + \frac{f[6, 5] - f[5, 1.8]}{6 - 1.8}(1.7)(-1.5)$$

$$\boxed{f_2(3.5) = 9.59375}$$

→ For the next x let's pick up 0, since it is the next closest to 3.5

x_i	$f(x_i)$	First-order	Second-order	Third-order
0	26	$f[1.8, 0]$	$f[5, 1.8, 0]$	$f[6, 5, 1.8, 0]$
1.8	16.415	$f[5, 1.8]$	$f[6, 5, 1.8]$	
5	5.375	$f[6, 5]$		
6	3.5			

$$f[1.8, 0] = \frac{f(1.8) - f(0)}{1.8 - 0} = \frac{16.415 - 26}{1.8} = -5.325$$

$$f[5, 1.8, 0] = \frac{f[5, 1.8] - f[1.8, 0]}{5 - 0} = \frac{-3.45 + 5.325}{5} = 0.375$$

$$f[6, 5, 1.8, 0] = \frac{f[6, 5, 1.8] - f[5, 1.8, 0]}{6 - 0} = \frac{0.375 - 0.375}{6} = 0$$

$$R_2 = \underbrace{f[6, 5, 1.8, 0]}_{0} (x - x_0)(x - x_1)(x - x_2)$$

$\boxed{R_2 = 0}$ → for next x is chosen as 0

→ If the next one is chosen as 8.2

x_i	$f(x_i)$	First-order	Second-order	Third-order
1.8	16.415	$f[5,1.8]$	$f[6,5,1.8]$	$f[8.2,6,5,1.8]$
5	5.375	$f[6,5]$	$f[8.2,6,5]$	
6	3.5	$f[8.2,6]$		
8.2	2.015			

$$f[8.2,6,5,1.8] = \frac{f[8.2,6,5] - f[6,5,1.8]}{8.2-1.8}$$

$$\frac{f[8.2,6] - f[6,5]}{8.2-5} = \frac{-0.675+1.875}{3.2} = 0.375$$

$$= \frac{0.375 - 0.375}{6.4}$$

$$= 0$$

$$R_2 = f[8.2,6,5,1.8](x-x_0)(x-x_1)(x-x_2)$$

$\boxed{R_2 = 0}$ → for next x is chosen as 8.2

→ If the next one is chosen as 9.2

$$f[9.2,6,5,1.8] = \frac{f[9.2,6,5] - f[6,5,1.8]}{9.2-1.8}$$

$$\frac{f[9.2,6] - f[6,5]}{9.2-5} = \frac{-0.3+1.875}{4.2} = 0.375$$

$$= \frac{0.375 - 0.375}{7.4} = 0$$

$$R_2 = f[9.2,6,5,1.8](x-x_0)(x-x_1)(x-x_2)$$

$\boxed{R_2 = 0}$ → for next "x" is chosen as 9.2.

→ If the next one is chosen as 12

$$f[12,6,5,1.8] = \frac{f[12,6,5] - f[6,5,1.8]}{12-1.8}$$

$$f[12,6,5] = \frac{f[12,6] - f[6,5]}{12-5} = \frac{0.75+1.875}{7} = 0.375$$

$$= \frac{0.375 - 0.375}{12-1.8}$$

$$= 0$$

$$R_2 = f[12,6,5,1.8](x-x_0)(x-x_1)(x-x_2)$$

$\boxed{R_2 = 0}$ → for next x is chosen as 12

→ It is shown that for the next step choosing any value of 0, 8.2, 9.2 or 12 error estimation will be 0 therefore it will make any contribution R_2 both negative or positive way, then we can stop.

$\boxed{f_2(3.5) = 9.59375}$ → is the exact value