MATH6001 – Mathematics Masters Project 3

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Project Final Report

Drill and Blast Optimisation for Open-pit Mining

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Declaration

The work presented in this report is my own work and all references are duly acknowledged.

This work has not been submitted, in whole or in part, in respect of any academic award at Curtin University or elsewhere.

Cem Alpaslan

16.10.2025

Acknowledgments

I would like to say thank you and present my respect to my project supervisor Dr. Sarang Kulkarni and the course coordinator Professor Yong Wu for their help, guidance and support throughout the entire process. Furthermore, with sincere appreciation to Ozan Perincek, Ryan Loxton, Sarang Kulkarni and Daniel Arthur for providing the paper called *Drill Pattern Optimisation for Large Complex Blasts to improve Fragmentation and Dig Efficiency* which served as the main source of my project.

I believe this project has enhanced my knowledge in mining industry and understanding of implementing mathematical optimisation techniques to drill and blast procedures in open-pit mines. I express my gratitude to my project supervisors, and to Curtin University for providing this unique project development opportunity as a part of the program.

Cem Alpaslan

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Abstract

Mining industry has been a significant and profitable business field for the Australia`s economy for many years. Over the years hundreds of mines have been established and extracting valuable minerals across the country. Open-pit mining method and drill and blast procedures have been used as the core techniques in the mining industry. Moreover, universities and giant companies have established powerful networks to optimize the drill and blast processes in open-pit mines. Furthermore, fragmentation has been studied as the primary optimisation objective for many years for its effects. Fragmentation objective has a huge impact on minimizing the total cost of the mining operations and reducing the environmental footprint. In addition, efficient fragmentation facilitates the loading and hauling processes of the mines. This paper focused on developing a new novel methodology to minimize certain fragmentation objectives such as total fragmentation and maximum fragmentation ratio per hole. A well-known optimisation algorithm which is called Particle Swarm Optimisation is examined, developed and used as the main optimisation approach in this study. Lastly, detailed analysis of the results obtained has been presented and investigated to provide significant insights for the mining professionals.

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1. Introduction

1.1 Problem Description and Background

Mining industry has a massive share in Australia's economy with over 350 active operations and 19 different precious minerals extraction spreading across the country (Geoscience Australia 2025). Furthermore, mining sector will maintain its dominance and contribution to the Australia's economy for many years. Minetek (2024) indicates that mining sector constitutes 13.7% of Australia's gross domestic product. Moreover, huge portion of Australia's international export is supplied by the mining operations and it provides employment opportunities for more than two hundred fifty thousand people (Minetek 2024). In addition, mining industry enhances the quality of the industries such as healthcare, defense, service and education by investing the generated income into those fields (Minetek 2024).

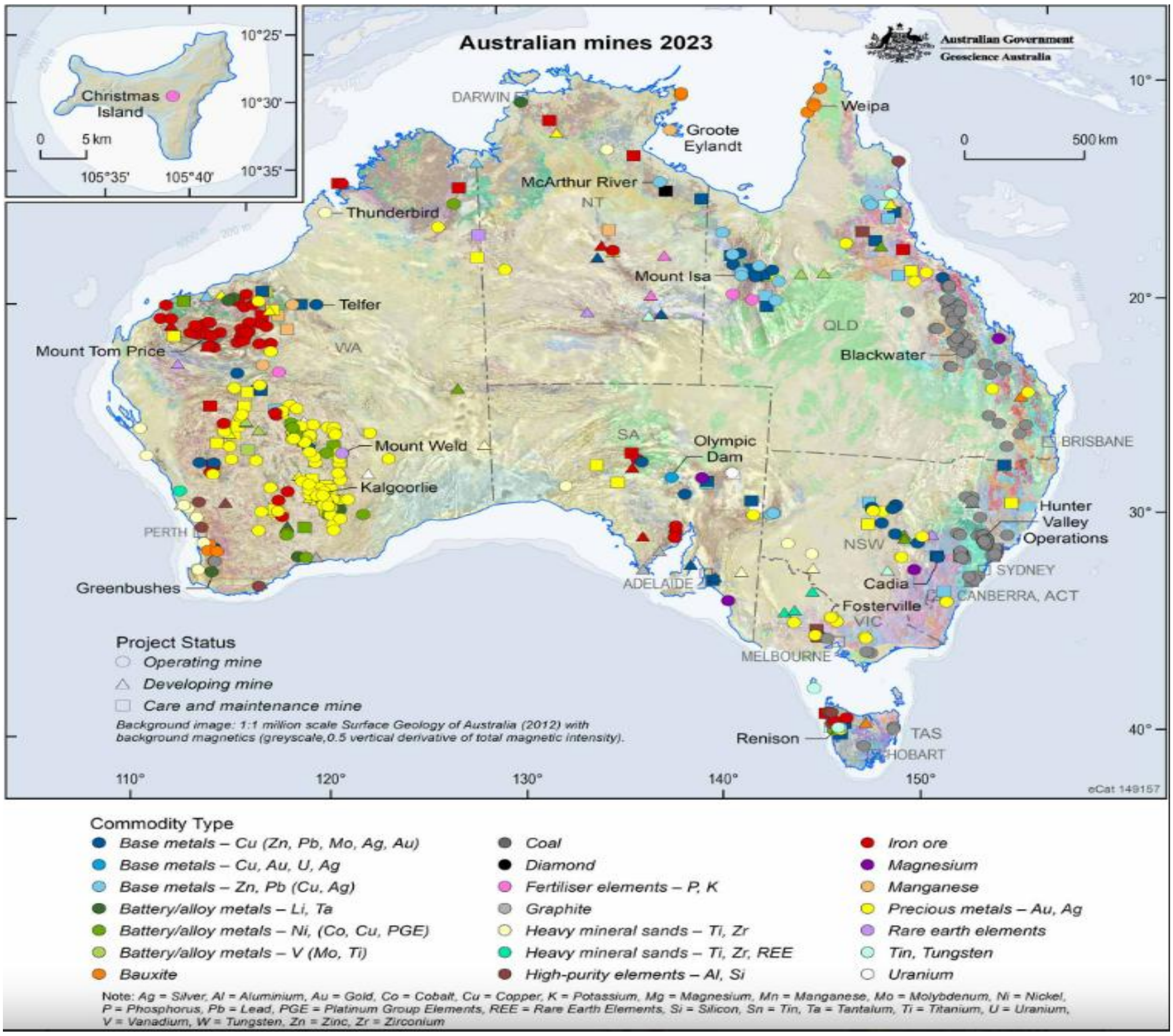


Figure 1. Major mining and mineral deposits in Australia in 2023

Source: Figure reproduced from Geoscience Australia (2025).

Mining is a multi-disciplinary activity which has a purpose of identifying, extracting and processing valuable minerals from the ground. There are various mining techniques depending on the mineral types and geographical conditions but the most common ones are underground mining and open-pit mining (Dam 2024). Underground mining has a purpose of reaching the minerals and mines which are located below the ground (Dam 2024). Advanced mining techniques such as vertical and horizontal tunneling are most used strategies to reach the resources (Dam 2024). Furthermore, Dam (2024) highlights that mine planning step is the most important stage of the underground mining operations, because it evaluates all the tasks in detail and avoids catastrophic events such as groundwater contamination and roof collapse. Lastly, underground mining needs more financial investment and is expensive, but it minimizes the environmental impact and provides high revenue opportunities for the investors compared to open-pit mines (Dam 2024). On the other hand, open-pit mining is another crucial and popular mining method in the industry. Open-pit mining is the oldest but efficient mining technique which is also known as the surface-based mining (Dam 2024). Main approach of open-pit mining is to create a bowl-shaped dig near to the surface to reach and obtain valuable minerals (Erizon 2024). Furthermore, Erizon (2024) mentions that digging huge holes and extracting the valuable minerals layer by layer is the main technique of surface mining. Moreover, open-pit mines have a unique design structure consisting of two segments (Erizon 2024). The incline section of the mine which is called batter has a responsibility of ensuring slope stability and preserving wall integrity (Erizon 2024). In addition, the flat segment which is called bench is serving as the mineral extraction section by applying drill and blast operations (Erizon 2024). Compared to underground mining surface mining is cost efficient and less risky. However, improperly designed open-pit mines have huge environmental impacts and high pollution possibility (Dam 2024).

As shown in Figure 2 on the next page, in open-pit mines drill and blast procedures have a lifecycle which starts with drill pattern design continues with blasting and ends with hauling. Hence, drill and blast play an essential role for open-pit mines to extract mines and minerals efficiently. Drill and blast are advanced mining methods where holes are bored into the rock and after that inserting blasting agents to break the rock into smaller fragments (Jouav 2024). Moreover, Jouav (2024) argues that burden and spacing configuration are very significant to ensure effective drilling patterns for open-pit mines. In addition, another crucial part of minimizing the fragmentation is to determine blasting pattern and sequence. Initiation and timing design between detonations is a fundamental technical concept and improves the efficiency of the blast. Singh (2007) claims that determining the appropriate explosion product according to hardness characteristics of the rock have a huge impact on optimizing the blast outcome. Moreover, implementing software systems for the drill and blast procedures in open-pit mines to design loading methods, calculate timing and delay and monitoring the bench optimizes the overall blast productivity (Singh 2007).

This project has focused on generating a synthetic bench and developing a novel algorithm to generate drill and blast pattern designs to perform drill and blast operations with a purpose of minimizing fragmentation objectives. Due to this project surface mines will be able to implement fragmentation-efficient drill and blast designs and it will enhance their performance of overall operations such as total cost minimization, less ecological impact and fast loading and hauling services.

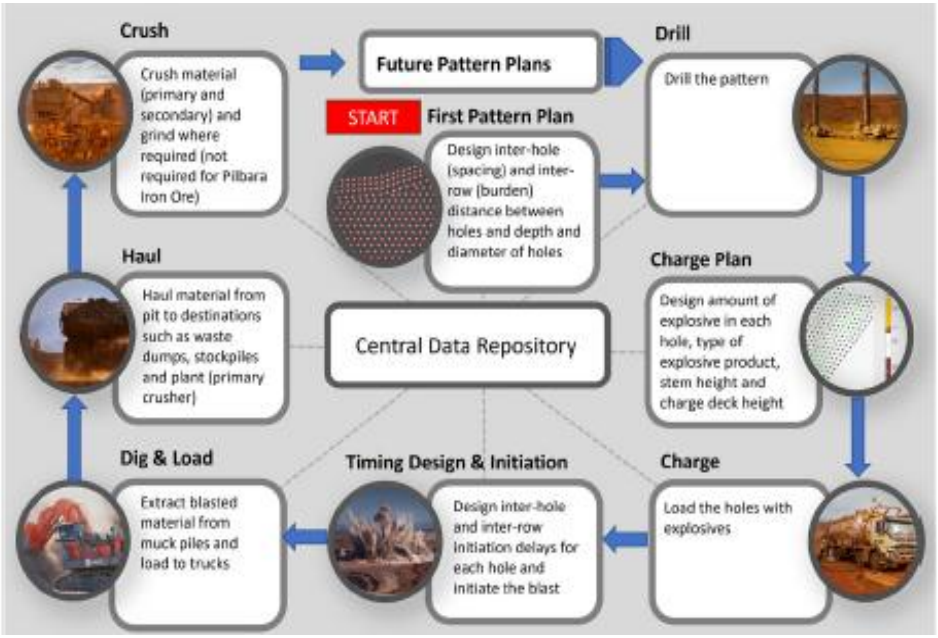


Figure 2. Ore extraction process in open-pit mining

Source: Figure reproduced from Perincek et al. (2025, 578).

1.2 Scope of the Project

The main concentration of this project is to provide optimal drill and blast design strategies for open-pit mines to the mining professionals. To improve the efficiency of the operations and minimize the total cost components fragmentation plays a crucial role for mining companies. Furthermore, fragmentation is very significant for reducing the environmental impacts and improving the efficiency of the hauling operations. Thus, minimizing the fragmentation objectives in drill and blast designs will improve the overall productivity of the open-pit mines. In addition, giant mining companies such as Rio Tinto, BHP, FMG and Roy Hill have active and under construction open-pit operations across the country; therefore, providing a new novel methodology for improving the fragmentation objectives will support their extraction processes. Moreover, this project will not only assist the open-pit mines across Australia, but it will also provide significant insights for the open-pit mines in whole over the world.

This study has used the mathematical formulation of the article called *Drill Pattern Optimisation for Large Complex Blasts to improve Fragmentation and Dig Efficiency* as the main optimisation model. Necessary modifications such as removing the dig efficiency objective and adding the fragmentation ratio objective have been implemented to the mathematical model. In addition, since the non-linearity and complexity of the mathematical formulation Particle Swarm Optimisation approach has been used as the main algorithm as the new approach in this study.

Lastly, the developed codes for synthetic data generation and Particle Swarm Optimisation algorithm have not been provided in the appendix section due to their length but are discussed in detail in the methodology section of the report. Furthermore, codes were written by using the Python programming language and will be available in my GitHub profile and it can be shared with interested academicians and professionals by request. The detailed dataset including all input parameters can be shared with the interested academicians and professionals as well.

1.3 Objectives, Significances and Contributions of the Project

The main objective of this project is to analyze the optimisation technique in the main source which is called *Drill Pattern Optimisation for Large Complex Blasts to improve Fragmentation and Dig Efficiency* to understand the approach and to provide more practical optimisation methodology for the drill and blast designs in open-pit mining studies. The primary target of this project is to use meta-heuristic techniques to optimize two objectives which are minimizing the total fragmentation and minimizing the maximum fragmentation ratio. This study seeks to:

- Analyze and understand the mathematical formulation and heuristic method in the main source.
- Update the mathematical model to minimize the total fragmentation and maximum fragmentation ratio.
- Detailly investigate and understand the Particle Swarm Optimisation algorithm to use as the new main optimisation technique for the problem.

- Adjust the Particle Swarm Optimisation algorithm according to the needs and the structure of the problem to provide a novel methodology for the industry.
- Provide theoretical and practical outcomes for the mining professionals in the Australia to optimize their drill and blast designs for open-pit mines.

This project is a significant project, because it provides both academic and practical contributions. The adjusted mathematical model and developed algorithm demonstrate theoretical significance. Moreover, the developed algorithm has been tested by synthetic data and this has generated both theoretical and practical outcomes. Furthermore, this project has contributed to a new novel optimisation algorithm to minimize fragmentation objectives. In addition, due to this study, meta-heuristic algorithms have been used in mining optimisation and the project illustrates the efficiency of the meta-heuristic algorithms for drill and blast patterns. Lastly, this research will be available for future academic researchers to improve and will provide crucial contributions to the academia.

1.4 Literature Review

Optimizing the drill and blast patterns have always been a crucial technique for the mining industry. Engineers, mathematicians and academicians have been developing optimisation techniques and algorithms to optimize certain objectives such as cost, dig efficiency, fragmentation, environmental impact, deviation and timing. Moreover, there are various optimisation fields in both open-pit mines and underground mines. Drill and blast optimisation, production scheduling optimisation, haulage and fleet management optimisation, processing and blending optimisation and maintenance and shutdown scheduling optimisation are primary optimisation fields in the mining industry. Furthermore, linear programming, mixed-integer linear programming, non-linear programming, quadratic programming, heuristics and meta-heuristics are mainly used optimisation techniques to enhance the performance of the systems. In addition, over the years industry giants and universities have established significant correlations to optimize the mining operations across the country. Drill and blast optimisation have always been a crucial research field for the academicians and mining companies because it enhances certain objectives. Perincek et al. (2025), have focused on minimizing the fragmentation and dig efficiency in open-pit mines in their study. They have developed mathematical formulation and a heuristic algorithm for drill and blast patterns to satisfy those objectives (Perincek et al. 2025). Moreover, Abbaspour et al. (2018) have concentrated on developing a dynamic model to determine the optimal blast parameters to minimize the total cost of the open pit mine. In addition, Afum and Temeng (2015) have focused on analyzing the drill and blast parameters for an open pit gold mine in Ghana in their case study. They have examined the blast parameters in three different pits to provide cost-effective recommendations to the mining company (Afum and Temeng 2015). Lastly, Silveira and Lima (2024) have concentrated on determining the optimal blast hole diameter in an open pit mine which is located next to inhabited area to minimize the environmental footprint in their study.

2. Methodology

2.1 Mathematical Modeling

As indicated in the previous sections of the report article called *Drill Pattern Optimisation for Large Complex Blasts to improve Fragmentation and Dig Efficiency* has been used as the primary source for this study. The developed mathematical model has been used as the main mathematical formulation for this study but necessary adjustments such as removing the dig efficiency objective and adding a maximum fragmentation ratio objective have been implemented (Perincek et al. 2025).

There are several difficulties and challenges in this mathematical formulation. Firstly, number of rows and columns are unknown (Perincek et al. 2025). Only the location of the bench and sensitive site are known; therefore, identifying the rows and columns is a significant challenge (Perincek et al. 2025). Moreover, number of rows and columns automatically affects the burden and spacing because as shown in figure 3 burden is the distance between two rows and spacing is the distance between two columns (Perincek et al. 2025). Furthermore, some parameters are function of the decision variables which increases the complexity of the problem (Perincek et al. 2025). Lastly, the mathematical model is in a non-linear form which makes commercial solver tools inefficient in terms of computation time and optimality gap (Perincek et al. 2025).

To solve those difficulties this project has implemented different methods and approaches which are explained in detail in the data generation, particle swarm optimisation and algorithm development sections of the report.

The input parameters proposed by Perincek et al. (2025), are given below.

Input Parameters :

H: bench height

η_{ij}: ground hardness

E_{ij}^{0}: energy concentration for maximum excavator efficiency

D_{ij}: distance to the sensitive site

$\varepsilon_{ij}^{\max}$: maximum energy intensity

$d_{ij}^{\min}$: minimum hole depth

$SDoB_{ij}$: scaled depth of burial

row $i \in \{1, \ldots, I\}$

column $j \in \{1, \ldots, J_i\}$

The decision variables proposed by Perincek et al. (2025), are given below.

Decision Variables :

B_{ij}: burden

S_{ij}: spacing

ϕ_{ij}: hole diameter

d_{ij}: hole depth

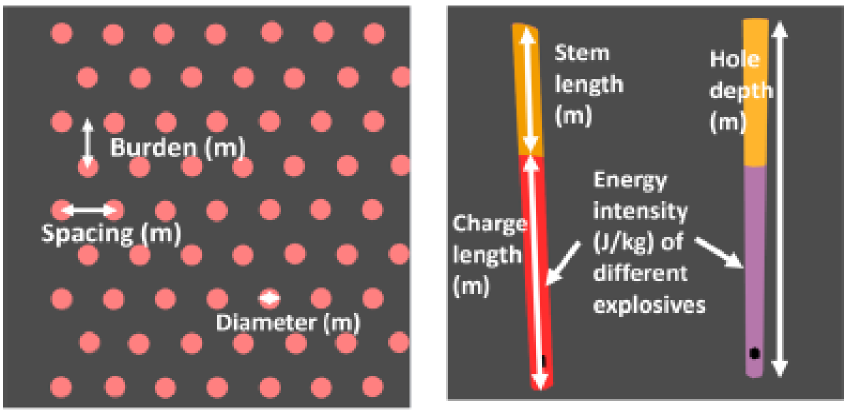
Sl_{ij}: stem length

Q_{ij}: mass of explosives

ρ_{ij}: density of explosive product

ε_{ij}: energy intensity of explosive product

R_{ij}: weight strength of the explosive product



(a) Top down view of pattern (b) Side profile view of two holes

Figure 3. Drill and blast decision variables

Source: Figure reproduced from Perincek et al. (2025, 578).

The objective function and constraints formed by Perincek et al. (2025), are given below.

Objective Functions:

Kuz Ram Equation:

$$x_m = AK^{-4/5}Q^{1/6}\left(\frac{115}{R}\right)^{19/20} \qquad (1)$$

Most widely recognized and accepted fragmentation formula which is equation 1 by Cunningham (2005) is used as the main fragmentation formula in this mathematical formulation (Perincek et al. 2025). x_m represents the average particle size, A is the rock factor, for presenting explosive powder factor K is used and R is the weight strength of the explosive product (Perincek et al. 2025). Moreover, $A = \gamma\eta_{ij}$and it has a linear relationship with hardness (Perincek et al. 2025).

Total Fragmentation Objective:

$$\min Z = \sum_{i=1}^{I}\sum_{j=1}^{J_i}\left[\hat{x}_m - \gamma\eta_{ij}\left(\frac{Q_{ij}}{d_{ij}\hat{B}_i\hat{S}_{ij}}\right)^{-4/5} Q_{ij}^{1/6}\left(\frac{115}{R_{ij}}\right)^{19/20}\right]^2 \qquad (2)$$

$\hat{x}_m$is used to represent the target particle size and the main purpose of this total fragmentation objective is to minimize the deviation in particle size (Perincek et al. 2025).

Kuz Ram Equation per hole:

$$x_{m,ij} = (\gamma\eta_{ij})\left(\frac{Q_{ij}}{d_{ij}\hat{B}_i\hat{S}_{ij}}\right)^{-4/5} Q_{ij}^{1/6}\left(\frac{115}{R_{ij}}\right)^{19/20} \qquad (3)$$

Equation 3 is derived from the main fragmentation formula which is equation 1 and it has been used in this study to calculate the fragmentation per hole.

Maximum Fragmentation Ratio Objective:

$$\min Z = \max_{i=1,\dots,I;\, j=1,\dots,J_i}\left|\frac{x_{m,ij} - \hat{x}_m}{\hat{x}_m}\right| \qquad (4)$$

This objective function minimizes the maximum fragmentation ratio for each hole.

Constraints :

Energy Intensity Constraint:

$$\varepsilon_{ij} \le \varepsilon_{ij}^{max}, \quad i = 1, \dots, I,\ j = 1, \dots, J_i \qquad (5)$$

This constraint ensures that the energy intensity of the explosive product does not exceed the maximum energy intensity (Perincek et al. 2025).

Ground Vibration:

Peak Particle Velocity (PPV) Formula:

$$PPV = \lambda \left(\frac{D}{\sqrt{W}} \right)^{\alpha} \qquad (6)$$

According to Perincek et al. (2025) λ and α represent the site factors obtained from the previously drilled and blasted holes, D is the distance to the sensitive site and W is used to represent Q_{ij}which is mass of explosives.

Ground Vibration Constraint:

$$\sqrt{Q_{ij}} \leq \frac{D_{ij}}{\sqrt[\alpha]{\frac{PPV}{\lambda}}}, \quad i = 1, \ldots, I, \ j = 1, \ldots, J_i \qquad (7)$$

This constraint ensures that sensitive site does not exceed the PPV limitations (Perincek et al. 2025).

Air Overpressure:

Air Overpressure Formula:

$$AOp = k \left(\frac{D}{\sqrt[3]{W}} \right)^{\beta} \qquad (8)$$

Widely accepted and popular air overpressure formula which is equation 8 by Kuzu et al. (2009) is used as the main air overpressure formula in this study. As reported by Perincek et al. (2025) k and β are site factors which are obtained from previous blast studies and AOp is used as the notation for air overpressure.

Air Overpressure Constraint:

$$\sqrt[3]{Q_{ij}} \leq \frac{D_{ij}}{\sqrt[\beta]{\frac{AOp}{k}}}, \quad i = 1, \ldots, I, \ j = 1, \ldots, J_i \qquad (9)$$

This constraint satisfies that maximum allowable mass per hole does not exceed the air overpressure limits of the sensitive site (Perincek et al. 2025).

Flyrock:

Scaled Depth of Burial (SDoB) Formula:

$$SDoB = \frac{Z_c}{\sqrt[3]{W_{10\Phi}}} \qquad (10)$$

Equation 10 is a well-known formula which limits the flyrock from the blasts (Perincek et al. 2025). $SDoB$ is the notation for scaled depth of burial and Z_c represents the distance between top of the hole and its center (Perincek et al. 2025).

Flyrock Constraint:

$$SDoB_{ij} \leq \frac{sl_{ij} + 5\Phi}{\sqrt[3]{\pi\left(\frac{\Phi}{2}\right)^2 \rho_{ij} 10\Phi}}, \quad i = 1, \dots, I, \; j = 1, \dots, J_i \tag{11}$$

This constraint ensures that scaled depth of burial ($SDoB_{ij}$) should be less than or equal to the allowable limit which prevents flyrock possibilities (Perincek et al. 2025).

Other Constraints:

$$1 \leq \frac{\hat{S}_{ij}}{\hat{B}_i} \leq 1.4, \quad i = 1, \dots, I, \; j = 1, \dots, J_i \tag{12}$$

This constraint limits the spacing-to-burden ratio within the desired range (Perincek et al. 2025).

$$\sum_{i=1}^{I} \hat{B}_i + y_0 = y_{max} \tag{13}$$

This constraint satisfies that rows are expanding and coordinating in the direction of the burden (Perincek et al. 2025).

$$\sum_{j=1}^{J_i} \hat{S}_{ij} + x_{i0} = x_i^{max}, \quad i = 1, \dots, I \tag{14}$$

This constraint makes sure that columns are expanding and coordinating in the direction of the spacing (Perincek et al. 2025).

$$\hat{B}_i \geq \hat{B}_{min}, \quad i = 1, \dots, I \tag{15}$$

This constraint ensures that burden at each row is greater or equal than the minimum burden value (Perincek et al. 2025). Moreover, this constraint ensures the grid geometry and facilitates the loading operations (Perincek et al. 2025).

$$\hat{S}_{ij} \geq \hat{S}_{min}, \quad i = 1, \dots, I, \; j = 1, \dots, J_i \tag{16}$$

This constraint satisfies that spacing between holes is greater or equal than the minimum spacing value (Perincek et al. 2025).Furthermore, this constraint also ensures the grid geometry as well and facilitates the loading operations (Perincek et al. 2025).

$$d_{ij} \geq sl_{ij}, \quad i = 1, \dots, I,\ j = 1, \dots, J_i \qquad (17)$$

This constraint makes sure that hole depth is greater or equal to the stem length (Perincek et al. 2025).

$$d_{ij} \geq d_{ij}^{min}, \quad i = 1, \dots, I,\ j = 1, \dots, J_i \qquad (18)$$

This constraint ensures that hole depth is greater or equal to minimum hole depth (Perincek et al. 2025).

2.2 Data Generation Approach

Accuracy of the data plays a crucial role in optimizing complex systems. Since a dataset has not been provided and found, this study has maintained a synthetic data generation approach to optimize the drill and blast designs in open-pit mines. Main purpose of this data generation algorithm is to generate a bench which consists of four different hardness levels and hole positions to perform drill and blast operations. To determine the hardness levels as proposed by International Gem Society (2025), Mohs Hardness Scale and Chart has been used as the primary source. Four different hardness profiles which are soft, medium, hard and extra hard have been determined. Soft hardness level is presented within a range between 1 to 1.5, medium between 1.6 to 2.5, hard between 2.6 to 3.5 and extra hard between 3.6 to 4. Moreover, as stated in the mathematical modeling section of the report to solve the complexity of the area identification burden and spacing values have been fixed in advance and number of rows and columns are calculated automatically. Furthermore, hardness zones have been created according to a pre-determined ratio. Soft zone consists of %5 of the bench, medium is %15, hard is %50 and extra hard is %30. To locate the zones efficiently on the bench map y-coordinates have been used to define the zone areas. Moreover, hardness level values have been selected randomly from the pre-defined hardness level ranges for the generated holes. In addition, generating holes effectively possible hole locations have been calculated by multiplying number of rows and number columns. After that, a probabilistic approach has been introduced for hole generation. Holes in calculated possible locations have been generated with 0.6 probability and with this approach more realistic and natural grid structure has been formed. Lastly, x and y coordinates of the holes have been calculated by adding the spacing value for x coordinates and the burden value for y coordinates for each generated hole. The dataset obtained has been converted into an excel file and provided in the appendix section. Furthermore, it has been used by the main optimisation algorithm to minimize certain fragmentation objectives.

Selected Input Parameters:	Hardness Levels:	Hardness Zone Ratios:
Bench Size: 200 (fixed)	Soft : 1 – 1.6	Soft: %5
Burden: 10 (fixed)	Medium: 1.6 – 2.5	Medium: %15
Spacing: 10 (fixed)	Hard: 2.6 – 3.5	Hard: %50
Rows: 200 / burden (10) = 20	Extra Hard: 3.6 – 4	Extra Hard: %30
Columns: 200 / spacing (10) = 20		

2.3 Particle Swarm Optimisation

Meta-heuristic algorithms have been playing a crucial role in optimizing complex systems in various industries. To understand the efficiency of the meta-heuristic algorithms the background and how they are formed should be investigated in detail. Meta-heuristic approaches are influenced by natural and social behaviors and provide approximate outcomes within small computation times to complex problems (Eurystic 2025). Furthermore, the difference between heuristic approaches and meta-heuristic algorithms are very crucial as well. Heuristic algorithms are useful methods when mathematical modelling is computationally ineffective, but the main problem of the heuristic algorithms are they are problem dependent (Eurystic 2025). On the other hand, meta-heuristic algorithms are not problem dependent and can be applied in any type of problem (Eurystic 2025). Moreover, meta-heuristic methods provide wide-ranging exploration strategies to reach global optimum with a purpose of avoid tackling local optimums (Eurystic 2025). In addition, the main disadvantages of meta-heuristics are high computation times for high dimensional problems and parameter selection has a huge impact on the quality of the outcome (Eurystic 2025).

There are various popular meta-heuristic methods such as Genetic Algorithm, Simulated Annealing and Particle Swarm Optimisation, but this study has used the Particle Swarm Optimisation as the main optimisation model and has developed an optimisation algorithm to minimize the fragmentation objectives for the drill and blast designs in open-pit mines.

Particle Swarm Optimisation algorithm was designed and presented by Dr. Eberhart and Dr. Kennedy in 1995 (Settles 2005). Algorithm has investigated the natural behavior and collective actions of birds and fish (Kennedy and Eberhart 1995). Kennedy and Eberhart (1995) point out that some motion rules such as moving in harmony and coordination, intelligence and instant direction change have been identified. With analyzing those findings and performing simulations a search mechanism which is called Particle Swarm Optimisation algorithm has been developed (Kennedy and Eberhart 1995). In addition, Kennedy and Eberhart (1995) have reached a conclusion that PSO technique performs efficiently in continuous non-linear functions. Since mathematical formulation is a non-linear continuous model, PSO has been used as the main optimisation framework for the development of the drill and blast pattern algorithm.

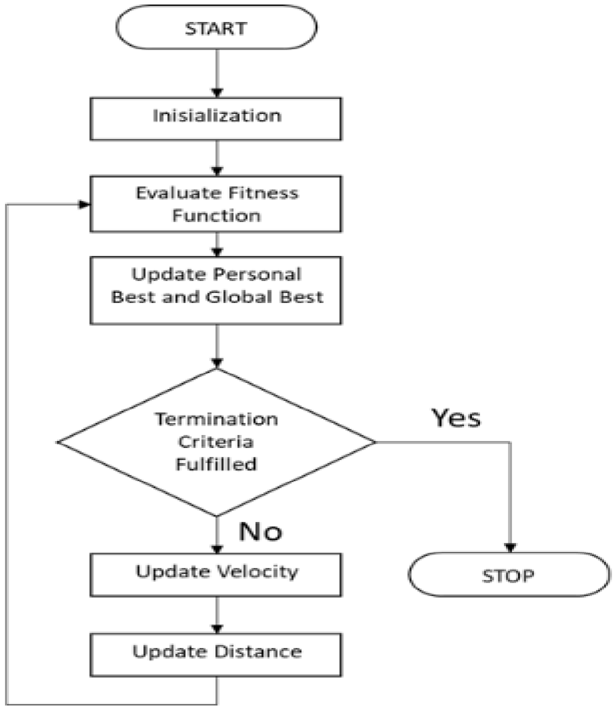


Figure 4. Flowchart of particle swarm optimisation algorithm

Source: Figure reproduced from Adyatama (2019).

The rate of position change (velocity) equation proposed by Adyatama (2019):

- $V_{id} = W * V_{id} + c_1 * rand[0,1] * (P_{id} - X_{id}) + c_2 * rand[0,1] * (G_{id} - X_{id})$ (1)

The new position of the particle equation formed by Adyatama (2019):

- $X_{id} = X_{id} + V_{id}$ (2)

PSO parameters for the equations defined by Adyatama (2019):

V_{id}: velocity of the particle

W: inertia

c_1: cognitive coefficient

c_2: social coefficient

$rand[0,1]$: random number between 0 and 1

X_{id}: current solution (position) from each individual

P_{id}: personal best, the best solution from each individual (particle)

G_{id}: global best, the best solution from the whole population (swarm) (Adyatama 2019)

Particle Swarm Optimisation algorithm is a powerful tool to optimize complex problems. In PSO terminology, candidate solutions (individuals) are represented by particles and population is represented by swarm (Adyatama 2019). The main purpose of the algorithm is to use the particles to seek the global optima of the problem by forming new generations (Adyatama 2019). Every particle holds its best outcome obtained so far and it is represented as personal best (Adyatama 2019). On the other hand, the best outcome within the personal best of every particle is known as the best solution of the swarm and it is represented as global best (Adyatama 2019). Moreover, using equation 1 and equation 2 velocity and position of each particle are calculated and updated in every iteration (Adyatama 2019). By using the obtained position value (X_{id}) each particle`s fitness function is calculated in every iteration and personal best and global best is updated according to the calculated fitness function value (Adyatama 2019). Furthermore, this population generation continues until reaching the pre-determined iteration parameter (Adyatama 2019). Most crucial factor of the PSO algorithm which has a huge impact on actions of the particles is determining the inertia, cognitive coefficient and social coefficient (Adyatama 2019). Adyatama (2019) argues that inertia has a significant impact on determining the extent of the search mechanism. To find the global optimum and avoid getting stuck in local optima linearly decreasing dynamic inertia outperforms static inertia approach, because it evaluates lots of neighbors in the beginning and towards end it only accepts better solutions (Adyatama 2019).

2.4 Algorithm Development

The developed algorithm has used the PSO approach as the main optimisation algorithm by updating the structure of the PSO according to the needs and conditions of the problem. Due to the non-linearity and computational complexity established mathematical model has been solved by using the PSO algorithm. Thus, the developed algorithm in this study is the adjusted version of the well-known PSO approach.

The two objective functions which are minimizing the total fragmentation and minimizing the maximum fragmentation ratio have been served as the fitness functions of the PSO algorithm. The most important part of the algorithm is the particle generation approach. Algorithm checks all the constraints and if all the constraints are satisfied then a particle has been generated . The constraints have not been added to the objective functions as penalties. Constraints have been used to create feasible particles for the algorithm. Moreover, constraints have been checked for every iteration and particle feasibility has been preserved in every generation by this algorithm. The algorithm starts with creating the initial population (swarm). All particles (individuals) have a position vector which stores all the decision variables of the mathematical model. Every particle stores its personal best value in their memory and swarm holds the global best value. After that, by using the well-known velocity and position equations of the PSO method, velocity and position computed and updated in every iteration. Then, algorithm calculates each particle`s fitness function value by using the position vector and updates the results of the personal best and the global best. Lastly, the algorithm has been run for the two fitness functions separately to optimize both objective functions and the outcomes have been published and evaluated in the results section of the report.

Adaptive parameter approach and restart mechanism have been implemented to this algorithm as the main enhancements. In the algorithm dynamic inertia, cognitive coefficient and social coefficient have been used. Inertia starts with 0.9 and gradually decreases to 0.4. This method enables avoid tackling local optima and increases the probability of reaching the global optima. With high inertia in the beginning, particles are able to do large-scale search and this prevents them getting stuck in local optima. Moreover, with lower inertia particles are able to do more precise search at the end. Furthermore, cognitive coefficient value starts with 2.5 and gradually decreases to 1.5. On the flip side, social coefficient value starts with 1.5 and gradually increases to 2.5. This method enhances exploration first and then focuses on exploitation. In addition, in the beginning of the algorithm it restarts itself in every 20 iterations to prevent getting stuck and at the end it restarts itself in every 50 iterations to focus on exploitation. These enhancements have improved the search mechanism of the developed method.

Selected PSO Algorithm Parameters:		
Swarm Size: 300	Inertia (W): 0.9 to 0.4	Cognitive Coefficient (C1): 2.5 to 1.5
Max Iteration: 500	Restart Threshold: 20 to 50	Social Coefficient (C2): 1.5 to 2.5

3. Results

3.1 Theoretical / Numerical Results and Analysis

Data Generation Approach Results:

```
🚀 Zone-Based Grid Hardness Generator
==================================================

📊 Generation Results:
   Soft         : 14 holes
   Medium       : 30 holes
   Hard         : 121 holes
   Extra Hard   : 72 holes

✅ Total: 237 holes generated from 20x20 possible positions
📏 Grid Parameters:
   Grid Size: 200m x 200m
   Burden: 10.0m (FIXED)
   Spacing: 10.0m (FIXED)
   Rows: 20 (calculated from 200/10.0)
   Columns: 20 (calculated from 200/10.0)
   Possible Positions: 400
   Hole Generation: Random (60% probability)
   Hardness Assignment: Percentage-based areas
🎯 Hardness Areas: Soft(5%), Medium(15%), Hard(50%), Extra Hard(30%)
```

Figure 5. Data generation results.

As shown in figure 5 above, the developed data generation algorithm has provided a natural and desired grid structure to perform drill and blast operations. As displayed in figure 5, 237 holes have been generated within the desired hardness zones. Totally 14 soft holes, 30 medium holes, 121 hard holes and 72 extra hard holes have been generated. Moreover, as demonstrated in figure 5, probabilistic approach for hole generation has been implemented efficiently. Furthermore, grid parameters have been applied correctly; therefore, a solid bench design has been obtained. In addition, visual representation of the generated bench has been provided in figure 6 and heatmap for the obtained hardness values has been shown in figure 7. Lastly, 3D visualization of the bench and the generated entire dataset have been provided in the appendix section of the report.

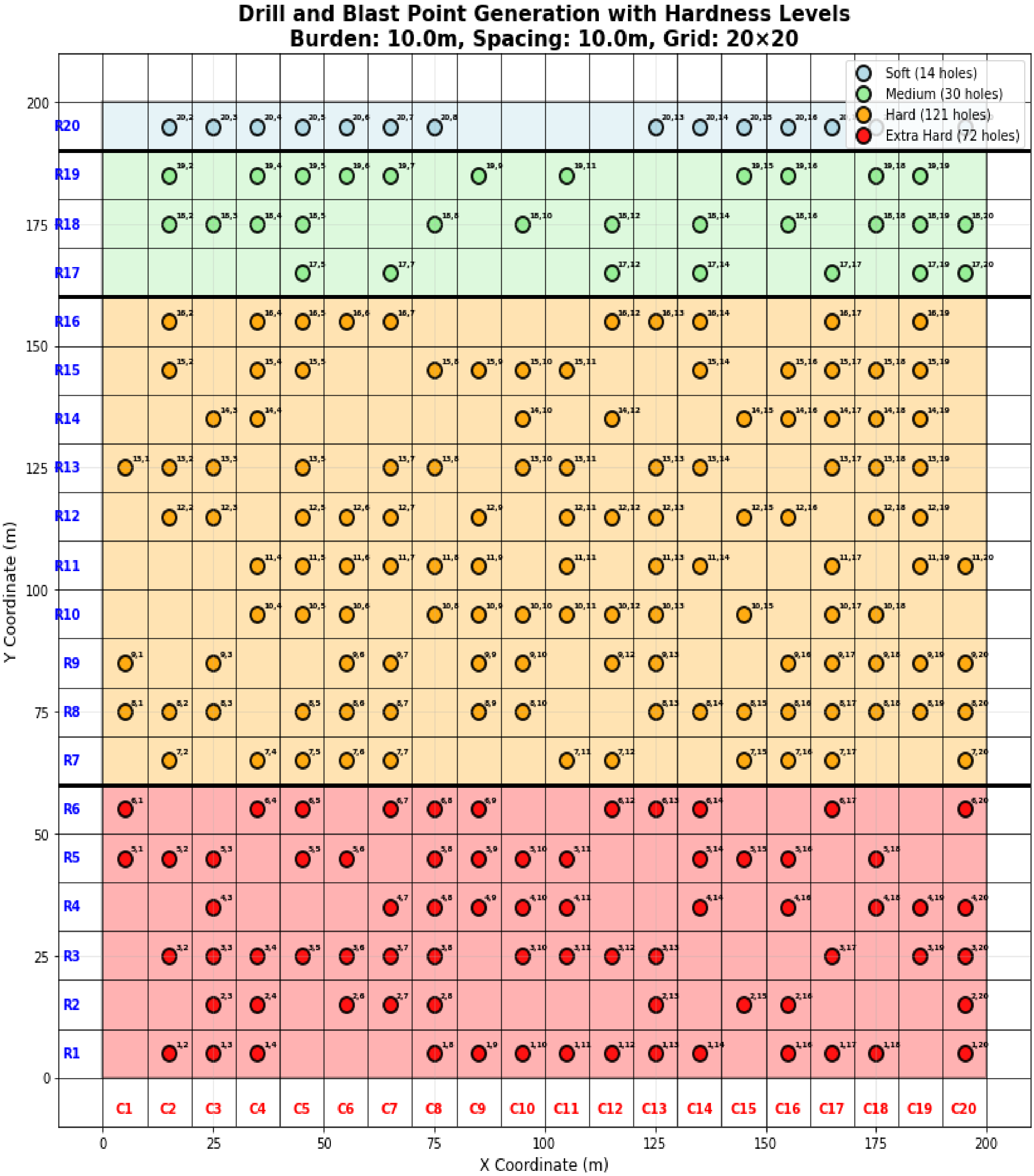


Figure 6. Visualization of the bench.

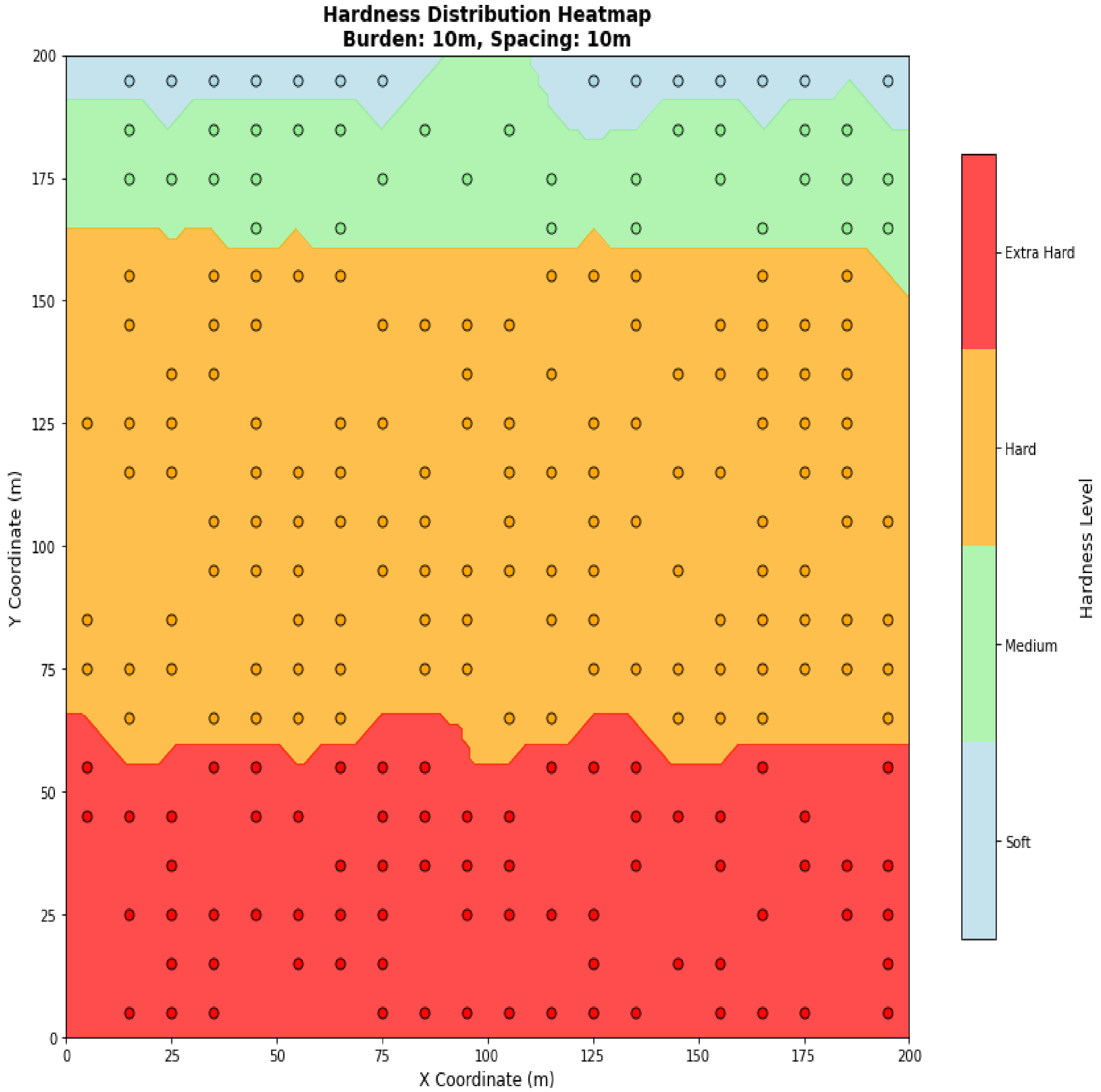


Figure 7. Heatmap of the generated hardness distribution.

Algorithm Development Results:

Minimizing the Total Fragmentation Results:

The first fitness function results of the developed PSO-based new algorithm are presented comprehensively in figure 8 and 9. Figure 8 shows the overall performance of the algorithm by using a line graph to visualize the fitness values over iterations. Moreover, figure 9 presents the outcome of the executed code in detail.

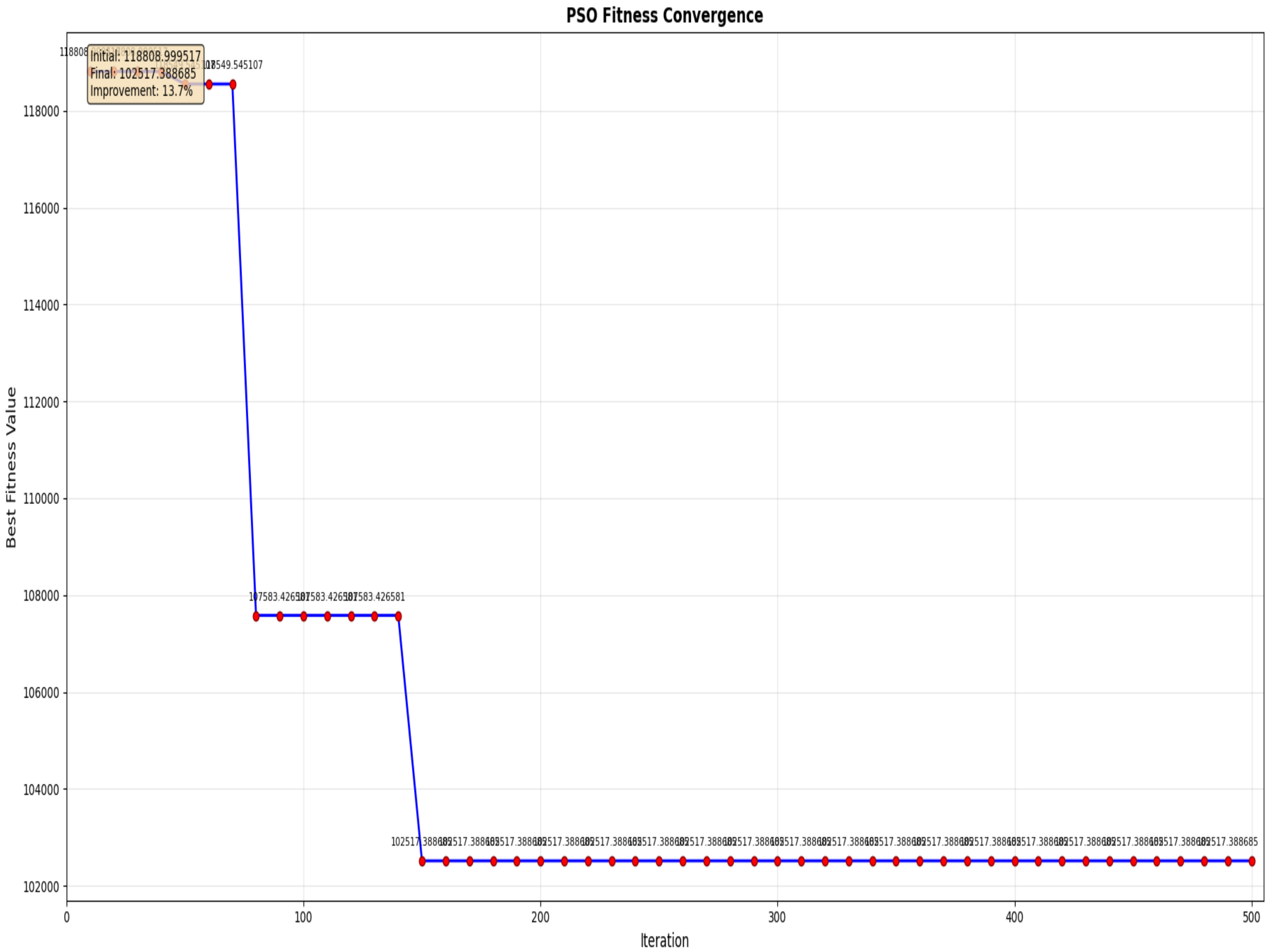


Figure 8. Line graph for fitness progression of total fragmentation objective.

The algorithm has been executed with 300 swarm size and 500 iteration parameters and 50 cm target particle size value. As shown in the line graph in figure 8 above, the algorithm initially found the fitness value as 118,809 for the initial population. After that, the fitness value has shown a gradual decline to 102,517 with an improvement of % 13.7. 102,517 shows the total fragmentation value of the 237 generated holes in terms of centimeters. Lastly, this total fragmentation optimization plays a crucial role for the open-pit mines and provides significant recommendations to the mining professionals to re-consider parameters such as type of explosive product, strength of the explosive product, hole depth and hole diameter for minimizing the total fragmentation efficiently.

```
PSO completed


Best solution found:
['1.000000', '1.000000', '1.000000', '1.000000', '0.458970', '1.000000', '0.630416', '1.000000', '1.000000', '1.000000']
✅ Generated feasible particle (attempt 1)
fitness of best solution = 102517.388685
Solution feasible: True

📊 Detailed Fitness Progression:
==================================================
Iter = 10 best fitness = 118809.000 (Initial)
Iter = 20 best fitness = 118809.000 (Improvement: +0.0%)
Iter = 30 best fitness = 118809.000 (Improvement: +0.0%)
Iter = 40 best fitness = 118809.000 (Improvement: +0.0%)
Iter = 50 best fitness = 118549.545 (Improvement: +0.2%)
Iter = 60 best fitness = 118549.545 (Improvement: +0.0%)
Iter = 70 best fitness = 118549.545 (Improvement: +0.0%)
Iter = 80 best fitness = 107583.427 (Improvement: +9.3%)
Iter = 90 best fitness = 107583.427 (Improvement: +0.0%)
Iter = 100 best fitness = 107583.427 (Improvement: +0.0%)
Iter = 110 best fitness = 107583.427 (Improvement: +0.0%)
Iter = 120 best fitness = 107583.427 (Improvement: +0.0%)
Iter = 130 best fitness = 107583.427 (Improvement: +0.0%)
Iter = 140 best fitness = 107583.427 (Improvement: +0.0%)
Iter = 150 best fitness = 102517.389 (Improvement: +4.7%)
Iter = 160 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 170 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 180 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 190 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 200 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 210 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 220 best fitness = 102517.389 (Improvement: +0.0%)
```

```
Iter = 230 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 240 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 250 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 260 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 270 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 280 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 290 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 300 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 310 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 320 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 330 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 340 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 350 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 360 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 370 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 380 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 390 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 400 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 410 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 420 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 430 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 440 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 450 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 460 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 470 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 480 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 490 best fitness = 102517.389 (Improvement: +0.0%)
Iter = 500 best fitness = 102517.389 (Final)
```

Figure 9. Results of the executed code for the total fragmentation objective.

As presented in figure 9 above, over the iterations fitness value has been showed a significant progress. The main reason for this improvement is implementing adaptive parameters and restarting mechanism approach to the algorithm. Those enhancements have increased the probability of reaching the global optima.

Minimizing the Maximum Fragmentation Ratio Results:

```
PSO completed

Best solution found:
['0.961139', '1.000000', '0.976145', '0.995082', '0.126828', '1.000000', '0.817004', '0.984848', '0.996473', '0.858693']
✅ Generated feasible particle (attempt 1)
fitness of best solution = 35.116666
Solution feasible: True

📊 Detailed Fitness Progression:
==================================================
Iter = 10 best fitness = 40.277 (Initial)
Iter = 20 best fitness = 40.277 (Improvement: +0.0%)
Iter = 30 best fitness = 40.277 (Improvement: +0.0%)
Iter = 40 best fitness = 39.929 (Improvement: +0.9%)
Iter = 50 best fitness = 39.929 (Improvement: +0.0%)
Iter = 60 best fitness = 39.929 (Improvement: +0.0%)
Iter = 70 best fitness = 39.929 (Improvement: +0.0%)
Iter = 80 best fitness = 39.929 (Improvement: +0.0%)
Iter = 90 best fitness = 39.929 (Improvement: +0.0%)
Iter = 100 best fitness = 39.929 (Improvement: +0.0%)
Iter = 110 best fitness = 39.929 (Improvement: +0.0%)
Iter = 120 best fitness = 39.929 (Improvement: +0.0%)
Iter = 130 best fitness = 39.929 (Improvement: +0.0%)
Iter = 140 best fitness = 39.929 (Improvement: +0.0%)
Iter = 150 best fitness = 39.058 (Improvement: +2.2%)
Iter = 160 best fitness = 39.058 (Improvement: +0.0%)
Iter = 170 best fitness = 39.058 (Improvement: +0.0%)
Iter = 180 best fitness = 39.058 (Improvement: +0.0%)
Iter = 190 best fitness = 39.058 (Improvement: +0.0%)
Iter = 200 best fitness = 39.058 (Improvement: +0.0%)
Iter = 210 best fitness = 39.058 (Improvement: +0.0%)
Iter = 220 best fitness = 39.058 (Improvement: +0.0%)
```

```
Iter = 230 best fitness = 39.058 (Improvement: +0.0%)
Iter = 240 best fitness = 37.549 (Improvement: +3.9%)
Iter = 250 best fitness = 37.506 (Improvement: +0.1%)
Iter = 260 best fitness = 37.049 (Improvement: +1.2%)
Iter = 270 best fitness = 37.049 (Improvement: +0.0%)
Iter = 280 best fitness = 36.998 (Improvement: +0.1%)
Iter = 290 best fitness = 36.998 (Improvement: +0.0%)
Iter = 300 best fitness = 36.633 (Improvement: +1.0%)
Iter = 310 best fitness = 36.633 (Improvement: +0.0%)
Iter = 320 best fitness = 36.087 (Improvement: +1.5%)
Iter = 330 best fitness = 35.978 (Improvement: +0.3%)
Iter = 340 best fitness = 35.707 (Improvement: +0.8%)
Iter = 350 best fitness = 35.707 (Improvement: +0.0%)
Iter = 360 best fitness = 35.117 (Improvement: +1.7%)
Iter = 370 best fitness = 35.117 (Improvement: +0.0%)
Iter = 380 best fitness = 35.117 (Improvement: +0.0%)
Iter = 390 best fitness = 35.117 (Improvement: +0.0%)
Iter = 400 best fitness = 35.117 (Improvement: +0.0%)
Iter = 410 best fitness = 35.117 (Improvement: +0.0%)
Iter = 420 best fitness = 35.117 (Improvement: +0.0%)
Iter = 430 best fitness = 35.117 (Improvement: +0.0%)
Iter = 440 best fitness = 35.117 (Improvement: +0.0%)
Iter = 450 best fitness = 35.117 (Improvement: +0.0%)
Iter = 460 best fitness = 35.117 (Improvement: +0.0%)
Iter = 470 best fitness = 35.117 (Improvement: +0.0%)
Iter = 480 best fitness = 35.117 (Improvement: +0.0%)
Iter = 490 best fitness = 35.117 (Improvement: +0.0%)
Iter = 500 best fitness = 35.117 (Final)
```

```
📊 Showing deviation analysis for existing holes only...

🎯 DEVIATION ANALYSIS SUMMARY:
   Target Particle Size: 50.0 cm
   Max Deviation Ratio: 35.117 (3511.7%)
   Mean Deviation Ratio: 16.060 (1606.0%)
   Total Existing Holes: 237
   Grid Size: 20 x 20 = 400 possible positions
   Hole Generation Rate: 59.2%

End particle swarm for drill and blast optimization
```

Figure 10. Results of the executed code for the fragmentation ratio objective.

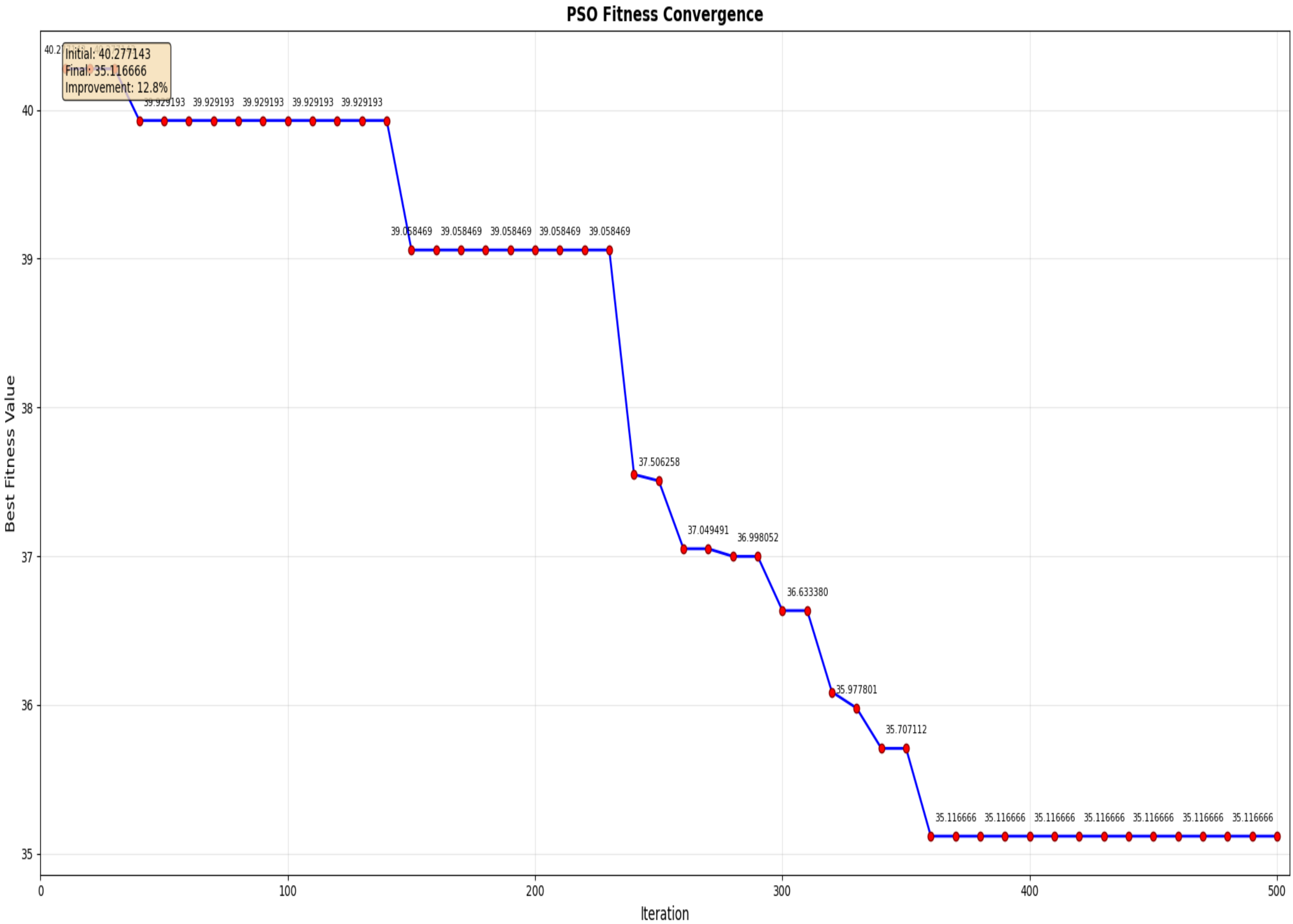


Figure 11. Line graph for fitness progression of fragmentation ratio objective.

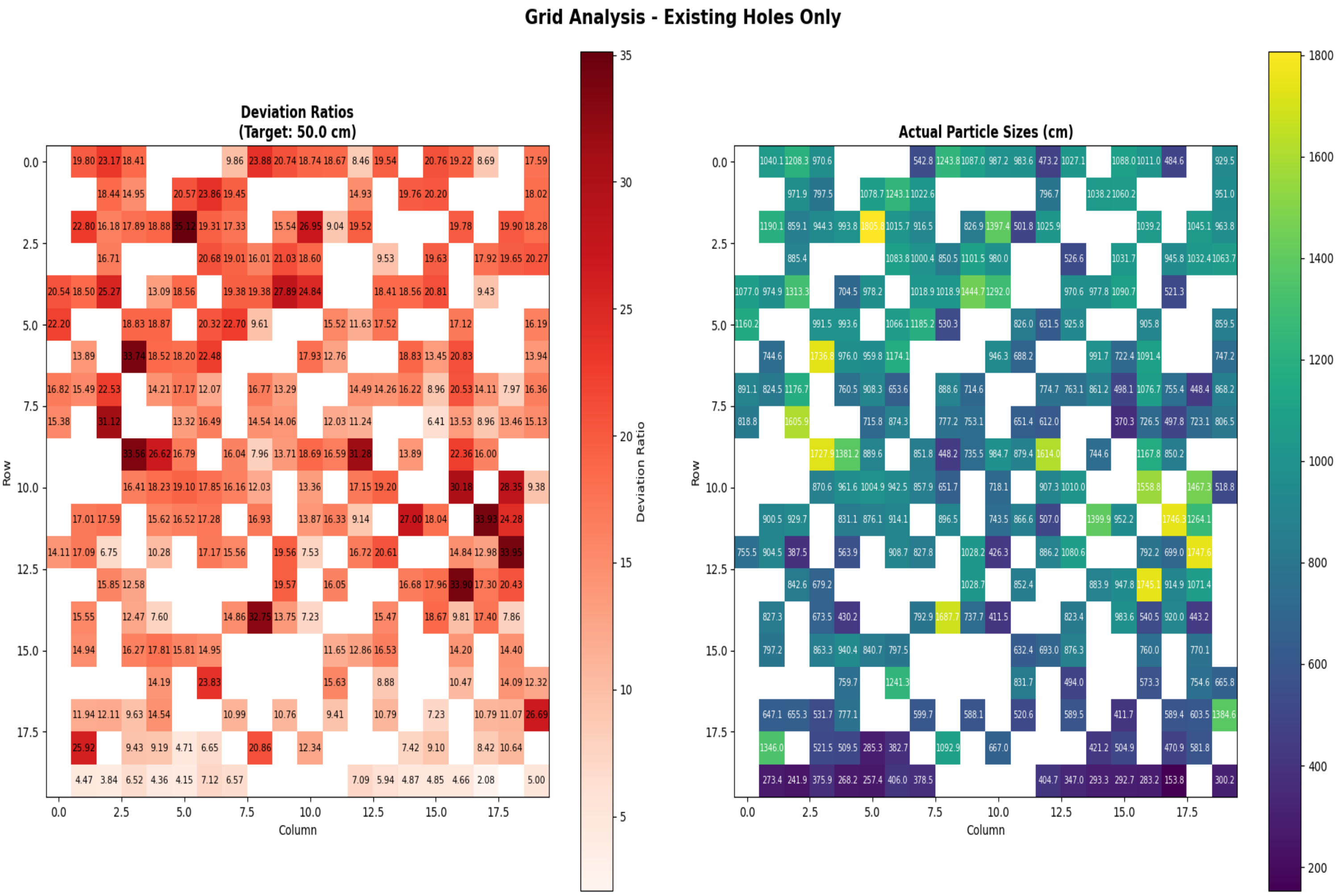


Figure 12. Grid analysis per each hole for fragmentation ratio objective.

As presented in figure 10 and 11, fitness value has been showed an impressive improvement. Adaptive parameter and restart mechanism approached have shown its impact on minimizing the maximum fragmentation ratio per hole as well. Initially it started with 40.277, and it has demonstrated significant development by reducing the maximum deviation ratio to 35.117 which corresponds to %12.8 improvement. Furthermore, figure 12 has provided a visual grid analysis to the readers for providing the deviation ratios and actual particle sizes for each hole. This grid visualization has been added to the study to provide meaningful recommendations about timing and pattern designs. Mining experts can analyze the fragmentation deviation ratio for each hole and can generate efficient blasting sequences. Lastly, minimizing the maximum fragmentation ratio is very crucial for open-pit mines because it can be used to optimize the cost components and environmental footprint of mine. Mining professionals can use these results to enhance the scope of the study.

4. Conclusion and Future Work

4.1 Significance of Findings

The project has presented a mathematical model to minimize certain fragmentation objectives such as minimizing the total fragmentation and minimizing the maximum fragmentation ratio to the readers. Moreover, Particle Swarm Optimisation Algorithm has been introduced comprehensively to the audience and a new novel methodology has been developed by using the PSO algorithm. In addition, this new algorithm has shown efficient results for improving the drill and blast design patterns in the open-pit mines. Furthermore, optimizing the fragmentation has huge impacts on the overall productivity of the open-pit mines. By minimizing the fragmentation total cost of the open-pit mines can be reduced, environmental footprints can be mitigated and loading and hauling operations can be accelerated. Thus, this research process consists of crucial outcomes and provides significant insights into the mining professionals.

4.2 Limitations

There are various limitations in this study. Firstly, the main article hasn’t provided any input parameters and any kind of data generation approach. Moreover, main article has used some input parameters from its previous studies and it has generated some data by using previously drilled holes in its previous studies. Furthermore, only the bench height was given in the article the other area parameters such as coordinates, number of rows, number of columns, burden and spacing were unknown. To overcome those issues a synthetic data generation algorithm has been developed and for facilitating the grid area calculations burden and spacing values have been fixed. In addition, this study has not considered the possibility of mineral occurrence. It is assumed that every hole generated has minerals inside. Lastly, this study hasn’t addressed the timing design and initiation operations.

4.3 Recommendations and Future Work

As indicated in the Project Proposal and Progress Report, Simulated Annealing algorithm was planned to be implemented, but Particle Swarm Optimisation algorithm has been implemented instead. Simulated Annealing algorithm can be designed according to the structure and conditions of the problem and it can be tested to provide meaningful comparisons between two algorithms. Moreover, in the current PSO based algorithm swarm size and number of iterations are fixed. Rather than fixing those PSO parameters, parameter tuning can be implemented into the code and the algorithm can be run with the optimal PSO parameters. Furthermore, other objective functions such as dig efficiency can be integrated into the algorithm and the scope of the study can be enhanced. In addition, rather than testing the developed PSO based algorithm with synthetic data, a real dataset can be gathered by industry connections and algorithm can be tested with an actual dataset to understand the efficiency of the developed methodology. Moreover, the developed algorithm can be tested on a computer with better Ram and CPU configurations. Additionally, rather than using Python as the programming language, complied languages such as C++ and C# can be used as the programming language to increase the speed. Lastly, to reduce the code complexity and execution speed, code of the developed algorithm can be shared with a computer scientist to implement methods such as multi-processing, multi-threading and data structures and algorithms focused revisions.

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6. Appendix

Hole_ID	X_Coordinate	Y_Coordinate	Row	Column	Hardness_Level	Hardness_Name	Hardness_Value
H_001	15	5	1	2	4	Extra Hard	3,78
H_002	25	5	1	3	4	Extra Hard	3,69
H_003	35	5	1	4	4	Extra Hard	3,79
H_004	75	5	1	8	4	Extra Hard	3,96
H_005	85	5	1	9	4	Extra Hard	3,92
H_006	95	5	1	10	4	Extra Hard	3,67
H_007	105	5	1	11	4	Extra Hard	3,63
H_008	115	5	1	12	4	Extra Hard	3,81
H_009	125	5	1	13	4	Extra Hard	3,85
H_010	135	5	1	14	4	Extra Hard	3,73
H_011	155	5	1	16	4	Extra Hard	3,93
H_012	165	5	1	17	4	Extra Hard	3,9
H_013	175	5	1	18	4	Extra Hard	3,87
H_014	195	5	1	20	4	Extra Hard	3,69
H_015	25	15	2	3	4	Extra Hard	3,68
H_016	35	15	2	4	4	Extra Hard	3,61
H_017	55	15	2	6	4	Extra Hard	3,7
H_018	65	15	2	7	4	Extra Hard	3,79
H_019	75	15	2	8	4	Extra Hard	3,94
H_020	125	15	2	13	4	Extra Hard	3,63
H_021	145	15	2	15	4	Extra Hard	3,77
H_022	155	15	2	16	4	Extra Hard	3,85
H_023	195	15	2	20	4	Extra Hard	3,68
H_024	15	25	3	2	4	Extra Hard	3,88
H_025	25	25	3	3	4	Extra Hard	3,8
H_026	35	25	3	4	4	Extra Hard	3,7
H_027	45	25	3	5	4	Extra Hard	3,86
H_028	55	25	3	6	4	Extra Hard	3,6
H_029	65	25	3	7	4	Extra Hard	3,9
H_030	75	25	3	8	4	Extra Hard	3,91
H_031	95	25	3	10	4	Extra Hard	3,64
H_032	105	25	3	11	4	Extra Hard	3,77
H_033	115	25	3	12	4	Extra Hard	3,67
H_034	125	25	3	13	4	Extra Hard	3,98
H_035	165	25	3	17	4	Extra Hard	3,81
H_036	185	25	3	19	4	Extra Hard	3,62

Hole_Data

	A	B	C	D	E	F	G	H
38	H_037	195	25	3	20	4	Extra Hard	3,7
39	H_038	25	35	4	3	4	Extra Hard	3,94
40	H_039	65	35	4	7	4	Extra Hard	3,78
41	H_040	75	35	4	8	4	Extra Hard	3,92
42	H_041	85	35	4	9	4	Extra Hard	3,87
43	H_042	95	35	4	10	4	Extra Hard	4
44	H_043	105	35	4	11	4	Extra Hard	3,84
45	H_044	135	35	4	14	4	Extra Hard	3,98
46	H_045	155	35	4	16	4	Extra Hard	3,96
47	H_046	175	35	4	18	4	Extra Hard	3,85
48	H_047	185	35	4	19	4	Extra Hard	3,89
49	H_048	195	35	4	20	4	Extra Hard	3,8
50	H_049	5	45	5	1	4	Extra Hard	3,93
51	H_050	15	45	5	2	4	Extra Hard	3,82
52	H_051	25	45	5	3	4	Extra Hard	3,96
53	H_052	45	45	5	5	4	Extra Hard	3,9
54	H_053	55	45	5	6	4	Extra Hard	3,79
55	H_054	75	45	5	8	4	Extra Hard	3,7
56	H_055	85	45	5	9	4	Extra Hard	3,7
57	H_056	95	45	5	10	4	Extra Hard	3,86
58	H_057	105	45	5	11	4	Extra Hard	3,91
59	H_058	135	45	5	14	4	Extra Hard	3,81
60	H_059	145	45	5	15	4	Extra Hard	3,85
61	H_060	155	45	5	16	4	Extra Hard	3,71
62	H_061	175	45	5	18	4	Extra Hard	3,63
63	H_062	5	55	6	1	4	Extra Hard	3,71
64	H_063	35	55	6	4	4	Extra Hard	3,71
65	H_064	45	55	6	5	4	Extra Hard	3,73
66	H_065	65	55	6	7	4	Extra Hard	3,82
67	H_066	75	55	6	8	4	Extra Hard	3,66
68	H_067	85	55	6	9	4	Extra Hard	3,69
69	H_068	115	55	6	12	4	Extra Hard	3,88
70	H_069	125	55	6	13	4	Extra Hard	3,88
71	H_070	135	55	6	14	4	Extra Hard	3,63
72	H_071	165	55	6	17	4	Extra Hard	3,76
73	H_072	195	55	6	20	4	Extra Hard	3,82
74	H_073	15	65	7	2	3	Hard	2,97

Hole_Data +

	A	B	C	D	E	F	G	H
75	H_074	35	65	7	4	3	Hard	2,79
76	H_075	45	65	7	5	3	Hard	2,98
77	H_076	55	65	7	6	3	Hard	3,41
78	H_077	65	65	7	7	3	Hard	3,13
79	H_078	105	65	7	11	3	Hard	3,23
80	H_079	115	65	7	12	3	Hard	3,37
81	H_080	145	65	7	15	3	Hard	3,29
82	H_081	155	65	7	16	3	Hard	2,94
83	H_082	165	65	7	17	3	Hard	2,61
84	H_083	195	65	7	20	3	Hard	2,92
85	H_084	5	75	8	1	3	Hard	3,28
86	H_085	15	75	8	2	3	Hard	3,37
87	H_086	25	75	8	3	3	Hard	3,46
88	H_087	45	75	8	5	3	Hard	2,98
89	H_088	55	75	8	6	3	Hard	3,27
90	H_089	65	75	8	7	3	Hard	3,09
91	H_090	85	75	8	9	3	Hard	3,14
92	H_091	95	75	8	10	3	Hard	2,8
93	H_092	125	75	8	13	3	Hard	2,8
94	H_093	135	75	8	14	3	Hard	2,99
95	H_094	145	75	8	15	3	Hard	2,63
96	H_095	155	75	8	16	3	Hard	2,9
97	H_096	165	75	8	17	3	Hard	3,21
98	H_097	175	75	8	18	3	Hard	2,96
99	H_098	185	75	8	19	3	Hard	2,75
100	H_099	195	75	8	20	3	Hard	3,02
101	H_100	5	85	9	1	3	Hard	2,71
102	H_101	25	85	9	3	3	Hard	3,16
103	H_102	55	85	9	6	3	Hard	2,62
104	H_103	65	85	9	7	3	Hard	2,95
105	H_104	85	85	9	9	3	Hard	3,11
106	H_105	95	85	9	10	3	Hard	2,62
107	H_106	115	85	9	12	3	Hard	3,18
108	H_107	125	85	9	13	3	Hard	2,72
109	H_108	155	85	9	16	3	Hard	3,02
110	H_109	165	85	9	17	3	Hard	2,65
111	H_110	175	85	9	18	3	Hard	2,94

Hole_Data +

	A	B	C	D	E	F	G	H
112	H_111	185	85	9	19	3	Hard	2,79
113	H_112	195	85	9	20	3	Hard	2,89
114	H_113	35	95	10	4	3	Hard	3,29
115	H_114	45	95	10	5	3	Hard	2,94
116	H_115	55	95	10	6	3	Hard	3,28
117	H_116	75	95	10	8	3	Hard	3,35
118	H_117	85	95	10	9	3	Hard	2,83
119	H_118	95	95	10	10	3	Hard	2,67
120	H_119	105	95	10	11	3	Hard	2,62
121	H_120	115	95	10	12	3	Hard	3,09
122	H_121	125	95	10	13	3	Hard	3,5
123	H_122	145	95	10	15	3	Hard	2,91
124	H_123	165	95	10	17	3	Hard	3,19
125	H_124	175	95	10	18	3	Hard	3,3
126	H_125	35	105	11	4	3	Hard	3,19
127	H_126	45	105	11	5	3	Hard	3,28
128	H_127	55	105	11	6	3	Hard	3,45
129	H_128	65	105	11	7	3	Hard	2,78
130	H_129	75	105	11	8	3	Hard	2,62
131	H_130	85	105	11	9	3	Hard	2,74
132	H_131	105	105	11	11	3	Hard	2,71
133	H_132	125	105	11	13	3	Hard	3,2
134	H_133	135	105	11	14	3	Hard	3,11
135	H_134	165	105	11	17	3	Hard	2,8
136	H_135	185	105	11	19	3	Hard	3,23
137	H_136	195	105	11	20	3	Hard	3,29
138	H_137	15	115	12	2	3	Hard	2,75
139	H_138	25	115	12	3	3	Hard	3,15
140	H_139	45	115	12	5	3	Hard	3,27
141	H_140	55	115	12	6	3	Hard	2,7
142	H_141	65	115	12	7	3	Hard	3,34
143	H_142	85	115	12	9	3	Hard	3,47
144	H_143	105	115	12	11	3	Hard	2,7
145	H_144	115	115	12	12	3	Hard	2,62
146	H_145	125	115	12	13	3	Hard	2,88
147	H_146	145	115	12	15	3	Hard	3,21
148	H_147	155	115	12	16	3	Hard	3,46

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	A	B	C	D	E	F	G	H
149	H_148	175	115	12	18	3	Hard	2,96
150	H_149	185	115	12	19	3	Hard	3,24
151	H_150	5	125	13	1	3	Hard	2,67
152	H_151	15	125	13	2	3	Hard	3,22
153	H_152	25	125	13	3	3	Hard	3,16
154	H_153	45	125	13	5	3	Hard	2,69
155	H_154	65	125	13	7	3	Hard	3,3
156	H_155	75	125	13	8	3	Hard	3,37
157	H_156	95	125	13	10	3	Hard	3,14
158	H_157	105	125	13	11	3	Hard	2,71
159	H_158	125	125	13	13	3	Hard	3,49
160	H_159	135	125	13	14	3	Hard	3,3
161	H_160	165	125	13	17	3	Hard	2,91
162	H_161	175	125	13	18	3	Hard	2,99
163	H_162	185	125	13	19	3	Hard	2,93
164	H_163	25	135	14	3	3	Hard	3,06
165	H_164	35	135	14	4	3	Hard	2,91
166	H_165	95	135	14	10	3	Hard	3,36
167	H_166	115	135	14	12	3	Hard	3,34
168	H_167	145	135	14	15	3	Hard	2,69
169	H_168	155	135	14	16	3	Hard	3,46
170	H_169	165	135	14	17	3	Hard	3,17
171	H_170	175	135	14	18	3	Hard	3,35
172	H_171	185	135	14	19	3	Hard	3,24
173	H_172	15	145	15	2	3	Hard	2,99
174	H_173	35	145	15	4	3	Hard	3,26
175	H_174	45	145	15	5	3	Hard	3,47
176	H_175	75	145	15	8	3	Hard	2,84
177	H_176	85	145	15	9	3	Hard	3,33
178	H_177	95	145	15	10	3	Hard	3,08
179	H_178	105	145	15	11	3	Hard	3,04
180	H_179	135	145	15	14	3	Hard	2,99
181	H_180	155	145	15	16	3	Hard	3,26
182	H_181	165	145	15	17	3	Hard	2,84
183	H_182	175	145	15	18	3	Hard	3,37
184	H_183	185	145	15	19	3	Hard	3,35
185	H_184	15	155	16	2	3	Hard	2,68

Hole_Data

	A	B	C	D	E	F	G	H
186	H_185	35	155	16	4	3	Hard	3,39
187	H_186	45	155	16	5	3	Hard	2,82
188	H_187	55	155	16	6	3	Hard	3,02
189	H_188	65	155	16	7	3	Hard	3,15
190	H_189	115	155	16	12	3	Hard	2,94
191	H_190	125	155	16	13	3	Hard	2,63
192	H_191	135	155	16	14	3	Hard	3,37
193	H_192	165	155	16	17	3	Hard	2,76
194	H_193	185	155	16	19	3	Hard	2,79
195	H_194	45	165	17	5	2	Medium	2,32
196	H_195	65	165	17	7	2	Medium	1,91
197	H_196	115	165	17	12	2	Medium	2,39
198	H_197	135	165	17	14	2	Medium	2,23
199	H_198	165	165	17	17	2	Medium	1,85
200	H_199	185	165	17	19	2	Medium	1,61
201	H_200	195	165	17	20	2	Medium	2,45
202	H_201	15	175	18	2	2	Medium	1,68
203	H_202	25	175	18	3	2	Medium	2,25
204	H_203	35	175	18	4	2	Medium	2,04
205	H_204	45	175	18	5	2	Medium	2,28
206	H_205	75	175	18	8	2	Medium	2,22
207	H_206	95	175	18	10	2	Medium	2,18
208	H_207	115	175	18	12	2	Medium	2,04
209	H_208	135	175	18	14	2	Medium	2,31
210	H_209	155	175	18	16	2	Medium	1,68
211	H_210	175	175	18	18	2	Medium	1,8
212	H_211	185	175	18	19	2	Medium	2,22
213	H_212	195	175	18	20	2	Medium	1,88
214	H_213	15	185	19	2	2	Medium	2,12
215	H_214	35	185	19	4	2	Medium	2,03
216	H_215	45	185	19	5	2	Medium	2,08
217	H_216	55	185	19	6	2	Medium	1,98
218	H_217	65	185	19	7	2	Medium	2,27
219	H_218	85	185	19	9	2	Medium	1,9
220	H_219	105	185	19	11	2	Medium	2,23
221	H_220	145	185	19	15	2	Medium	1,84
222	H_221	155	185	19	16	2	Medium	1,83

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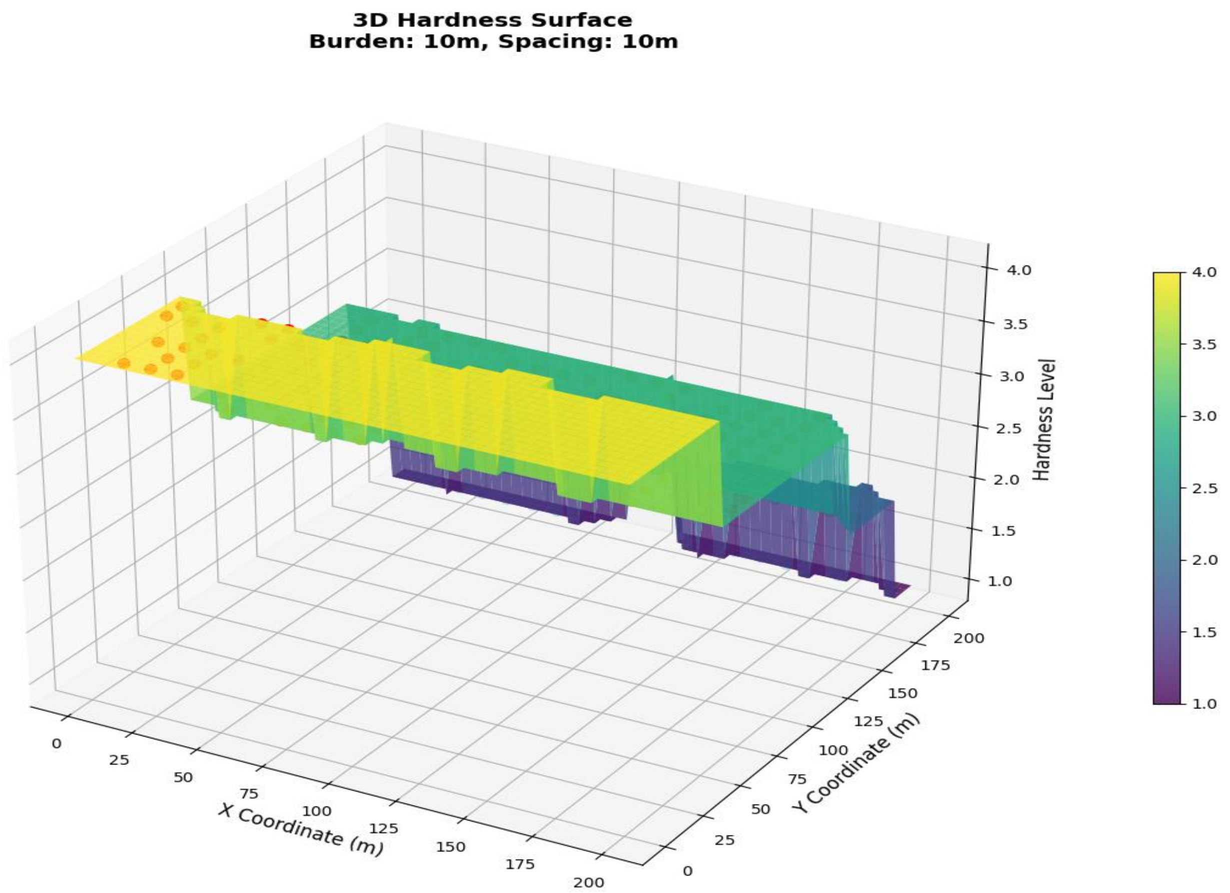
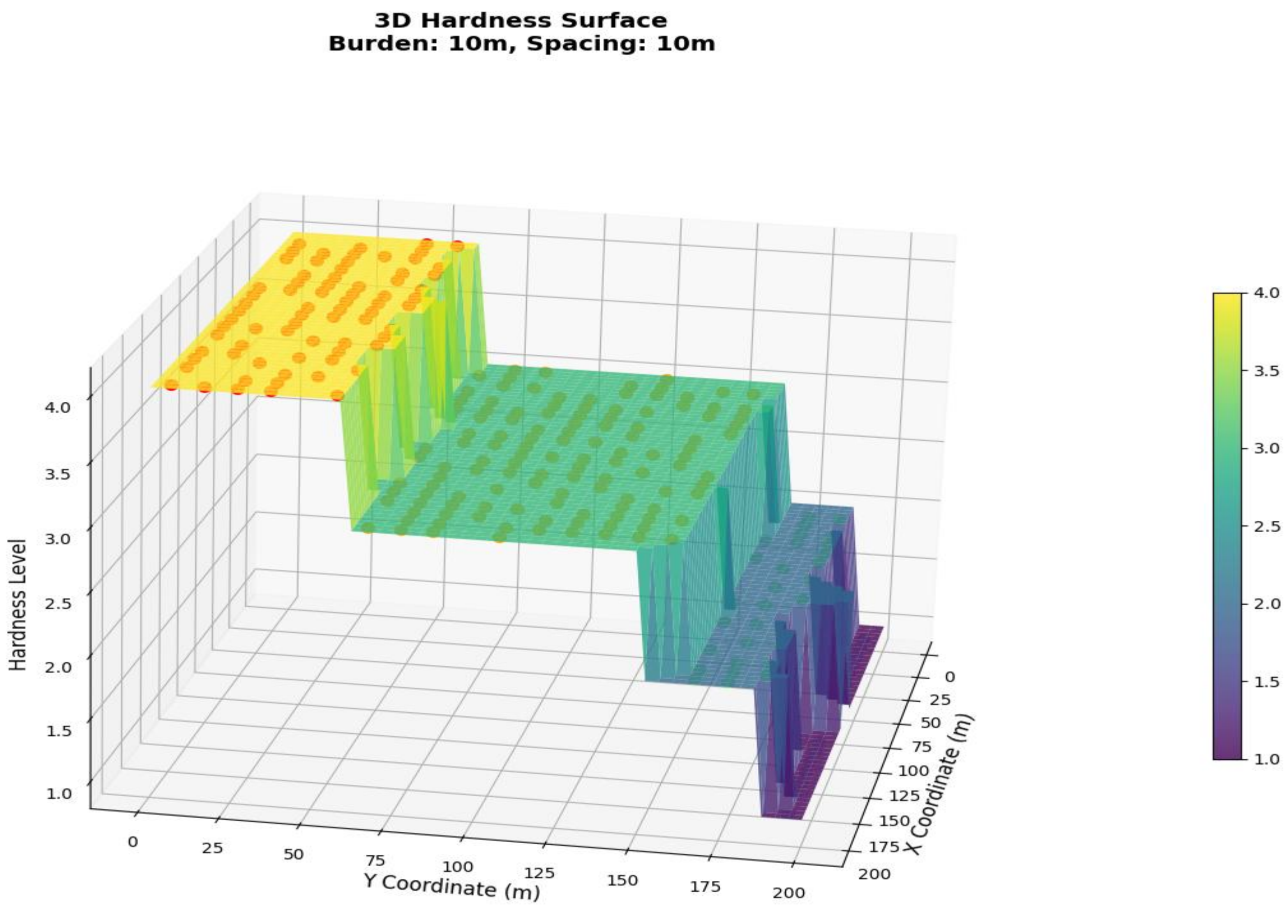
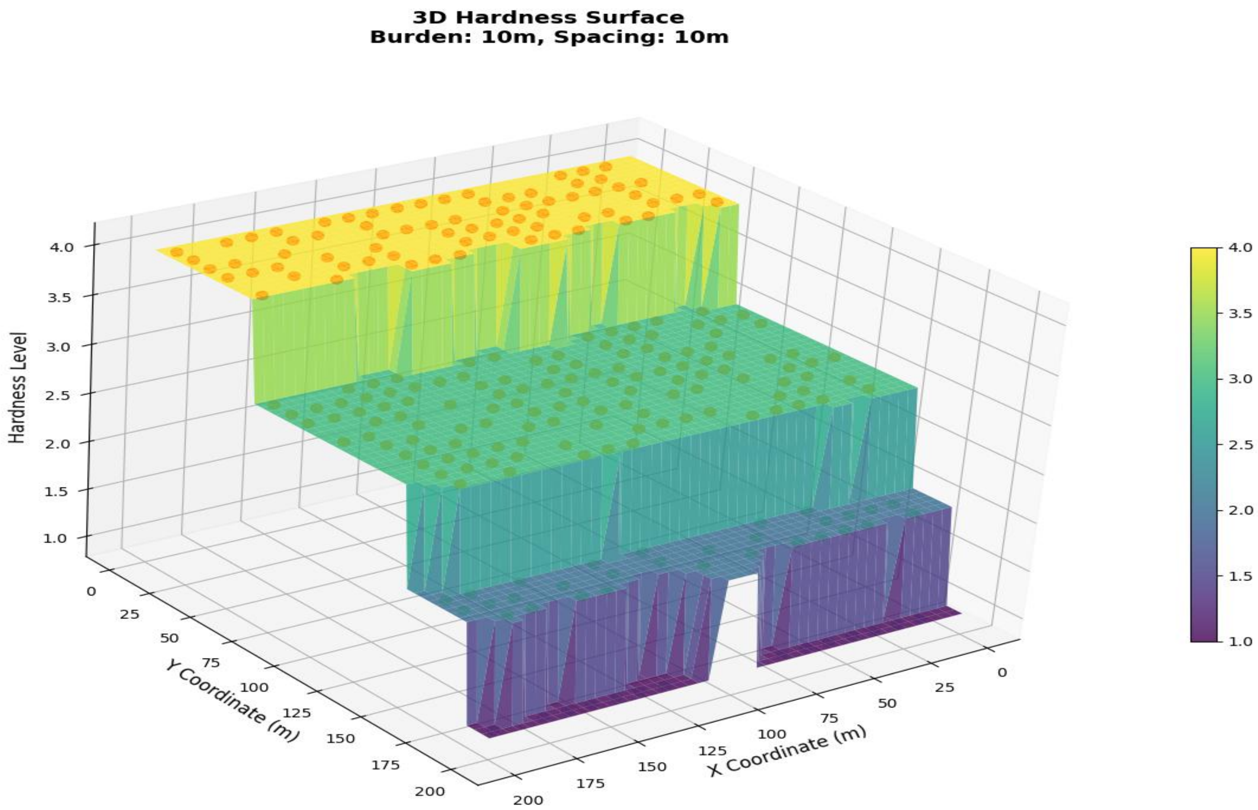
223	H_222	175	185	19	18	2	Medium	1,71
224	H_223	185	185	19	19	2	Medium	1,77
225	H_224	15	195	20	2	1	Soft	1,06
226	H_225	25	195	20	3	1	Soft	1,27
227	H_226	35	195	20	4	1	Soft	1,38
228	H_227	45	195	20	5	1	Soft	1,09
229	H_228	55	195	20	6	1	Soft	1,11
230	H_229	65	195	20	7	1	Soft	1,24
231	H_230	75	195	20	8	1	Soft	1,36
232	H_231	125	195	20	13	1	Soft	1,49
233	H_232	135	195	20	14	1	Soft	1,26
234	H_233	145	195	20	15	1	Soft	1,14
235	H_234	155	195	20	16	1	Soft	1,05
236	H_235	165	195	20	17	1	Soft	1,1
237	H_236	175	195	20	18	1	Soft	1,11
238	H_237	195	195	20	20	1	Soft	1,09

Figure 13. Entire dataset created by the data generation algorithm.


3D Hardness Surface
Burden: 10m, Spacing: 10m
Hardness Level
X Coordinate (m)
Y Coordinate (m)


3D Hardness Surface
Burden: 10m, Spacing: 10m
Hardness Level
Y Coordinate (m)
X Coordinate (m)



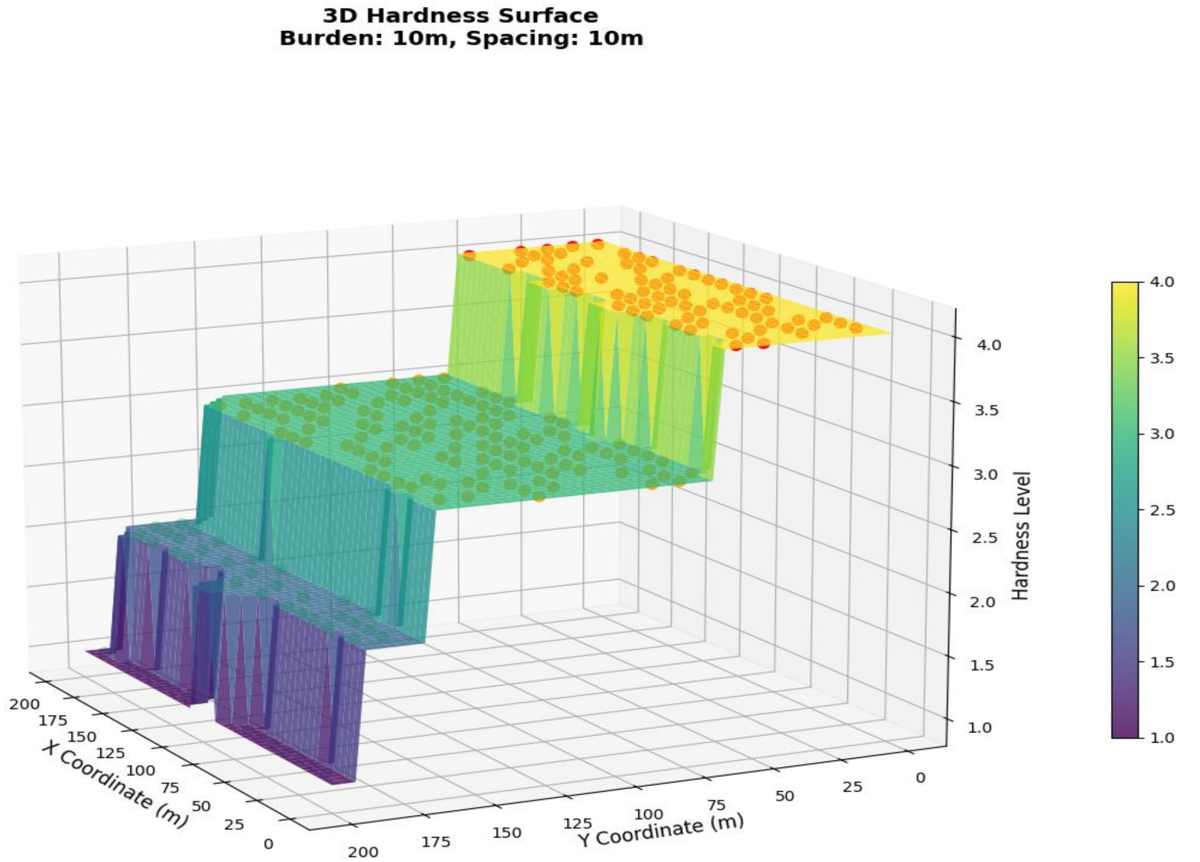


Figure 14. 3D Visualization of the generated bench.