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DRO: $\inf_{x \in X} \underbrace{\sup_{\mu \in \mathfrak{M}} E_\mu[\Psi(x,\xi)]}$

<u>inner problem</u>

Primal: $V_P := \sup_{\mu \in P(\Xi)} \left\{ \int_\Xi \Psi(\xi)\mu(d\xi) : W_p(\mu,\nu) \le \theta \right\}$

Dual: $V_D := \inf_{\lambda \ge 0} \left\{ \lambda\theta^p - \int_\Xi \inf_{\xi \in \Xi} [\lambda d^p(\xi,\zeta) - \Psi(\xi)\nu(d\zeta)] \right\}$

=> Finitely supported nominal dist

$$\nu = \frac{1}{N}\sum_{i=1}^N \delta_{\hat{\xi}^i}$$

$$V_P = V_D = \min_{\lambda \ge 0} \left\{ \lambda\theta^p + \frac{1}{N}\sum_{i=1}^N \sup_{\xi \in \Xi} [\Psi(\xi) - \lambda d^p(\xi,\hat{\xi}^i)] \right\}$$

also

$$V_P = V_D = \sup_{\substack{\underline{\xi}^i, \bar{\xi}^i \in \Xi \\ i=1,\dots,N \\ q_1, q_2 \ge 0 \\ q_1 + q_2 \le 1}} \left\{ \frac{1}{N}\sum_{i=1}^N [q_1\Psi(\underline{\xi}^i) + q_2\Psi(\bar{\xi}^i)] : \frac{1}{N}\sum_{i=1}^N [q_1 d^p(\underline{\xi}^i, \hat{\xi}^i) + q_2 d^p(\bar{\xi}^i,\hat{\xi}^i)] \le \theta^p \right\}$$

=> Robust program approx: $L, M \ge 0$ $|\Psi(\xi) - \Psi(\zeta)| < L d(\xi,\zeta) + M$

$$V_k := \sup_{(\xi^{ik})_{i,k} \in \mathfrak{M}_k} \frac{1}{NK}\sum_{i=1}^N \sum_{k=1}^K \Psi(\xi^{ik})$$

$$\mathfrak{M}_k := \left\{ (\xi^{ik})_{i,k} : \frac{1}{NK}\sum_{i=1}^N\sum_{k=1}^K d^p(\xi^{ik}, \hat{\xi}^i) \le \theta^p,\ \xi^{ik} \in \Xi,\ \forall i,k \right\}$$

=> $V_k \uparrow \sup_{\mu \in \mathfrak{M}} E_\mu[\Psi(\xi)]$ as $K \to \infty$

=> $\mathfrak{M}_k \subseteq \mathfrak{M}$ that contains all distributions supported on NK points with equal prob $\frac{1}{NK}$