Decision tree

Original presentation from Jeff Howbert

Example of a decision tree

nominal nominal ratio class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

splitting nodes Refund Yes NO MarSt Single, Divorced Married TaxInc NO < 80k >= 80K YES classification nodes

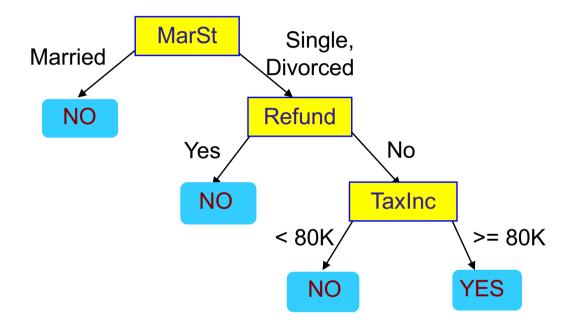
training data

model: decision tree

Another example of decision tree

nominal nominal ratio class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



There may be more than one tree that matches the same data!

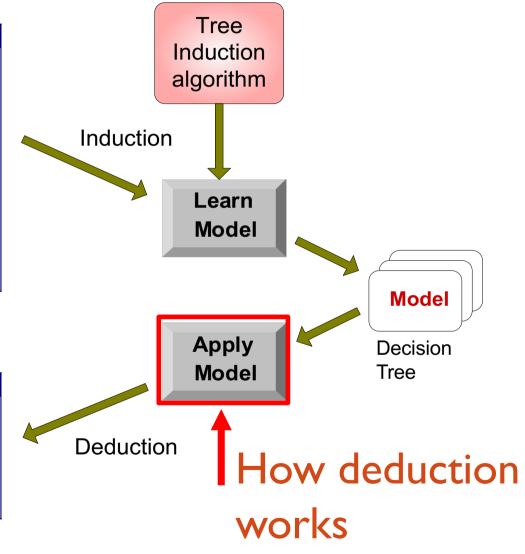
Decision tree classification task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

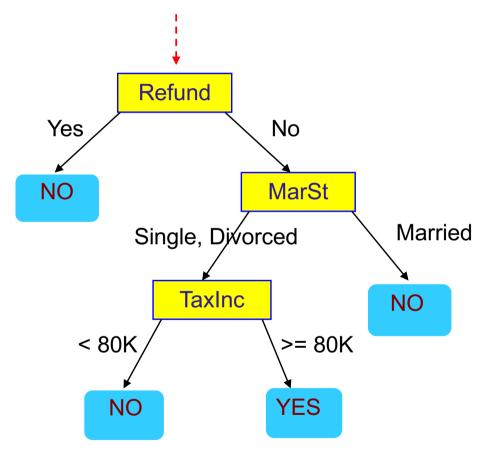
Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



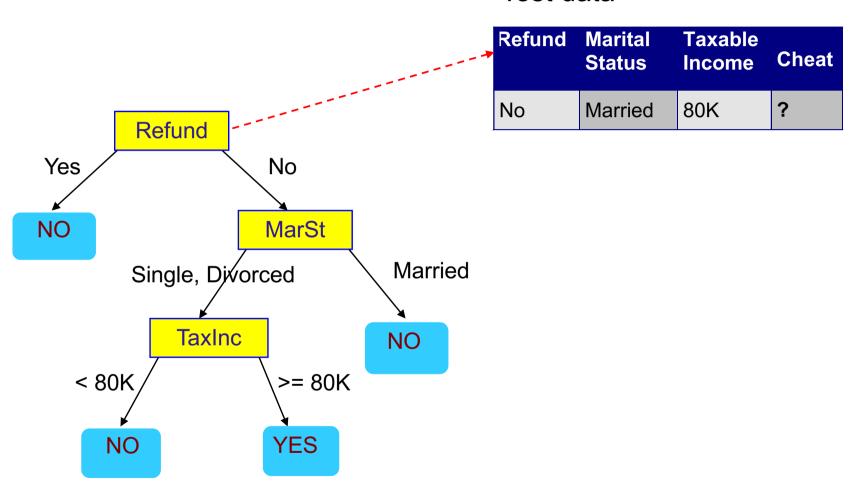
Start from the root of tree.



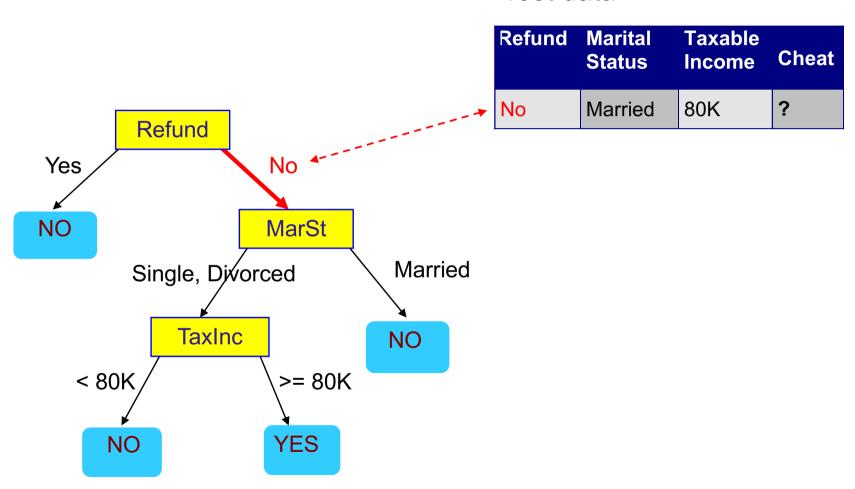
Test data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

Test data



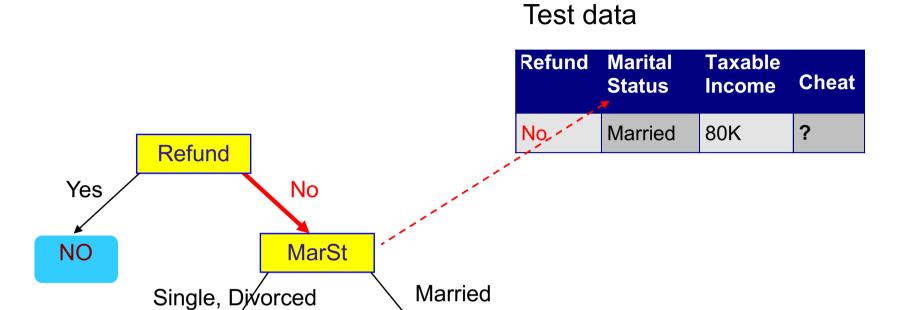
Test data



TaxInc

< 80K

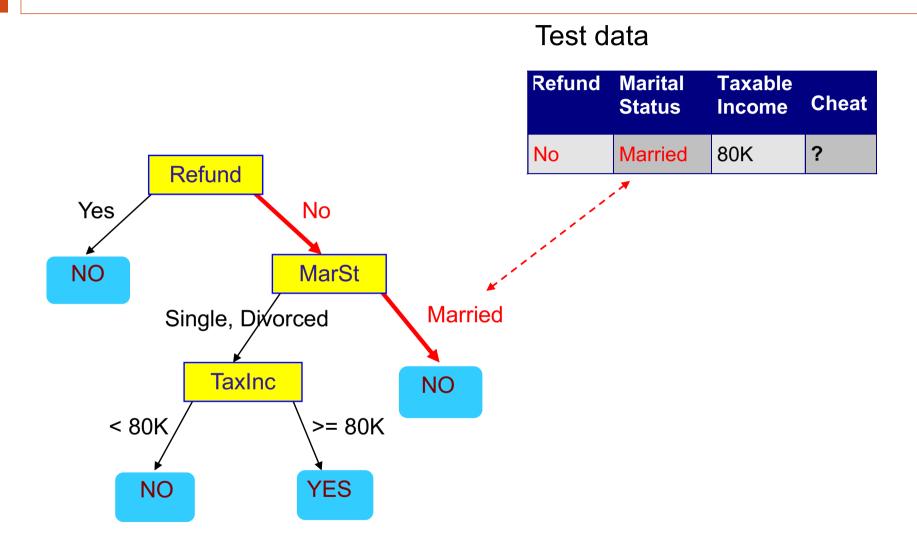
NO

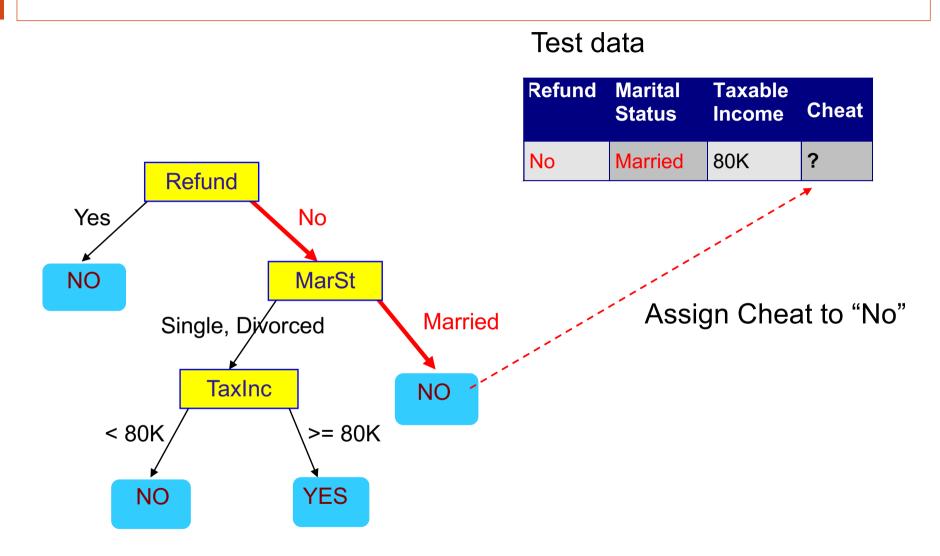


NO

>= 80K

YES





Decision tree induction

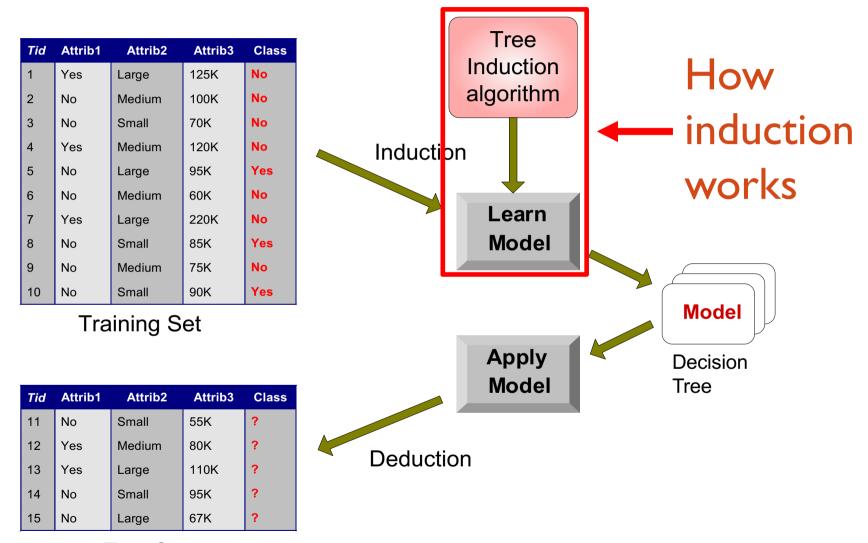
Deduction

- Very easy to execute
- Readable / Explainable

Induction

- How to build a decision tree
- Many algorithms:
 - Hunt's algorithm (one of the earliest)
 - ▶ CART
 - ▶ ID3, C4.5
 - ▶ SLIQ, SPRINT
- As our goal is not to build a competitor at sklearn, we're just going to look at a few principles of Hunt's algorithm

Decision tree classification task

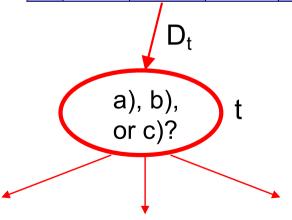


Test Set

General structure of Hunt's algorithm

- Hunt's algorithm is recursive.
- General procedure:
 - Let D_t be the set of training records that reach a node t.
 - I. If all records in D_t belong to the same class y_t , then t is a leaf node labeled as y_t .
 - 2. If D_t is an empty set, then t is a leaf node labeled by the default class as y_d .
 - 3. If D_t contains records that belong to more than one class, use an attribute test to **split** the data into smaller subsets, then apply the procedure to each subset.

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes





Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Select the majority classe Affect all node to this class

If the node is pure or empty

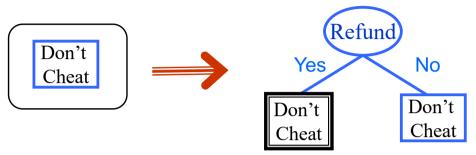
→ It's finished

Else select an attribute a

split the node

Black box = pure node (identical class labels for all samples)

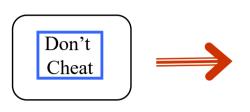
Blue box = impure node

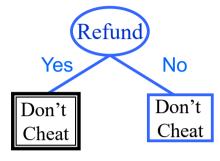


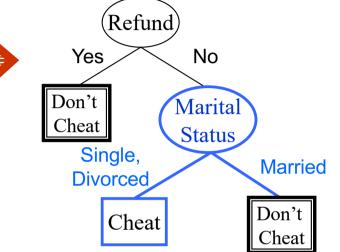
Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Black box = pure node (identical class labels for all samples)

Blue box = impure node



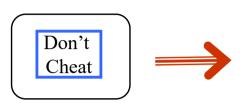




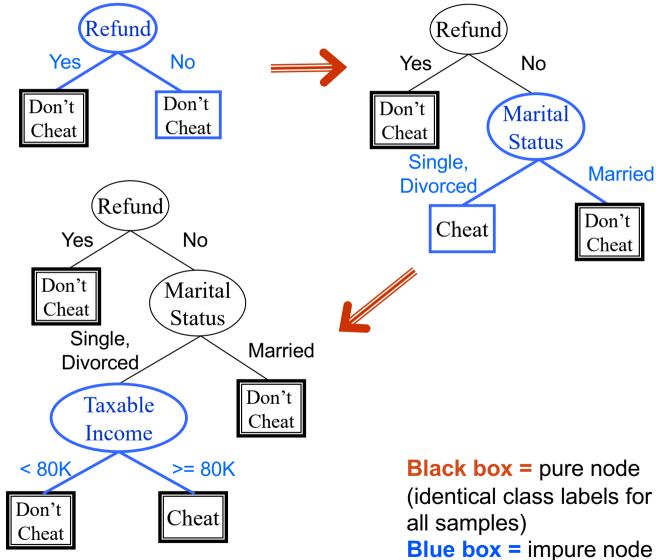
Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Black box = pure node (identical class labels for all samples)

Blue box = impure node



Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



- Step 1: establish majority (aka default) class for root node
 - ▶ 7/10 samples have label = No
- Step 2: choose Refund as split criterion for 10 samples
 - ▶ Refund = yes \rightarrow 3 samples, all with label = No
 - ▶ Refund = no \rightarrow 7 samples, 4 with label = No, 3 with label = Yes
- Step 3: choose Marital Status as split criterion for 7 samples have to decide how to group attribute values during split
 - Marital Status = single/divorced → 4 samples, 3 with label = Yes, 1 with label = No
 - Marital Status = married → 3 samples, all with label = No
- Step 4: choose Taxable Income as split criterion for 4 samples –
 have to decide value on which to split attribute
 - ► Taxable Income < 80K → 3 samples, all with label = No
 - Taxable Income ≥ 80K → 1 sample, with label = Yes

Tree induction

Greedy strategy

> Split the records at each node based on an attribute test that optimizes some chosen criterion.

Issues

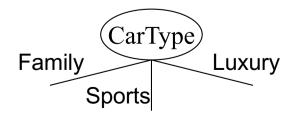
- Determine how to split the records
 - How to specify the test condition?
 - ▶ How to determine the best split?
- Determine when to stop splitting

Specifying test condition

- Depends on attribute type
 - Nominal single, married
 - Ordinal small, medium, large
 - Continuous (interval or ratio)
- Depends on number of ways to split
 - ▶ Binary (two-way) split



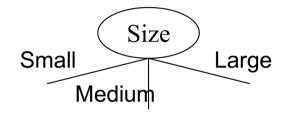
Multi-way split



Splitting based on ordinal attributes

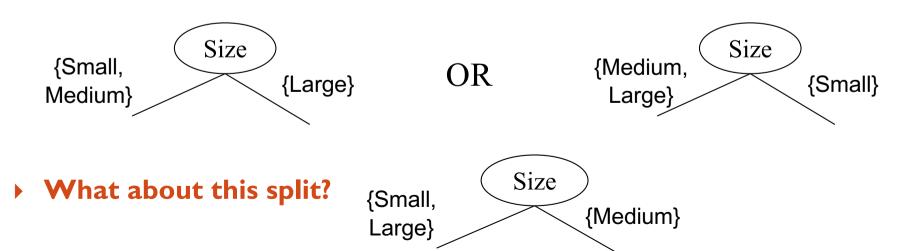
Multi-way split:

Use as many partitions as distinct values.



Binary split:

- Divides values into two subsets.
- Need to find optimal partitioning.

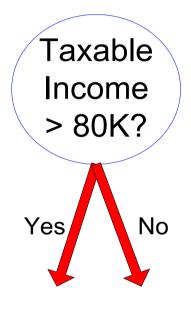


Splitting based on continuous attributes

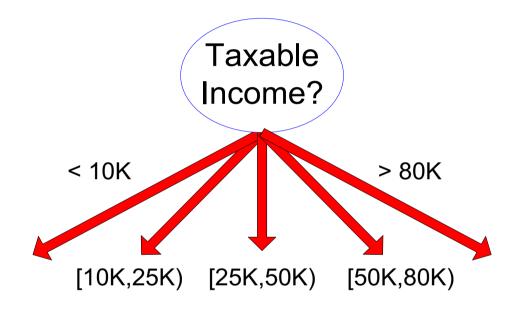
- Different ways of handling
 - Discretization to form an ordinal attribute
 - static
 - □ discretize once at the beginning
 - dynamic
 - □ ranges can be found by equal interval bucketing, equal frequency bucketing (percentiles), or clustering.
 - ▶ Threshold decision: (A < v) or $(A \ge v)$
 - consider all possible split points v and find the one that gives the best split
 - can be more compute intensive

Splitting based on continuous attributes

Splitting based on threshold decision



(i) Binary split



(ii) Multi-way split

Tree induction

Greedy strategy

▶ Split the records at each node based on an attribute test that optimizes some chosen criterion.

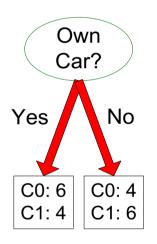
Issues

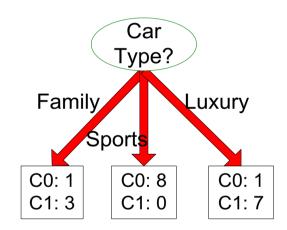
- Determine how to split the records
 - ▶ How to specify the test condition?
 - ▶ How to determine the best split?
- Determine when to stop splitting

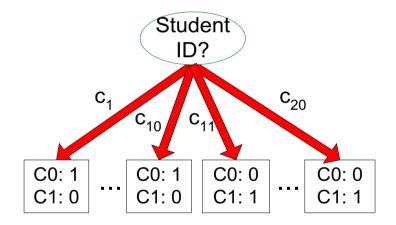
Determining the best split

Before splitting: 10 records of class 0

10 records of class 1







Which attribute gives the best split?

Determining the best split

- Greedy approach:
 - Nodes with homogeneous class distribution are preferred.
- Need a measure of node impurity:

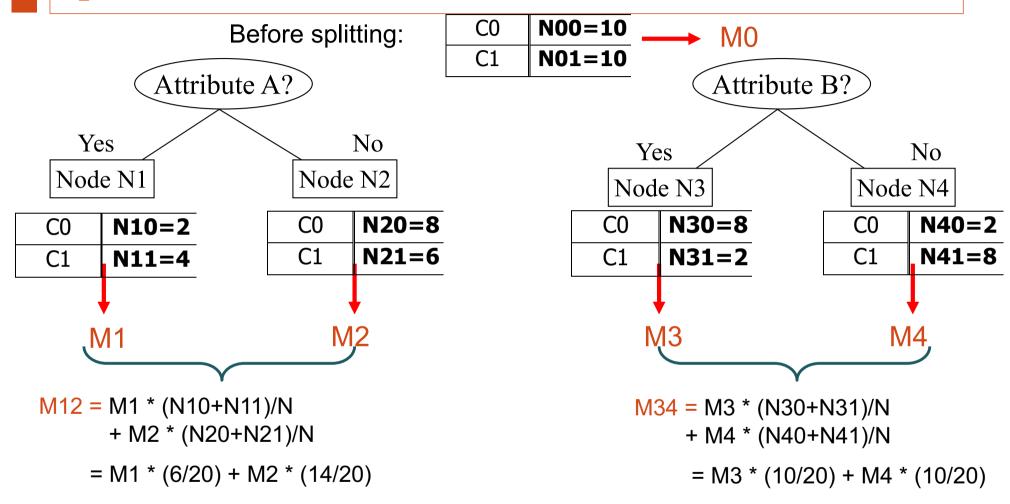
Non-homogeneous, high degree of impurity

C0: 9 C1: 1

Homogeneous, low degree of impurity

- Measures of node impurity
 - Gini index $G(t) = 1 \sum_{i=0}^{c-1} [p(i|t)]^2$
 - Entropy $E(t) = -\sum_{i=0}^{c-1} p(i|t) \log_2 p(i|t)$
 - Classification error = $1 \max_{i}[p(i|t)]$
 - Where
 - p(i|t) denote the fraction of records belonging to class i at a given node t.
 - c is the number of class
 - ▶ $0 \log_2 0 = 0$ in entropy calculation

Using a measure of impurity to determine best split



- Choose splitting attribute that maximizes gain : Gain = M0 M12 vs. M0 M34
 - M0: Measure of impurity before splitting
 - M1, M2, M3, M4: Measure of impurity for each node after splitting

Measure of impurity: Gini index Example

Gini index for a given node t:

$$GINI(t) = 1 - \sum [p(j|t)]^2$$

- p(j | t) is the relative frequency of class j at node t
- Maximum = I I / c c = number of classes

- when records are equally distributed among all classes, implying least amount of information
- \blacktriangleright Minimum = 0.0
 - when all records belong to one class, implying most amount of information.

C1	0
C2	6
Gini=	0.000

Gini=	0.278
C2	5
C1	1

C1	2	
C2	4	
Gini=0.444		

$$p(C1) = 2/6$$
 $p(C2) = 4/6$
Gini = 1 - (2/6)² - (4/6)² = 0.444

$$p(C1) = 1/6$$
 $p(C2) = 5/6$

Gini =
$$1 - (1/6)^2 - (5/6)^2 = 0.278$$

$$p(C1) = 0/6 = 0$$
 $p(C2) = 6/6 = 1$

Gini =
$$1 - p(C1)^2 - p(C2)^2 = 1 - 0 - 1 = 0$$

Tree induction

Greedy strategy

> Split the records at each node based on an attribute test that optimizes some chosen criterion.

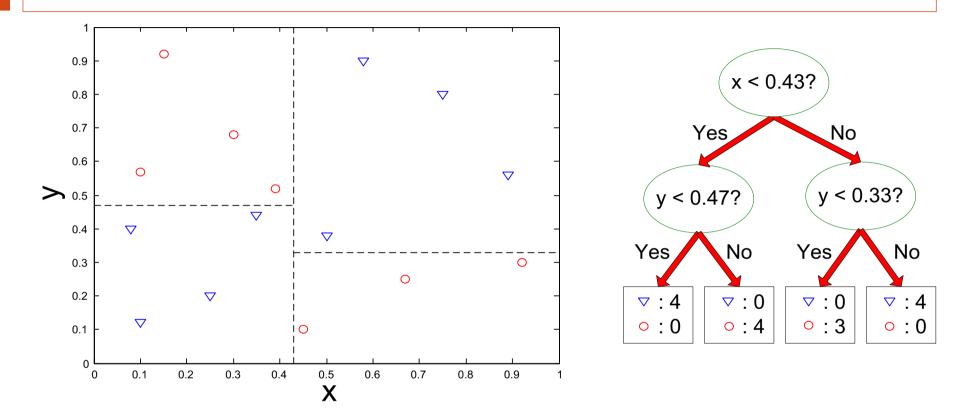
Issues

- Determine how to split the records
 - How to specify structure of split?
 - What is best attribute / attribute value for splitting?
- Determine when to stop splitting

Stopping criteria for tree induction

- Stop expanding a node when all the records belong to the same class
- Stop expanding a node when all the records have identical (or very similar) attribute values
 - No remaining basis for splitting
- Early termination (also known as pruning)
 - Pre-pruning or Post-pruning.

Decision trees: decision boundary



- Border between two neighboring regions of different classes is known as decision boundary.
- In decision trees, decision boundary segments are always parallel to attribute axes, because test condition involves one attribute at a time.

Decision trees: addressing overfitting

- ▶ Pre-pruning (early stopping rules) before constructing new leafs
 - Stop the algorithm before it becomes a fully-grown tree
 - ▶ At each stage of splitting the tree, we check the cross-validation error
 - If the error does not decrease significantly enough then we stop
 - General stopping conditions for a node:
 - Stop if all instances belong to the same class
 - Stop if all the attribute values are the same
 - Early stopping conditions (more restrictive):
 - Stop if number of instances is less than some user-specified threshold
 - Stop if class distribution of instances are independent of the available features (e.g., using χ 2 test)
 - Stop if expanding the current node does not improve impurity measures (e.g., Gini or information gain).

Decision trees: addressing overfitting

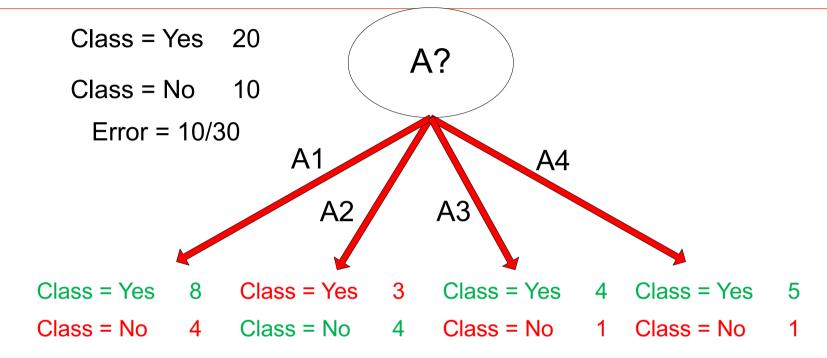
▶ Post-pruning — after constructing new leafs

- As the name implies, pruning involves cutting back the tree.
- After a tree has been built (and in the absence of early stopping discussed below) it may be overfitted.
 - The final subsets (known as the *leaves* of the tree) each consist of only one or a few data points.
 - The tree has learned the data exactly, but a new data point that differs very slightly might not be predicted well.
- Pruning strategies,
 - Minimum error. The tree is pruned back to the point where the cross-validated error is a minimum.
 - Cross-validation is the process of building a tree with most of the data and then using the remaining part of the data to test the accuracy of the decision tree.
 - ▶ Smallest tree. The tree is pruned back slightly further than the minimum error.
 - Prune if the estimated generalization error is bigger than the error on the test set (optimistic or pessimistic approach)

Estimating Generalization Errors

- Frror on training (Σ e(t))
- Generalization errors: error on testing $(\Sigma e'(t))$
- Methods for estimating generalization errors:
 - Optimistic approach: e'(t) = e(t)
 - Pessimistic approach:
 - For each leaf node: e'(t) = (e(t)+0.5)
 - ▶ Total errors: $e'(T) = e(T) + N \times 0.5$ (N: number of leaf nodes)
 - Example: For a tree with 30 leaf nodes and 10 errors on training (out of 1000 instances):
 - ▶ Training error = 10/1000 = 1%
 - ▶ Optimistic generalization error = 1% (the same)
 - ▶ Optimistic generalization error = $(10 + 30 \times 0.5)/1000 = 2.5\%$

Example of post-pruning



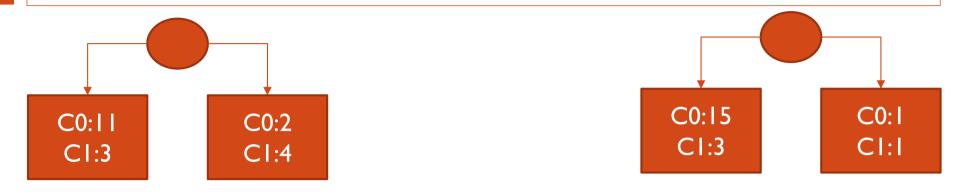
Before splitting

- Training error (before splitting) = 10/30
- Pessimistic error (before splitting) = (10+0.5)/30

After training

- Training error (after splitting) = (4+3+1+1)/30 = 9/30
- Pessimistic error (after splitting) = (9+4*0.5)/30 = 11/30

Exemple of post-pruning



	Case I	Case 2
Error train (before splitting)	7/20=0,35	4/20=0.2
Pessimistic error (before training)	(7+1*0.5)/20=0.375	(4+0.5)/20=0.225
Error train (after splitting)	(3+2)/20=0.25	(3+1)/20=0.2
Pessimistic error (after splitting)	(5+2*0.5)/20=0.3	(4+2*0.5)/20=0.25
Optimistic error	No prune	No prune
Pessimistic error	No prune	Prune

Classification with decision trees

Advantages:

- Inexpensive to construct
- Extremely fast at classifying unknown records
- Easy to interpret for small-sized trees
- Can be combined with other decision techniques.
- Accuracy comparable to other classification techniques for many simple data sets

Disadvantages:

- They are unstable, meaning that a small change in the data can lead to a large change in the structure of the optimal decision tree.
- They are often relatively inaccurate. Many other predictors perform better with similar data. This can be remedied by replacing a single decision tree with a <u>random forest</u> of decision trees, but a random forest is not as easy to interpret as a single decision tree.
- Decision boundary restricted to being parallel to attribute axes
- Easy to overfit

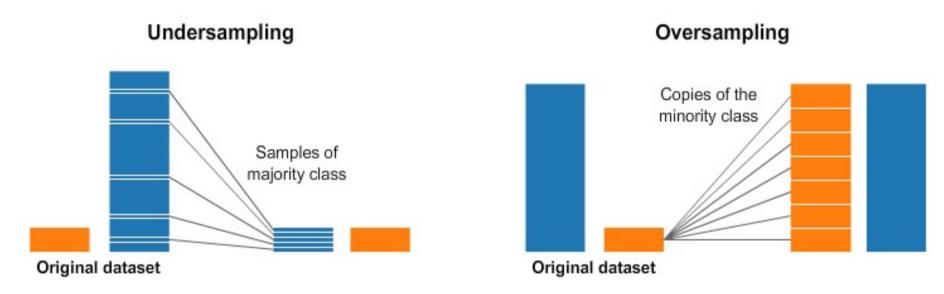
Decision tree in Python

- https://scikit-learn.org/stable/modules/tree.html
- DecisionTreeClassifier()
 - ► To perform multi-class classification
- DecisionTreeRegressor ()
 - To resolve regression problems
- Some tips
 - Performing dimension reduction (PCA, ICA) before to construct trees,
 - gives a better chance of finding features that are discriminative.
 - Use max_depth to control the size of the tree to prevent overfitting.
 - Use min_samples_split or min_samples_leaf to ensure that multiple samples inform every decision in the tree, by controlling which splits will be considered
 - A very small number will usually mean the tree will overfit, whereas a large number will prevent the tree from learning the data.
 - ▶ For classification with few classes, min_samples_leaf= I is often the best choice.
 - **Balance your dataset before training** to prevent the tree from being biased toward the classes that are dominant.

Balance your dataset before training

Two methods:

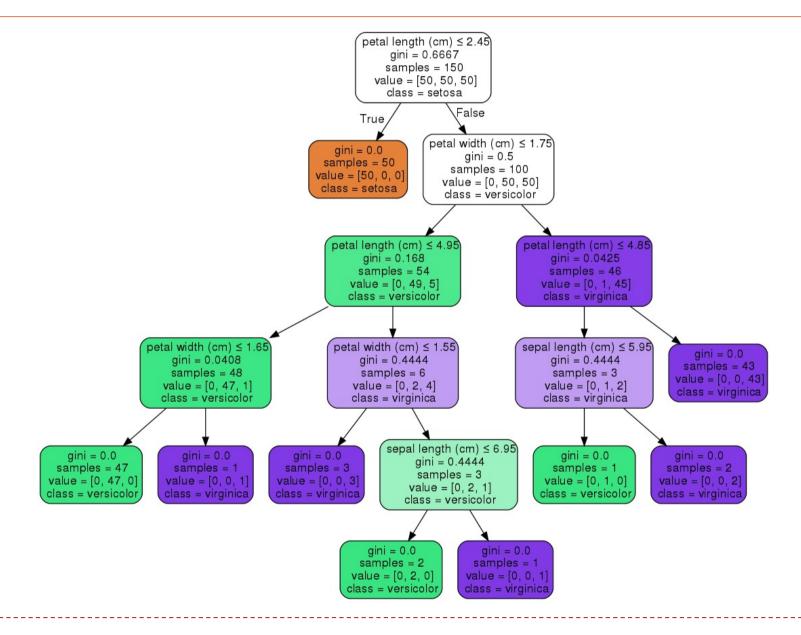
- Undersampling: select only some of the data from the majority class
- Oversampling: create copies of our minority class in order to have the same number of examples as the majority class has



Visualize your tree

- # Initialize our decision tree object
- from sklearn import tree
- classification tree = tree.DecisionTreeClassifier()
- #Train our decision tree (tree induction + pruning)
- classification_tree = classification_tree.fit(X, y)
- # plot the tree
- import graphviz
- dot_data = tree.export_graphviz(classification_tree, feature_names=feature_names, class_names=target_names)
- graph = graphviz.Source(dot_data)
- graph.render("iris")

Visualize your tree



PROs decision tree

Easy to understand and interpret.

- Not require any statistical knowledge to read and interpret them.
- Its graphical representation is very intuitive and users can easily relate their hypothesis.
- Require very little data preparation.
 - All that remains to be done is to adjust a few hyperparameters such as the depth of the tree
 - It is not influenced by outliers and missing values to a fair degree.
- The cost of using the tree for inference is logarithmic in the number of data points used to train the tree.
 - This is a very great advantage because if we add data, the learning time changes little.
- Data type is not a constraint:
 - It can handle both numerical and categorical variables.
- Useful in Data exploration:
 - One of the fastest way to identify most significant variables and relation between two or more variables.
 - With the help of decision trees, we can create new variables / features that has better power to predict target variable (cf. <u>Trick to enhance power of regression model</u>)
- Non Parametric Method:
 - Decision tree is considered to be a non-parametric method. This means that decision trees have no assumptions about the space distribution and the classifier structure.

CONs decision tree

Overfitting is quite common

- ▶ The reduction of dimensionality (via PCA) allows to circumvent this problem.
- Do pruning
- Use random forest

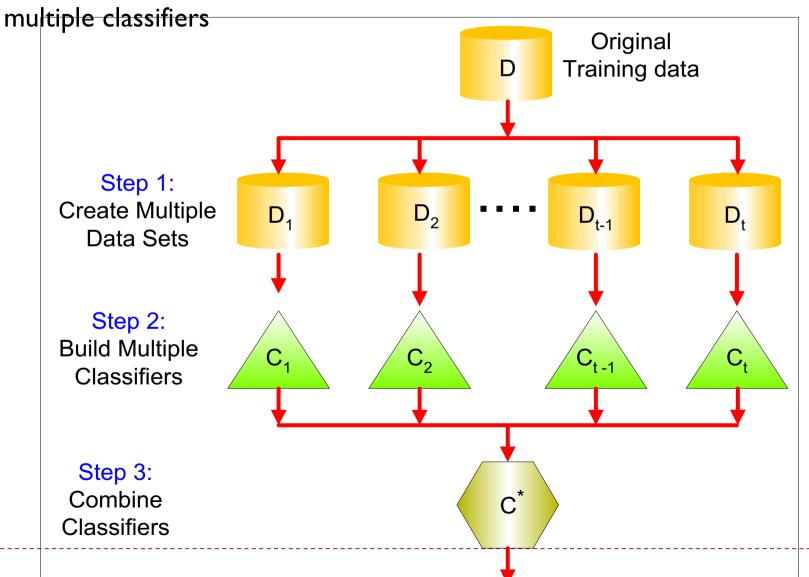
Not fit for continuous variables:

- While working with continuous numerical variables, decision tree looses information when it categorizes variables in different categories.
- Works poorly with unbalanced dataset
 - Always balance the classes if necessary.

Ensemble methods

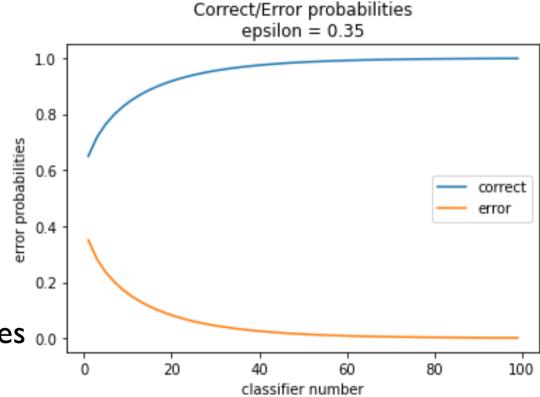
Ensemble Methods

Predict class label of test records by combining the predictions made by



Why Ensemble Methods work?

- Suppose there are 25 base classifiers
 - Each classifier has error rate, $\varepsilon = 0.35$
 - Assume errors made by classifiers are uncorrelated
 - Vote for the result
- Probability that the ensemble classifier makes 0.0 a wrong prediction:



$$P(X \ge 13) = \sum_{i=13}^{25} {25 \choose i} \varepsilon^{i} (1 - \varepsilon)^{25 - i} = 0.06$$

Or 94 % gives a correct prediction with a correct rate of 0.65

Ensemble methods

- Useful for classification or regression
 - For classification, aggregate predictions by voting.
 - For regression, aggregate predictions by averaging.
- Model types can be:
 - Heterogeneous
 - ▶ Example: neural net combined with SVM combined with decision tree combined with ...
 - ▶ Homogeneous most common in practice
 - Individual models referred to as base classifiers (or regressors)
 - ► Example: ensemble of 1000 decision trees
- Base classifiers: important properties
 - Computationally fast: usually need to compute large numbers of classifiers
 - Accuracy: error rate of each base classifier better than random
 - Diversity (lack of correlation)

Base classifiers: important properties

Diversity

- Predictions vary significantly between classifiers
- Usually attained by using unstable classifier
 - small change in training data (or initial model weights) produces large change in model structure
- Examples of unstable classifiers:
 - decision trees
 - neural nets
 - rule-based
- Examples of stable classifiers:
 - ▶ Linear models: logistic Regression
 - ▶ Linear discriminant

How to create diverse base classifiers

- Random initialization model parameters
 - Network weights with neural networks
- Use random projection of the dataset on a lower-dimentional space
- Resample / subsample training data
 - Sample instances
 - Disjoint partitions
 - Randomly without replacement
 - Randomly with replacement (e.g. bagging)
 - Sample features (random subspace approach)
 - Randomly prior to training
 - Randomly during training (e.g. random forest)

Random Forest

Original presentation from Jeff Howbert

From decision tree to random forest

- Decision trees are greedy
 - They choose which variable to split on using a greedy algorithm that minimizes error
 - sensitivity of single trees to the order of predictors,
 - Overfitting, it's easy
- Combining predictions from multiple trees should work better.
- Random forest changes the algorithm for the way that the sub-trees are learned so that the resulting predictions from all of the subtrees have less correlation
- Principle: the decision tree forest algorithm learns about multiple decision trees driven by slightly different subsets of data.

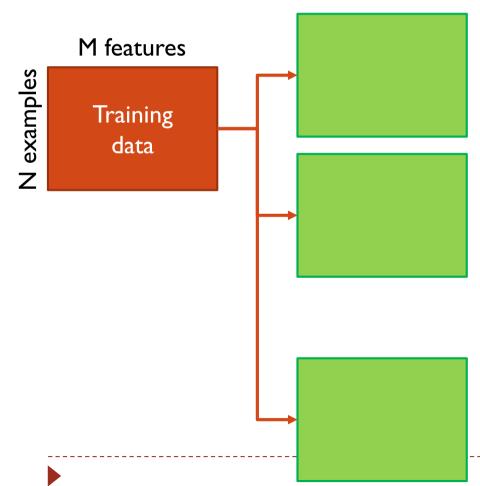
M features

N examples

Training data

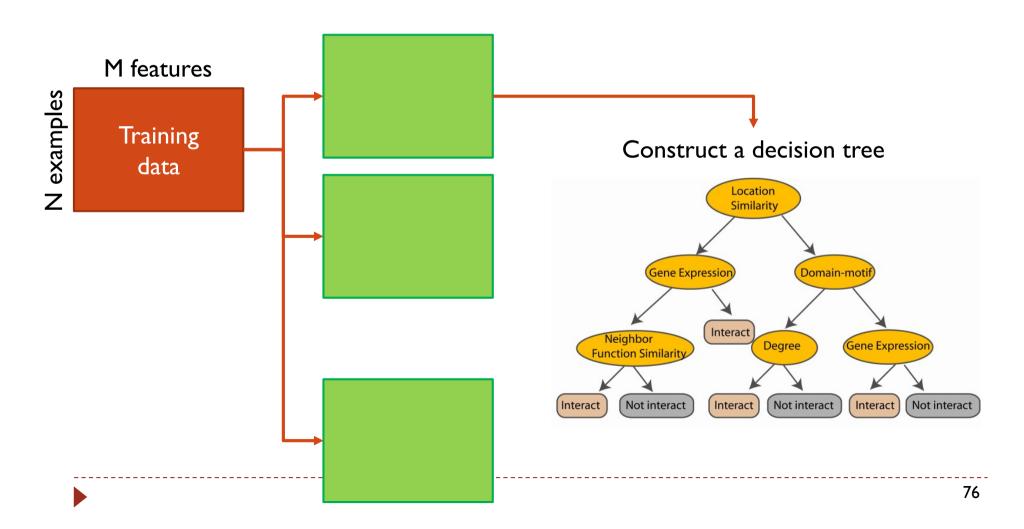
Create many (100) samples from the training data

- with a same number of observations M identical of the original data ((random sample with replacement. technique known as bootstrap)
- with m random features (generally $m < \sqrt{M}$) I

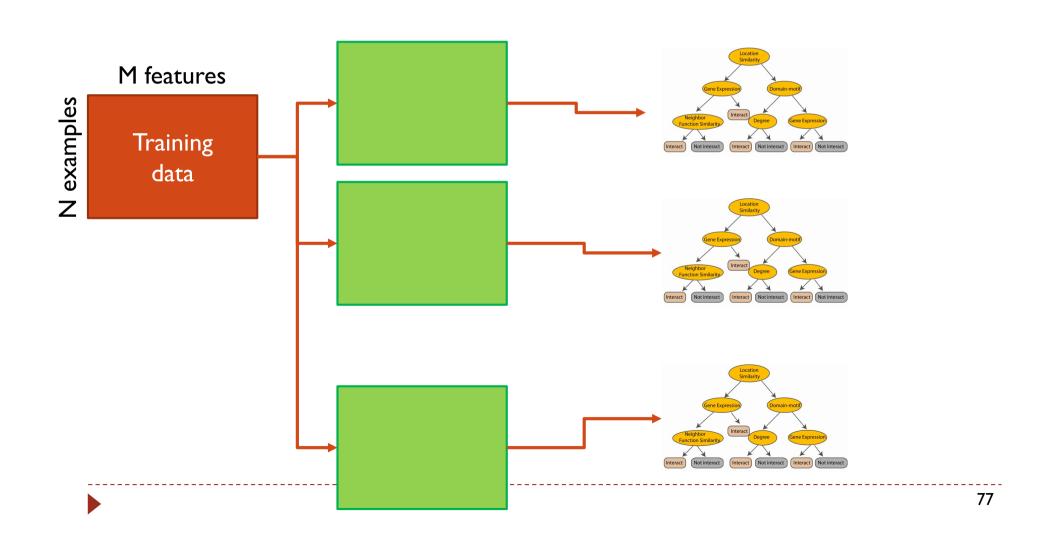


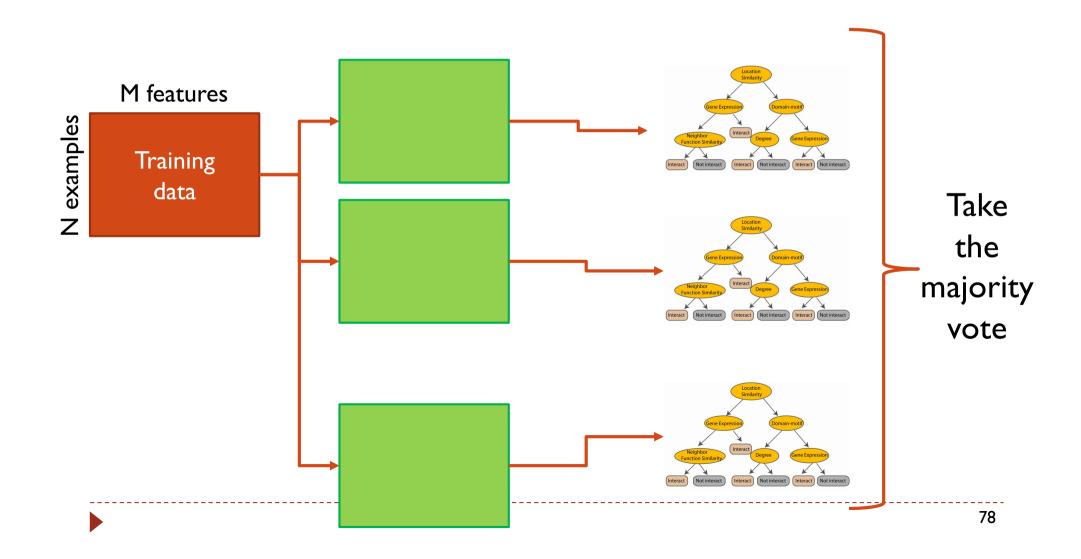
Create many (100) random sub-samples of the training data

- random sample with replacement technique known as bootstrap
- and randomly select for each training data only m features (generally $m < \sqrt{M}$) I



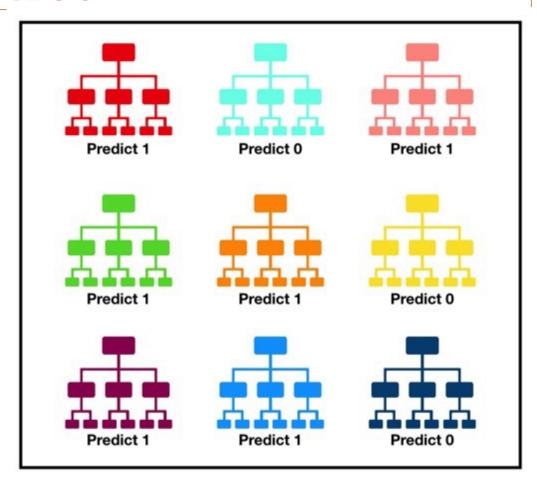
Create decision tree from each bootstrap sample





Why many trees give a more valuable result than one tree

- The reason for this wonderful effect is that the trees protect each other from their individual mistakes.
- While some trees may be wrong, many others will be right.
- Remember
 - With 25 classifier
 - \blacktriangleright Error rate = 0.35
 - ▶ 94 % of correct prediction



Tally: Six 1s and Three 0s

Prediction: 1

Random Forest in Python

- Use RandomForestClassifier or RandomForestRegressor
- Main parameters
 - n_estimators : integer, optional (default= 10)
 - ▶ The number of trees in the forest.
 - max_depth : integer or None, optional (default=None)
 - ▶ The maximum depth of the tree.
 - If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min_samples_split samples.
 - min_samples_split : int, float, optional (default=2)
 - ▶ The minimum number of samples required to split an internal node
 - min_samples_leaf : int, float, optional (default=1)
 - ▶ The minimum number of samples required to be at a leaf node.
 - A split point at any depth will only be considered if it leaves at least min_samples_leaf training samples in each of the left and right branches.

Random forest

Others parameters

- max_features : int, float, string or None, optional (default="auto")
 - ▶ The number of features to consider when looking for the best split
- ▶ min_impurity_decrease : float, optional (default=0.)
 - A node will be split if this split induces a decrease of the impurity greater than or equal to this value.
- min_impurity_split : float, (default= l e-7)
 - ▶ Threshold for early stopping in tree growth.
 - ▶ A node will split if its impurity is above the threshold, otherwise it is a leaf.

PROs of Random Forest Algorithm

- I. It can be used in classification and regression problems.
- 2. It solves the problem of overfitting as output is based on majority voting or averaging.
- 3. It performs well even if the data contains null/missing values.
- 4. Each decision tree created is independent of the other thus it shows the property of parallelization.
- 5. It is highly stable as the average answers given by a large number of trees are taken.
- 6. It maintains diversity as all the attributes are not considered while making each decision tree though it is not true in all cases.
- 7. It is immune to the curse of dimensionality. Since each tree does not consider all the attributes, feature space is reduced.
- 8. We don't have to segregate data into train and test as there will always be 30% of the data which is not seen by the decision tree made out of bootstrap.

CONs of Random Forest Algorithm

- I. Random forest is highly complex when compared to decision trees where decisions can be made by following the path of the tree.
- 2. Training time is more compared to other models due to its complexity. Whenever it has to make a prediction each decision tree has to generate output for the given input data.