

Student Information

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Answer 1

a.

$M=(K, \Sigma, \delta, s, \{h\})$

$\Sigma = \{a, b, \sqcup, \triangleright\}$

$\delta(q, \triangleright) = (q, \rightarrow)$ for all $q \in K-h$. $K = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, h\}$

$s = \{q_0\}$

q	σ	$\delta(q, \sigma)$
q_0	\triangleright	$\delta(q_0, \rightarrow)$
q_0	a	$\delta(q_1, \rightarrow)$
q_0	b	$\delta(q_1, \rightarrow)$
q_0	\sqcup	$\delta(q_1, \rightarrow)$
q_1	\triangleright	$\delta(q_1, \rightarrow)$
q_1	a	$\delta(q_2, \sqcup)$
q_1	b	$\delta(q_3, \sqcup)$
q_1	\sqcup	$\delta(h, \sqcup)$
q_2	\triangleright	$\delta(q_2, \rightarrow)$
q_2	a	$\delta(q_2, \leftarrow)$
q_2	b	$\delta(q_2, \leftarrow)$
q_2	\sqcup	$\delta(q_4, \leftarrow)$
q_3	\triangleright	$\delta(q_3, \rightarrow)$
q_3	a	$\delta(q_2, \leftarrow)$
q_3	b	$\delta(q_2, \leftarrow)$
q_3	\sqcup	$\delta(q_5, \leftarrow)$
q_4	\triangleright	$\delta(q_4, \rightarrow)$
q_4	a	$\delta(q_4, \leftarrow)$
q_4	b	$\delta(q_4, \leftarrow)$
q_4	\sqcup	$\delta(q_6, a)$
q_5	\triangleright	$\delta(q_5, \rightarrow)$
q_5	a	$\delta(q_5, \leftarrow)$
q_5	b	$\delta(q_5, \leftarrow)$
q_5	\sqcup	$\delta(q_6, b)$
q_6	\triangleright	$\delta(q_6, \rightarrow)$
q_6	a	$\delta(q_6, \rightarrow)$
q_6	b	$\delta(q_6, \rightarrow)$
q_6	\sqcup	$\delta(h, \sqcup)$

b.

i.

$(s, \triangleright \sqcup \sqcup \sqcup \underline{bab})$
 $(q_0, \triangleright \sqcup \sqcup \underline{bab}) \vdash_M (q_1, \triangleright \sqcup \sqcup \underline{bab}) \vdash_M (q_3, \triangleright \sqcup \sqcup \underline{ba\sqcup}) \vdash_M (q_5, \triangleright \sqcup \sqcup \underline{ba\sqcup}) \vdash_M (q_5, \triangleright \sqcup \sqcup \underline{ba\sqcup}) \vdash_M$
 $(q_5, \triangleright \sqcup \sqcup \underline{ba\sqcup}) \vdash_M (q_6, \triangleright \sqcup \sqcup \underline{bba\sqcup}) \vdash_M (q_6, \triangleright \sqcup \sqcup \underline{bba\sqcup}) \vdash_M (q_6, \triangleright \sqcup \sqcup \underline{bba\sqcup}) \vdash_M (q_6, \triangleright \sqcup \sqcup \underline{bba\sqcup}) \vdash_M (h, \triangleright \sqcup \sqcup \underline{bba\sqcup})$

ii.

$(s, \triangleright \underline{aaa})$
 $(q_0, \triangleright \underline{aaa}) \vdash_M (q_1, \triangleright \underline{aaa}) \vdash_M (q_2, \triangleright \underline{aa\sqcup}) \vdash_M (q_4, \triangleright \underline{aa\sqcup}) \vdash_M (q_4, \triangleright \underline{aa\sqcup}) \vdash_M (q_4, \triangleright \underline{aa\sqcup}) \vdash_M (q_4, \triangleright \underline{aa\sqcup}) \vdash_M$
... infinite loop. goes right and left infinitely.

iii.

$(q_0, \triangleright \underline{a\sqcup bb}) \vdash_M (q_1, \triangleright \underline{a\sqcup bb}) \vdash_M (h, \triangleright \underline{a\sqcup bb})$

Answer 2

$(\triangleright \sqcup \underline{bab\bar{c}}) \rightarrow (\triangleright \sqcup \underline{bab\bar{c}}) \rightarrow (\triangleright \sqcup \underline{bab\bar{c}\sqcup}) \rightarrow (\triangleright \sqcup \underline{bab\bar{c}\sqcup}) \rightarrow (\triangleright \sqcup \underline{bab\bar{c}\sqcup}) \rightarrow (\triangleright \sqcup \underline{bab\bar{c}\sqcup}) \rightarrow (\triangleright \sqcup \underline{bab\bar{c}\sqcup\sqcup}) \rightarrow$
 $(\triangleright \sqcup \underline{bab\bar{c}\sqcup\bar{b}}) \rightarrow (\triangleright \sqcup \underline{bab\bar{c}\sqcup\bar{c}}) \rightarrow (\triangleright \sqcup \underline{bab\bar{c}\sqcup\bar{c}\sqcup}) \rightarrow (\triangleright \sqcup \underline{bab\bar{c}\sqcup\bar{c}\sqcup}) \rightarrow (\triangleright \sqcup \underline{bab\bar{c}\sqcup\bar{c}\sqcup\sqcup}) \rightarrow (\triangleright \sqcup \underline{bab\bar{c}\sqcup\bar{c}\sqcup\bar{b}})$
The string should start with a blank symbol . There should be a "c" just before the first blank symbol. After these conditions satisfied, the five characters of the string (starting from the "c" which is located just before the first blank symbol) becomes $\bar{c}\sqcup\bar{c}\sqcup x$, where x is the initial letter of the string.

Answer 3

a.

$L = \{ (a \cup b)^* \}$ ie. if $w \in L$, w has disordered a's and b's. The TM M , regularize the string and make it in the format $a^n b^m$ where length of the string is equal to m+n. It puts all a's at the LHS and put all b's at RHS.

The input alphabet $\Sigma_0 = \{ a, b \}$

b.

say $w = (a \cup b)^*$, $w \in \{ a, b \}^*$ where length of w is k.
 $f(w) = a^n b^m$ where $n+m=k$.

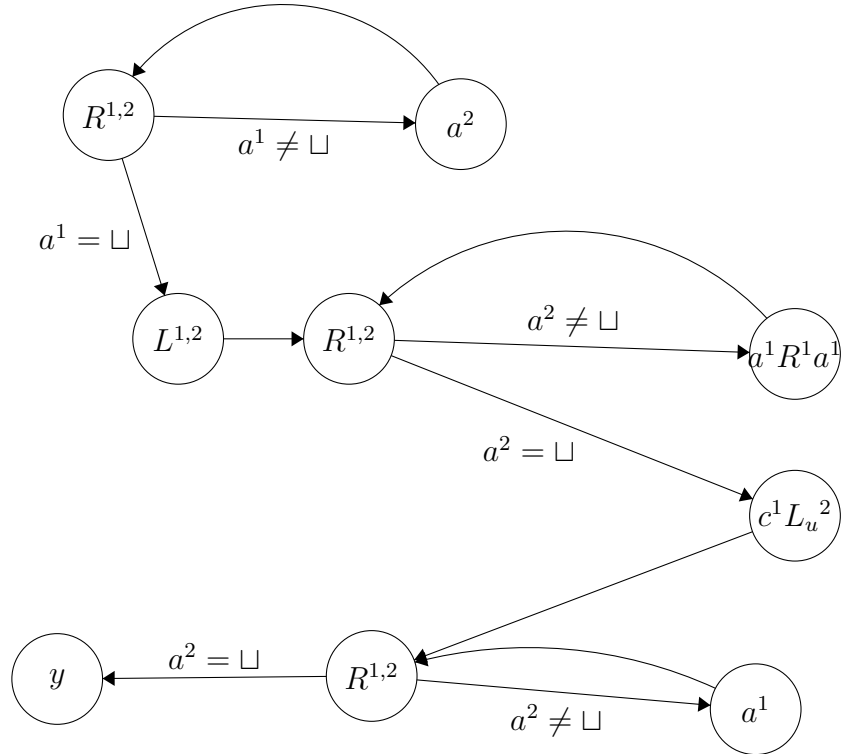
f: $\{a, b\}^* \rightarrow \{a, b\}^*$
and $M(w) = f(w)$.

Answer 4

Two tape TM $M = (K, \Sigma, \delta, s, H)$.

K is finite set of states.

$\Sigma = \{a, b, c\}$ and δ is transition function. s is initial state, H is halting state



Answer 7

a.

insert delete TM $M = (K, \Sigma, \delta, s, H)$

K is set of states, Σ is alphabet, δ is a function where it can delete/insert (new) characters in the input string on the tape, only at the front and rear positions.

s is initial state and H is halting state(s).

- for all $q \in K - H$ if $\delta(q, \triangleright) = (p, b)$ then $b = \rightarrow$

- for all $q \in K - H$ and $a \in \Sigma$, if $\delta(q, a) = (p, b)$ then $b = \triangleright$

b.

$(q, \triangleright \underline{a}w)$ ($a \in \Sigma, w \in \Sigma^*$) is a valid configuration of the insert-delete TM. It starts with the left end symbol and underlined character is the currently scanned symbol.

c.

$$(q_1, w_1 \underline{a_1} u_1) \vdash_M (q_2, w_2 \underline{a_2} u_2)$$

$$(a_1, a_2 \in \Sigma)$$

if this is insertion at the front, $w_1 = e$ since insertion can be only done in front also $w_2 = e$. $u_2 = a_1 u_1$ and a_2 is inserted at the front of the string.

if this is insertion at the back of the string, $u_1 = e, u_2 = e$. $w_2 = w_1 a_1$ and a_2 is inserted in the string.

When insertion is done, length of the string increases 1.

$$(q_1, w_1 \underline{a_1} u_1) \vdash_M (q_2, w_2 \underline{a_2} u_2)$$

if this is deletion at the front of the string, $w_1 = e, a_1$ is the character that will be deleted, $w_2 = e$ also.

$$a_2 u_2 = u_1.$$

if this is deletion at the back of the string, $u_1 = e, u_2 = e, a_1$ is the character that will be deleted and $w_2 a_2 = w_1$. $u_2 = e$ also.

d.

Insertion at the front of the string is equivalent in standard TM to shifting the string to the right (opening a blank symbol at the beginning of the string) and writing on this newly opened blank symbol the new character.

Again, insertion at the back of the string is shifting left and writing a character at the RHS of the string (since there can be a left end symbol, if shifting is not possible just before the beginning of the string, only write the new character at the end is also equivalent to insertion operation).

In deletion case, if deletion is happening at the front of the string (next to the left end symbol), make the unwanted symbol a blank symbol and shift the string left. In deletion at the back, again make the unwanted symbol a blank symbol but shifting is not necessary.

Answer 9

First prove that languages which are recursively enumerable are closed under concatenation. So, if L_1 and L_2 are recursively enumerable then $L_1 L_2$ is also. Let's say there are two TMs that semidecide L_1 and L_2 , say M_1 and M_2 .

$$M_1 = (K_1, \Sigma, \delta_1, s_1, H_1) \text{ and } M_2 = (K_2, \Sigma, \delta_2, s_2, H_2).$$

We want to prove that there is a TM $M = (K, \Sigma, \delta, s, H)$ that semidecides $L_1 L_2$.

This TM basically guess which part of the string belongs to L_1 and which part belongs to L_2 , it knows the split and when it finishes operations δ_1 on L_1 , it will nondeterministally continue with the L_2 .

$$K = K_1 \cup K_2$$

$$s = s_1, H = H_2, \text{ and}$$

$$\delta = \delta_1 \cup \delta_2 \cup \{((h_1, \sigma), (s_2, \rightarrow))\}$$

As a result, M is the turing machine to semidecides $L_1 L_2$.