

# Student Information

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## Answer 1

a.

$$\begin{aligned} \text{(i)} \quad G &= (V, \Sigma, R, S) \\ V &= \{a, b, S, K\} \\ \Sigma &= \{a, b\} \\ R &= \{ S \rightarrow aSa | bK \\ &\quad K \rightarrow aKa | b \\ &\quad \} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad G &= (V, \Sigma, R, S) \\ V &= \{a, b, S, K_1, L_1, K_2, L_2\} \\ \Sigma &= \{a, b\} \\ R &= \{ S \rightarrow aK_1a | bL_1b \\ &\quad K_1 \rightarrow bK_1a | bK_1b | aK_1a | aK_1b \\ &\quad K_1 \rightarrow bK_2a | bK_2b | aK_2a | aK_2b \\ &\quad K_2 \rightarrow a \\ &\quad L_1 \rightarrow bL_1a | bL_1b | aL_1a | aL_1b \\ &\quad L_1 \rightarrow bL_2a | bL_2b | aL_2a | aL_2b \\ &\quad L_2 \rightarrow b \\ &\quad \} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad G &= (V, \Sigma, R, S) \\ V &= \{a, b, S, S_1, S_2, R_1, C_1, R_4, A_1, R_2, R_3\} \\ \Sigma &= \{a, b\} \\ R &= \{ S \rightarrow S_1 | S_2 \\ &\quad S_1 \rightarrow aR_1C_1 | R_4bC_1 \\ &\quad R_1 \rightarrow aR_1b | e | aR_1 \\ &\quad R_4 \rightarrow aR_4b | e | aR_4 \\ &\quad C_1 \rightarrow C_1c | e \\ &\quad S_2 \rightarrow A_1bR_2 | R_4bC_1 \\ &\quad R_2 \rightarrow bR_2c | e | bR_2 \\ &\quad R_3 \rightarrow bR_3c | e | R_3c \\ &\quad A_1 \rightarrow aA_1 | e \\ &\quad \} \end{aligned}$$

(iv)

**b.**

## Answer 2

**a.**

$$G = (V, \Sigma, R, K_1)$$

$$V = \{a, b, K_1, A, B\}$$

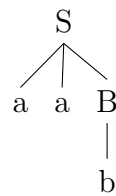
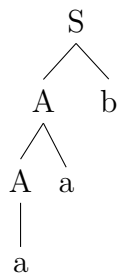
$$\Sigma = \{a, b\}$$

$$R = \{ K_1 \rightarrow Ab | aaB$$

$$A \rightarrow a | Aa$$

$$B \rightarrow b$$

$\}$



Left most and right most derivations of the string "aab" are given above respectively.

$$S \rightarrow Ab \rightarrow Aab \rightarrow aab \text{ (left)}$$

$$S \rightarrow aaB \rightarrow Aabaab \text{ (right)}$$

**b.**

$$S_1 \rightarrow A_1 b$$

$$A_1 \rightarrow aA_1 | a$$

$$G'(V_1, \Sigma_1, R_1, S_1) \text{ where}$$

$$V_1 = \{a, b, S_1, A_1\},$$

$$\Sigma_1 = \{a, b\},$$

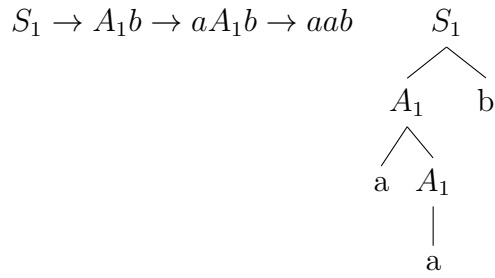
$$R_1 = \{ S_1 \rightarrow A_1 b, A_1 \rightarrow aA_1 | a \}$$

$$S_1 = \text{initial symbol}$$

**c.**

String s = "aab"

unique left most derivation of string s:



## Answer 3

a.

In the Push Down Automaton ,

# of aa's which are in the string is equal to # of x's which are in the stack . Also when there is bbb , x's are popped from stack. To accept the string , all x's should be popped from stack, that means # of aa's and # of bbb's are the same in the string.

Also empty string is accepted by Push Down Automaton.

$$L(M) = \{ (aa)^m (bbb)^m , m \geq 0 \}$$

b.

$$G = (V, \Sigma, R, S_1)$$

$$V = \{ a, b, S_1, K, L \} ,$$

$$\Sigma = \{ a, b \},$$

$$R = \{$$

$$S_1 \rightarrow K | L | KL | LK,$$

$$K \rightarrow a | aKb | aKa | bKb | bKa,$$

$$L \rightarrow b | aLa | aLb | bLa | bLb | bLa \}$$

the push down automaton=

$$M = (\Sigma, \Gamma, \Delta, p, \{ f \}, \{ p, f \})$$

$$\Sigma = \{ a, b \}$$

$$\Gamma = \{ S_1, K, L, e, a, b \}$$

$$\Delta = \{ ((p, e, e), (f, S_1)),$$

$$((f, e, S_1), (f, K)),$$

$$((f, e, S_1), (f, L)),$$

$$((f, e, S_1), (f, KL)), ((f, e, S_1), (f, LK)),$$

$$((f, e, K), (f, a)), ((f, e, K),$$

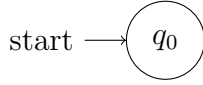
$$(f, aKa)), ((f, e, K), (f, bKa)), ((f, e, K), (f, aKb)), ((f, e, K), (f, bKb)),$$

$$((f, e, L), (f, b)), ((f, a, a), (f, e)),$$

$$((f, e, L), (f, aLa)),$$

$((f,e,L),(f,aLb)), ((f,e,L),(f,bLb)),$   
 $((f,e,L),(f,bLa)),$   
 $((f,b,b),(f,e)) \}$

**c.**



(i)



(ii)

State	Input	Stack	Transition
$q_0$	aabcc	e	-
			Accepts/Rejects.

State	Input	Stack	Transition
$q_0$	bac	e	-
			Accepts/Rejects.

## Answer 4

**a.**

$\Delta = \{$

$((p,e,e),(q,E)),$   
 $((q,e,E),(q,E+T)),$   
 $((q,e,E),(q,T)),$   
 $((q,e,T),(q,TxF)),$   
 $((q,e,T),(q,F)),$   
 $((q,e,F),(q,(E))),$   
 $((q,e,F),(q,a)),$   
 $((q,a,a),(q,e)),$   
 $((q,+,+),(q,e)),$   
 $((q,x,x),(q,e)),$   
 $((q,(,),(,),(q,e)),$   
 $((q,),),), (q,e))$   
 $\}$

$V = \{ a, +, x, (, ), E, T, F \}$

$E \rightarrow E + T | T$

$T \rightarrow TxF | F$

$F \rightarrow (E) | a$

$M = \{ \{ p, q \}, \Sigma, V, \Delta, p, \{ q \} \}$

$$\Sigma = \{ a, +, x, (, ) \}$$

b.

## Answer 5

a.

- (i) Let  $G_1 = (V_1, \Sigma_1, R_1, S_1)$  and Let  $G_2 = (V_2, \Sigma_2, R_2, S_2)$   
 $G = (V_1 \cup V_2 \cup \{S\}, \Sigma_1 \cup \Sigma_2, R_1 \cup R_2 \cup \{S \rightarrow S_1 S_2\}, S)$  where  
 $V_1 = \{a, b, S_1\}$  and  $V_2 = \{a, b, S_2\}$

$$\begin{aligned} \Sigma_1 &= \{ a, b \} \text{ and } \Sigma_2 = \{ a, b \} \\ R_1 &= \{ S_1 \rightarrow a S_1 b | e \} \\ &\} \\ R_2 &= \{ S_2 \rightarrow b S_2 a | e \} \\ &\} \end{aligned}$$

- (ii) This language is equal to  
 $\Sigma^* a \cup a \Sigma^* \cup \Sigma^* b b \Sigma^* \cup \Sigma^* \cup \Sigma^* \{ b a^n b a^m, n+1 \text{ is not equal to } m \} \Sigma^*$   
 Since each of these languages above are CF, their union is also CF  
 CFG's for each language above, respectively:

$$\begin{aligned} G_1 &= (V_1, \Sigma_1, R_1, S_1) \\ V_1 &= \{a, b, S_1, K_1\} \\ \Sigma_1 &= \{ a, b \} \\ R_1 &= \{ S_1 \rightarrow K_1 a \\ K_1 &\rightarrow e | a K_1 | b K_1 \} \\ &\} \end{aligned}$$

$$\begin{aligned} G_2 &= (V_2, \Sigma_2, R_2, S_2) \\ V_2 &= \{a, b, S_2, K_2\} \\ \Sigma_2 &= \{ a, b \} \\ R_2 &= \{ S_2 \rightarrow a K_2 \\ K_2 &\rightarrow e | a K_2 | b K_2 \} \\ &\} \end{aligned}$$

$$\begin{aligned} G_3 &= (V_3, \Sigma_3, R_3, S_3) \\ V_3 &= \{a, b, S_3, K_3\} \\ \Sigma_3 &= \{ a, b \} \\ R_3 &= \{ S_3 \rightarrow b b K_3 b b \\ K_3 &\rightarrow e | a K_3 | b K_3 \} \\ &\} \end{aligned}$$

b.

- (i) choose pumping length  $m$ ,  
there are possible cases here.  
First let  $w=uvxyz$  where  $|vxy| \leq m$  and  $|vy| \geq 1$  and for every  $i \geq 0$ ,  $uv^i xy^i z \in L$   
case1:  $u = a^{m-j}$ ,  $vxy = a^j b^{n-r}$  and  $z = b^r$   
for  $i=2$ , string  $w$  becomes  $a^{m-j}$
- (ii)

## Answer 6

- (i) (T/F)? True because of from book theorem 3.5.4  
From book
- (ii) (T/F)? False since  $L_1 = \{ a^n b^n c^m : m, n \geq 0 \}$  and  $L_2 = \{ a^m b^n c^n : m, n \geq 0 \}$  are both CF but their intersection  $\{ a^n b^n c^n : m, n \geq 0 \}$  is not CF.
- (iii) (T/F)? True  
 $L_1 - L = L_1 \cup L_2^-$   
where  $L_1$  is CFL and  $L_2$  is a RL and also  $L_2^-$  is a RL. RLs are close under complementation and intersection of a RL with a CFL is also a CFL from book Theorem 3.5.2
- (iv) (T/F)? False  
For example  $a^n b^n c^n$  is not a CFL but  $a^n b^n$  is a CFL.