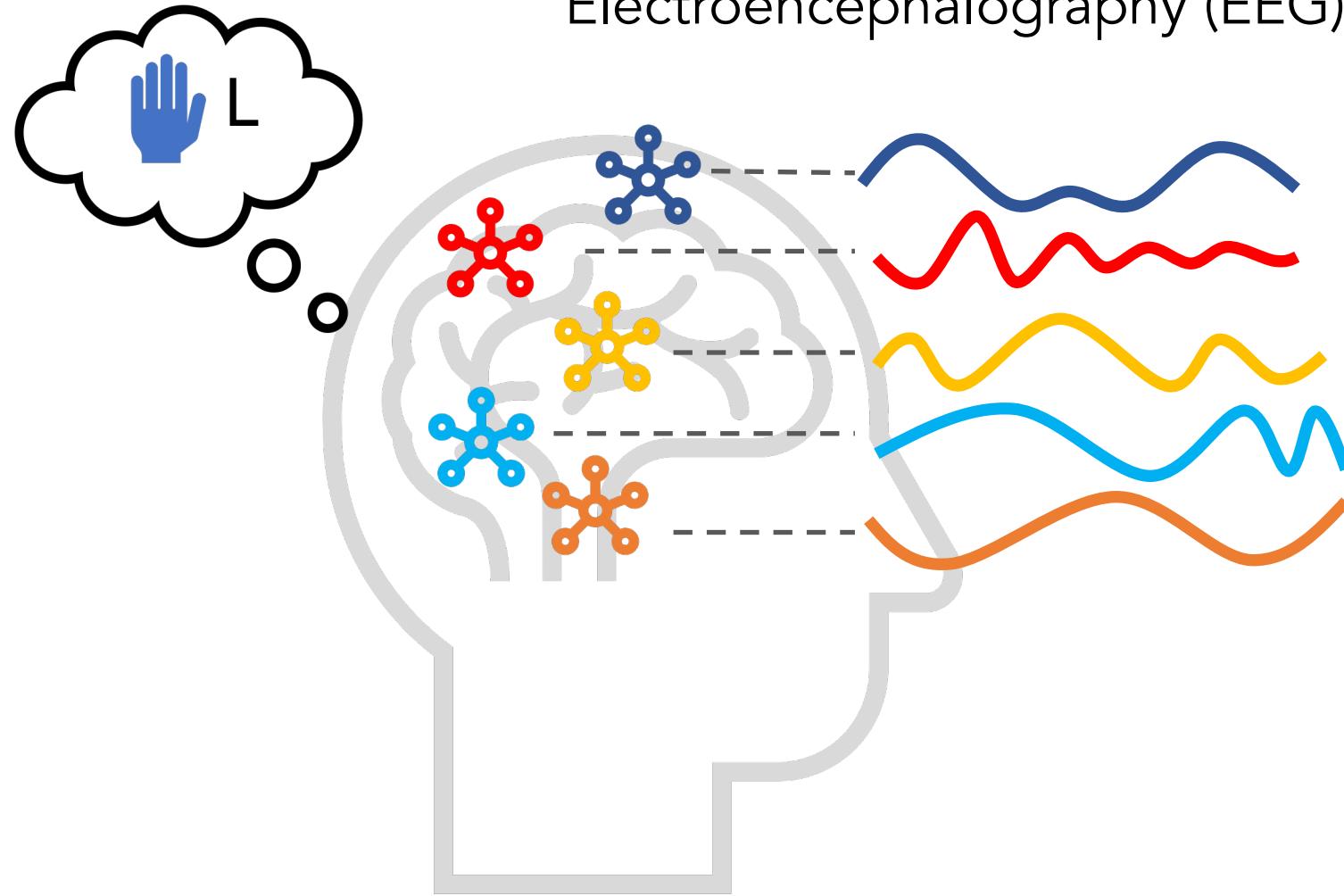
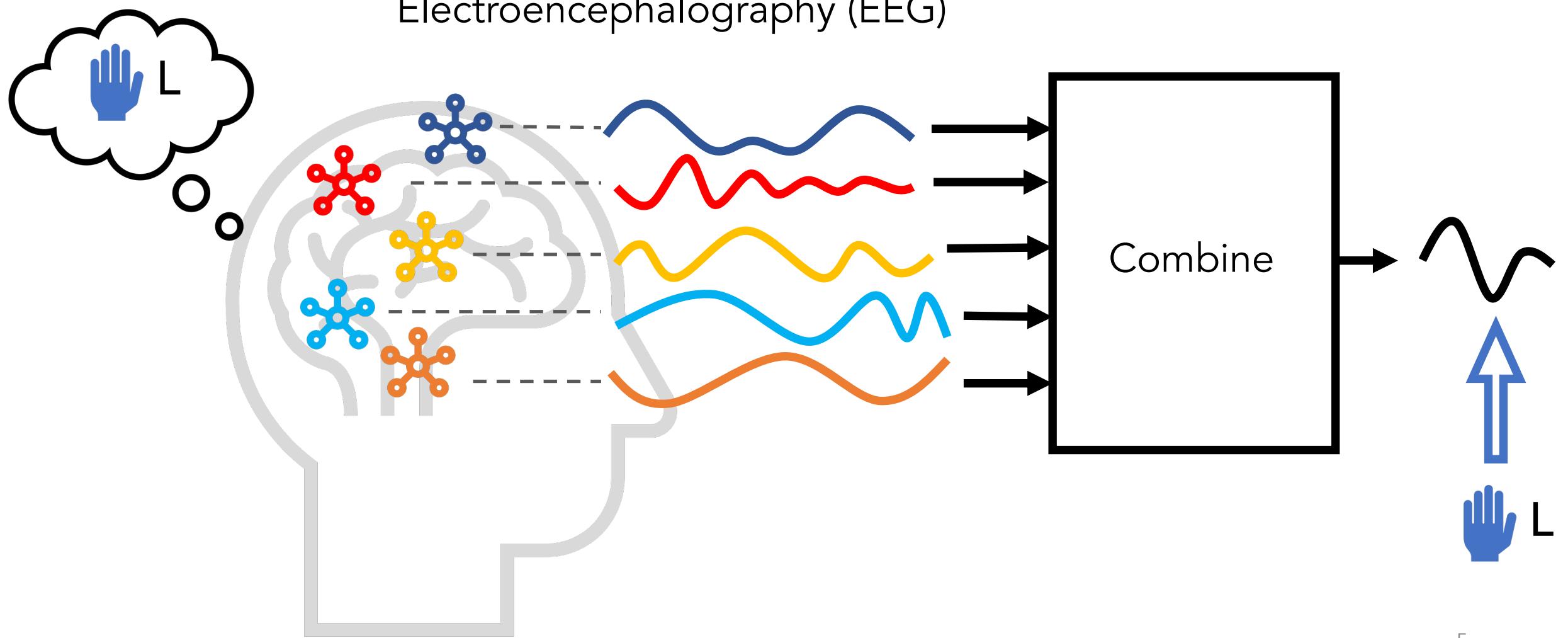
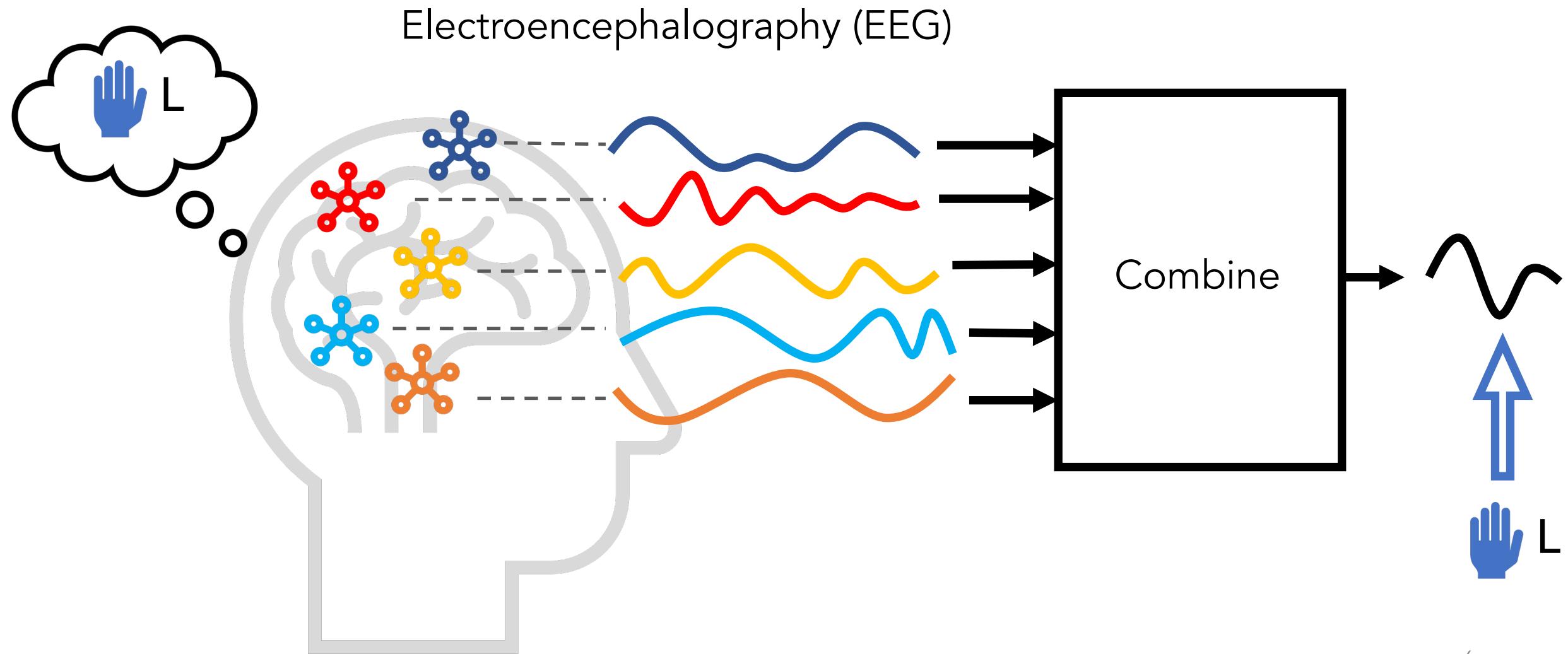


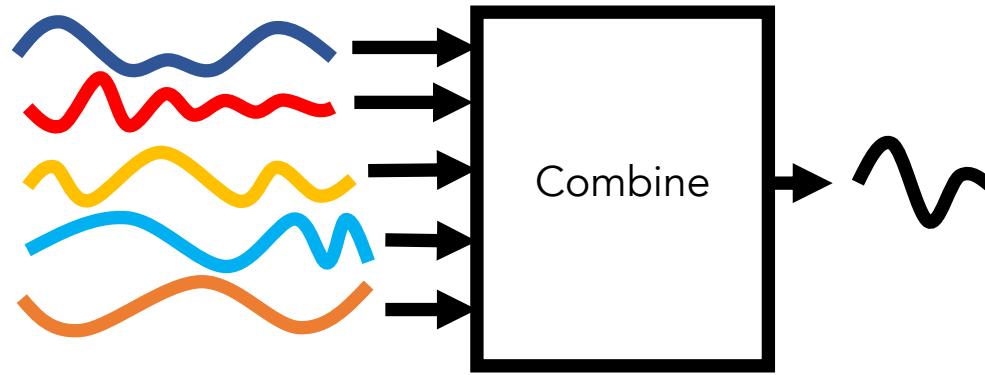
Electroencephalography (EEG)





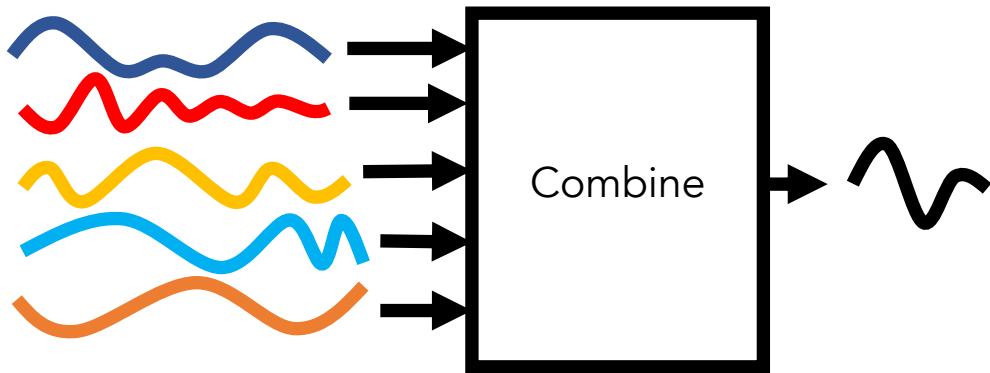
Motor imagery





Seizure detection

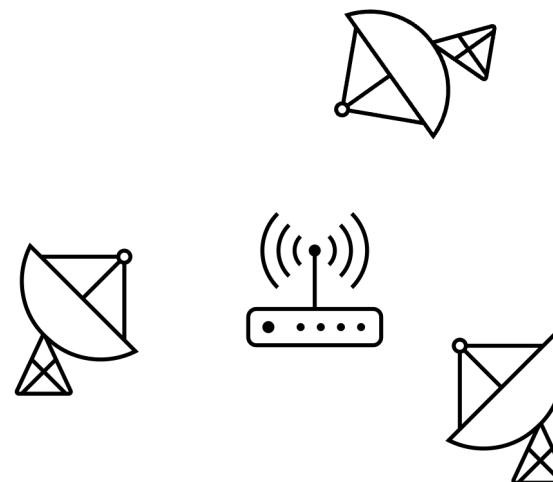




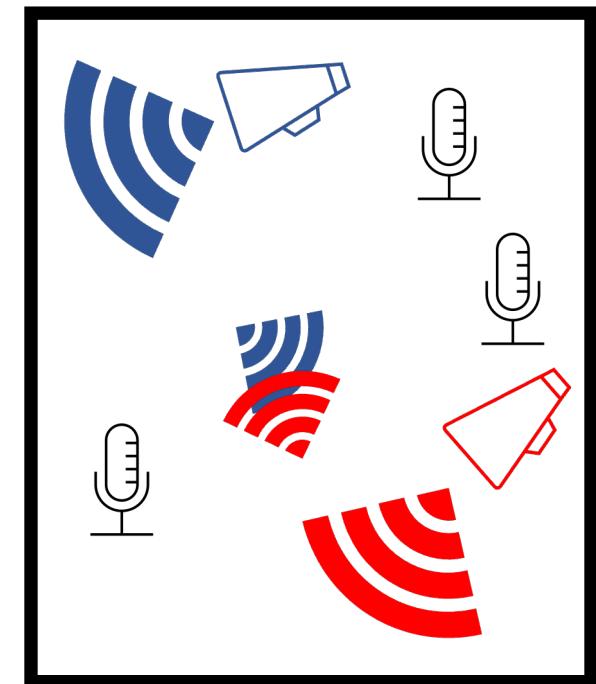
Seizure detection

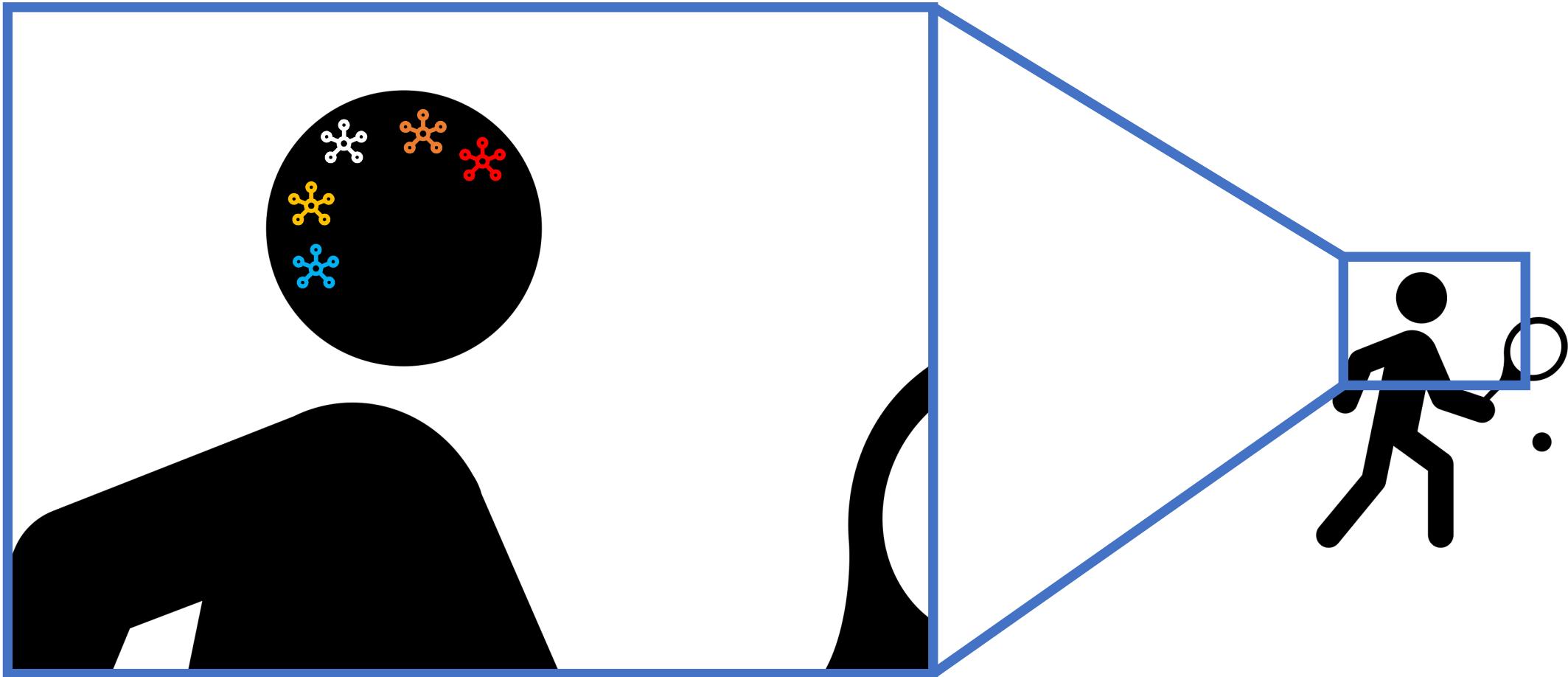


Source localization



Source separation



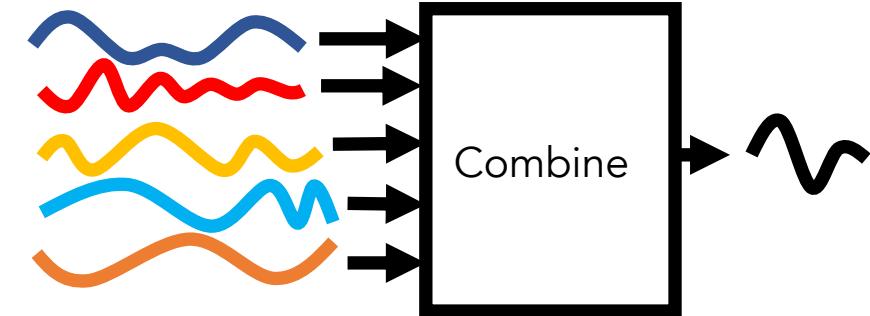
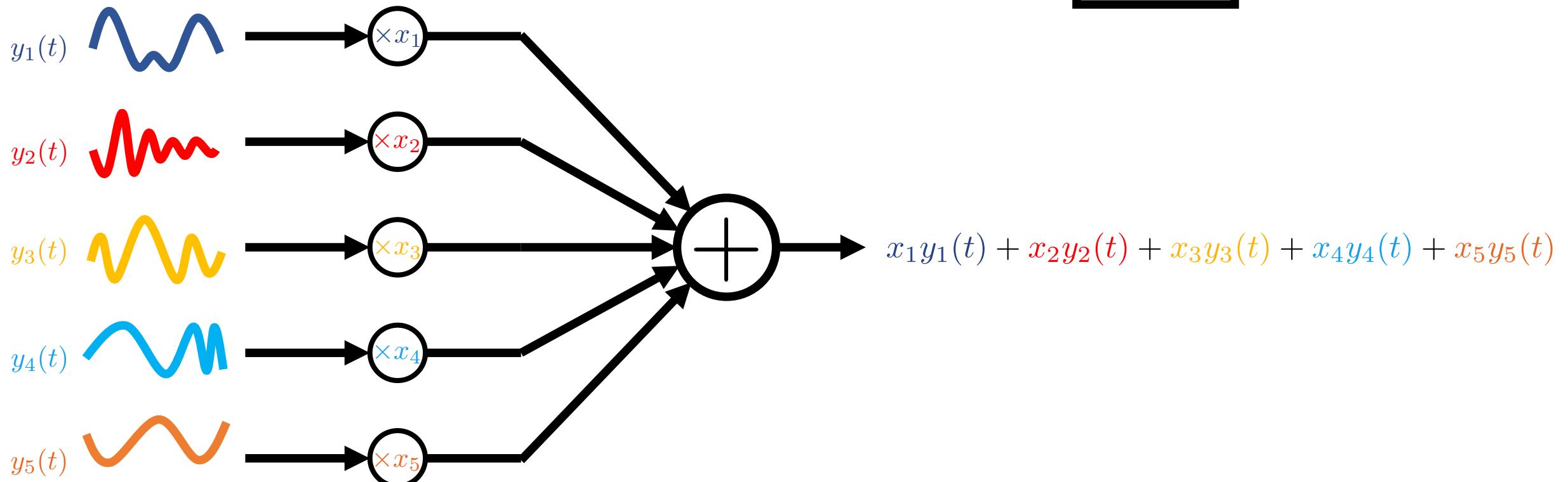


Distributed Spatial Filtering in Wireless Sensor Networks

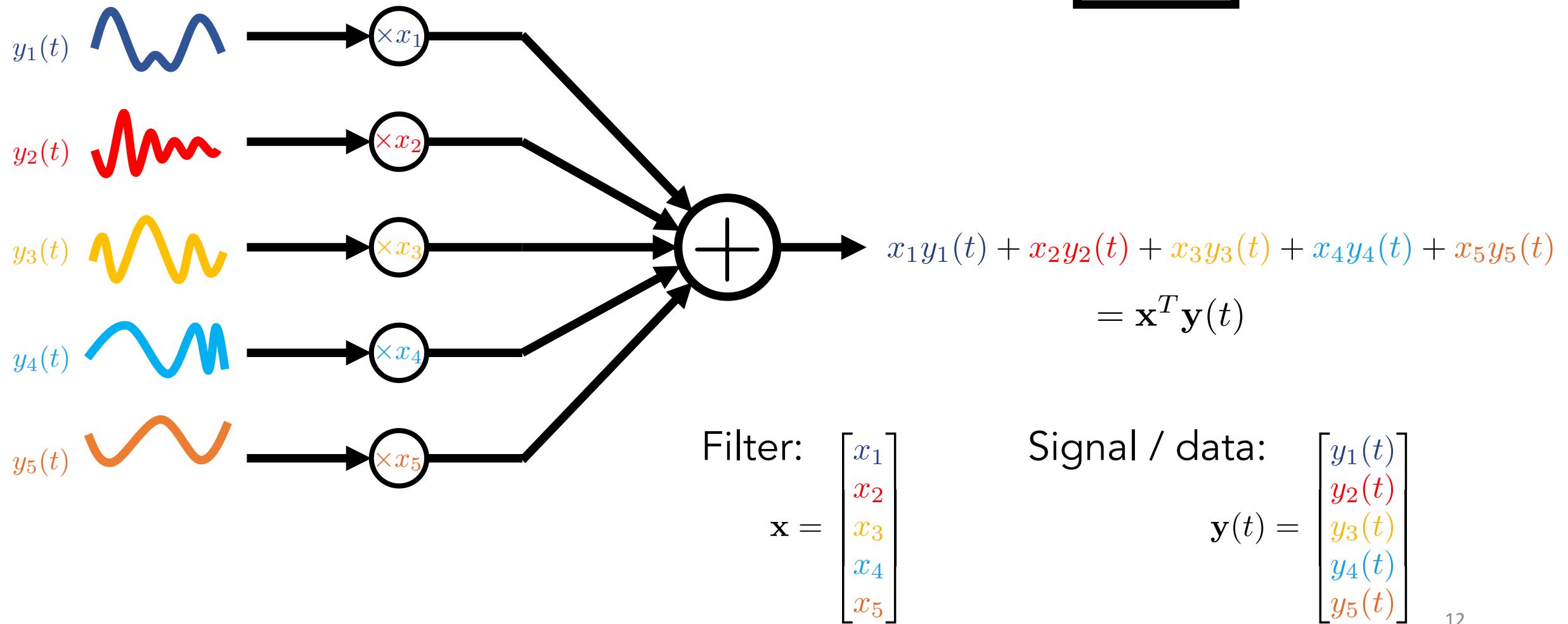
Cem Ates Musluoglu

Supervisor: Prof. dr. ir. Alexander Bertrand

Spatial filtering



Spatial filtering



Choosing the right filter

Choose the **best** possible!

Choosing the right filter

Choose the **best** possible!

Best?

Different for each **task**

Choosing the right filter

Choose the **best** possible!

Best?

Different for each **task**

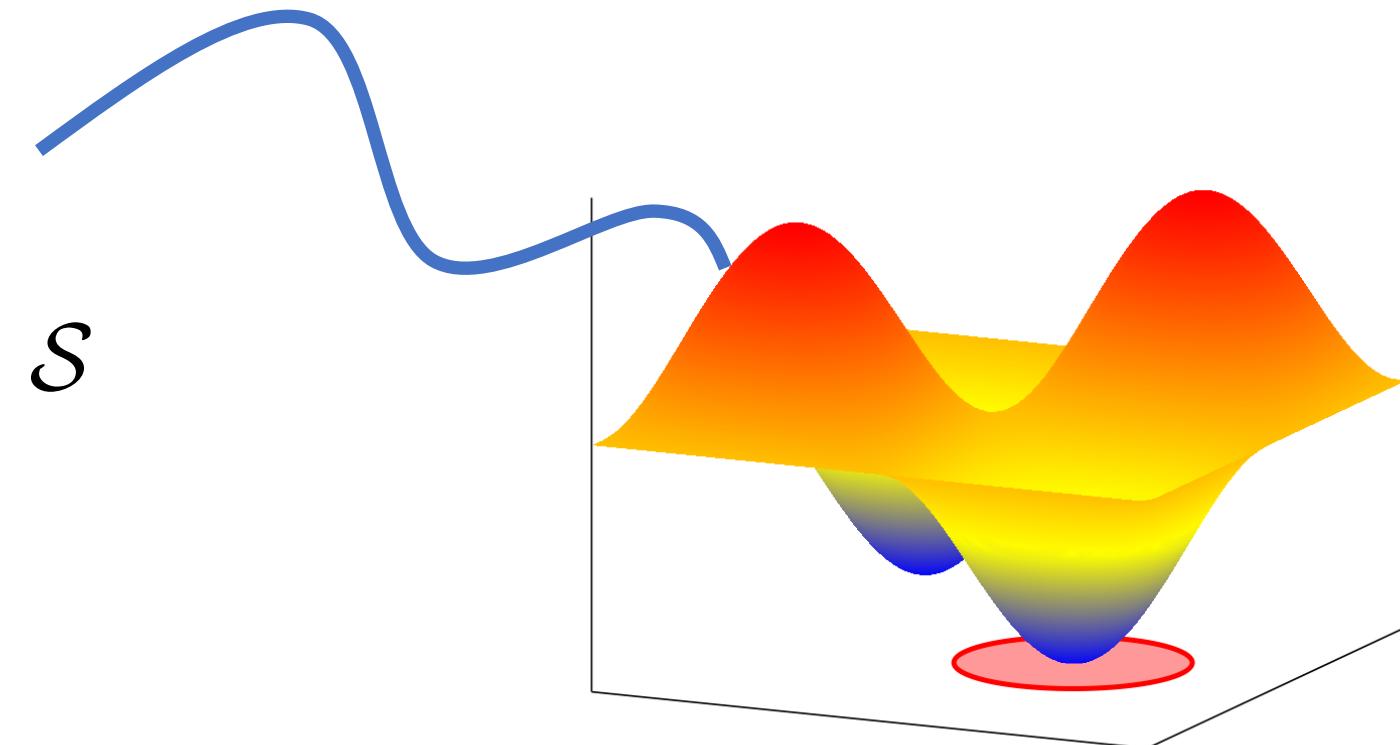
Tasks: What are we aiming to do?

MMSE, PCA, LCMV, GEVD, TRO, CCA,...

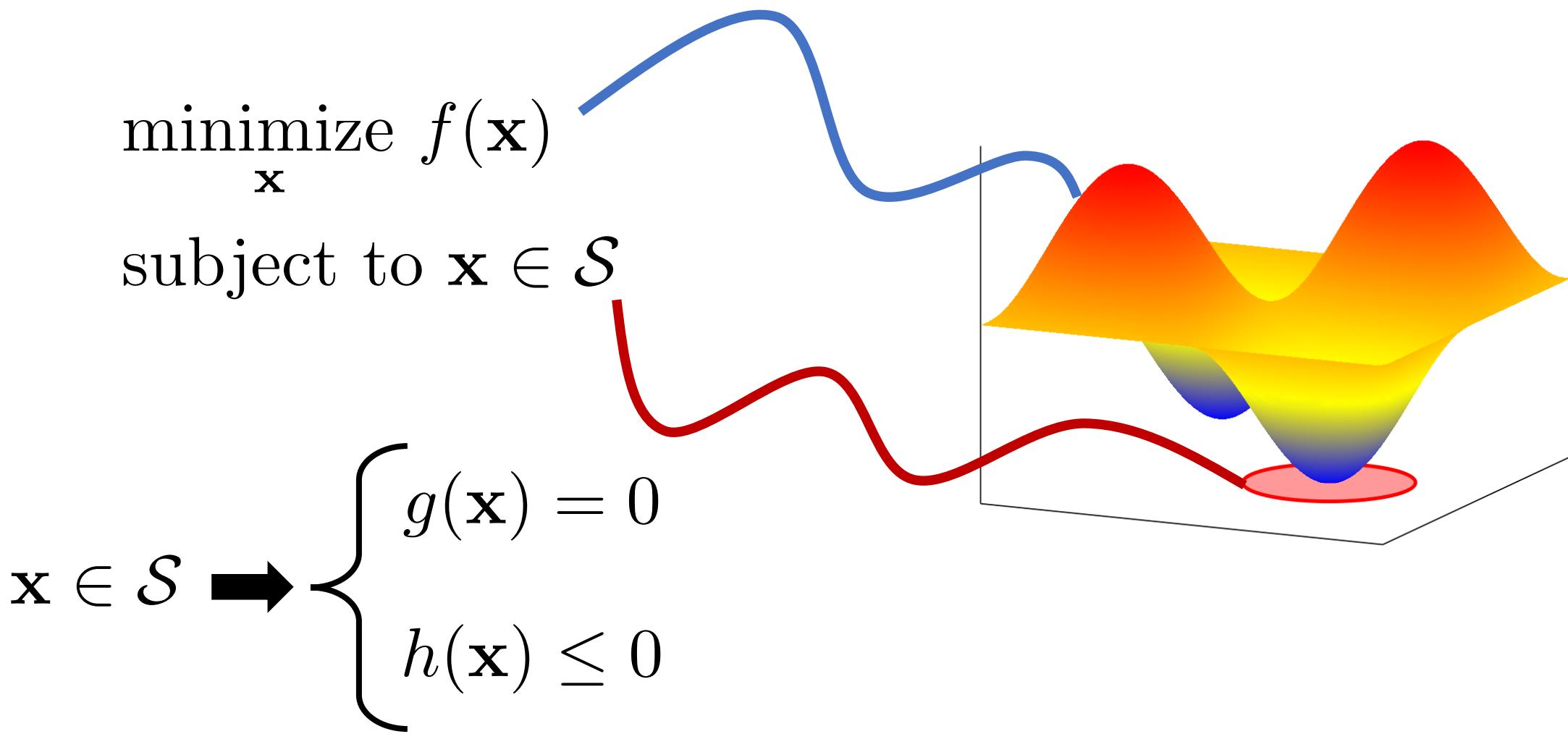
Tasks: Optimization problems

minimize $f(\mathbf{x})$

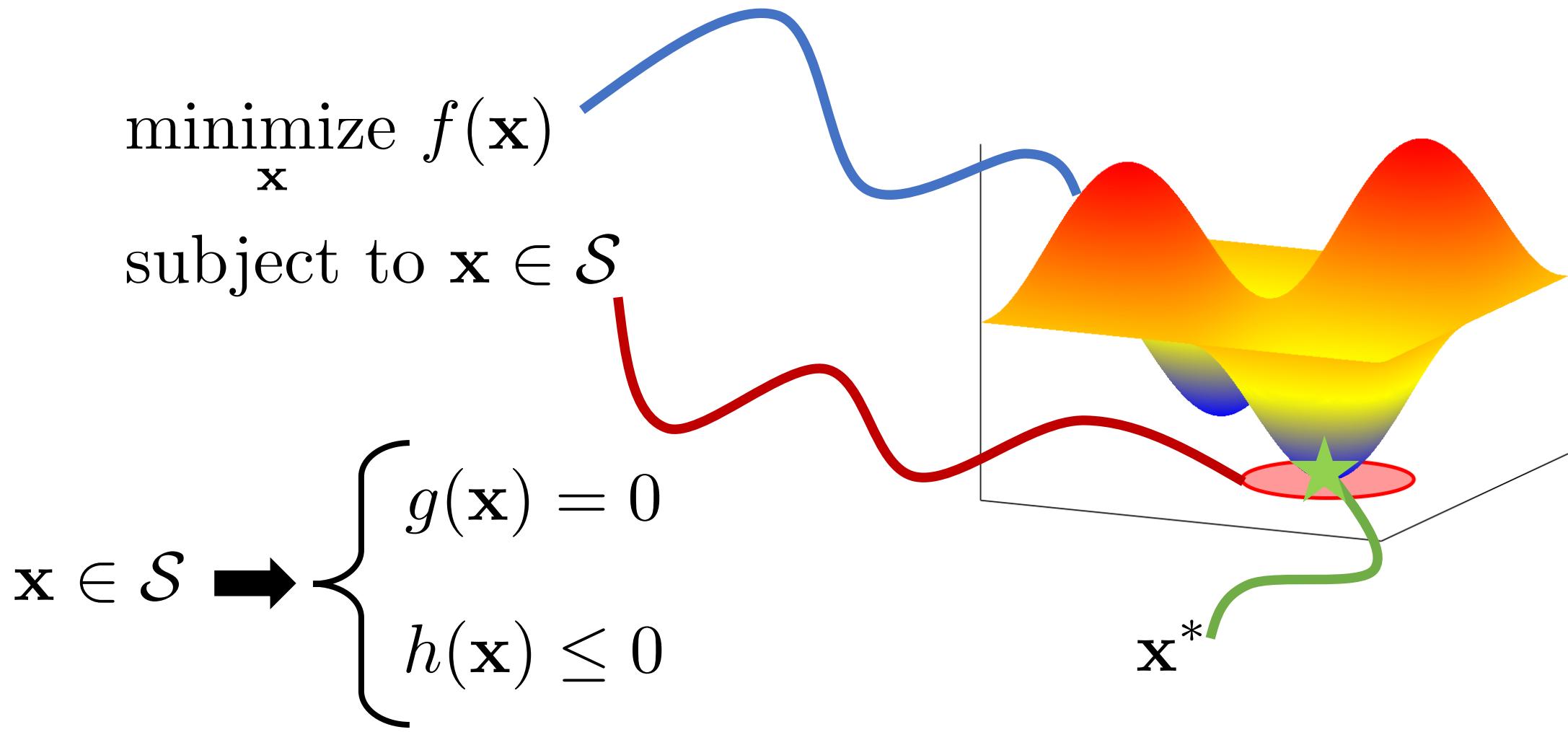
subject to $\mathbf{x} \in \mathcal{S}$



Tasks: Optimization problems

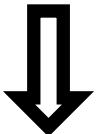


Tasks: Optimization problems



Spatial filtering tasks

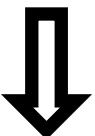
$\mathbf{x}^T \mathbf{y}(t)$: Main quantity of interest



$f(\mathbf{x}^T \mathbf{y}(t))$

Spatial filtering tasks

$\mathbf{x}^T \mathbf{y}(t)$: Main quantity of interest

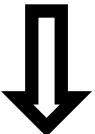


$$f(\mathbf{x}^T \mathbf{y}(t))$$

\mathbf{x}^* : Depends on the data

Spatial filtering tasks

$\mathbf{x}^T \mathbf{y}(t)$: Main quantity of interest

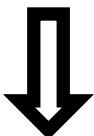


$f(\mathbf{x}^T \mathbf{y}(t))$

\mathbf{x}^* : Depends on the data  Contains uncertainty

Spatial filtering tasks

$\mathbf{x}^T \mathbf{y}(t)$: Main quantity of interest



$f(\mathbf{x}^T \mathbf{y}(t))$

\mathbf{x}^* : Depends on the data  Contains uncertainty

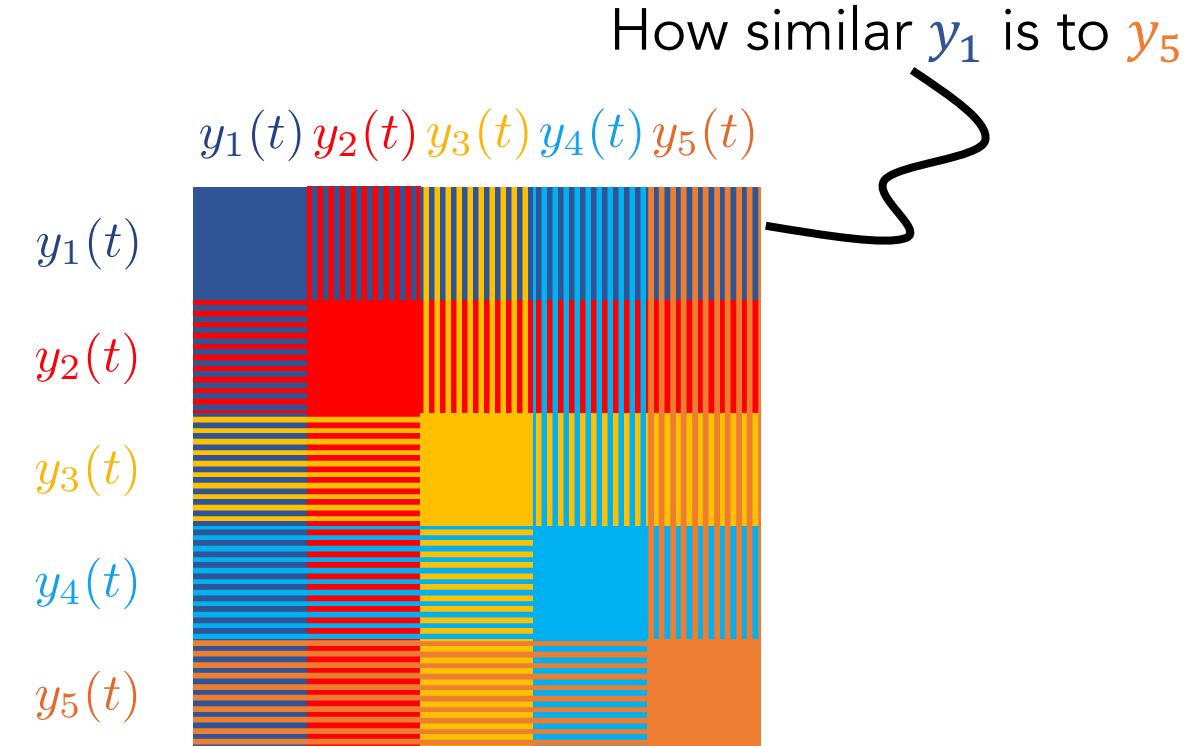
Use averages: $E[\cdot]$



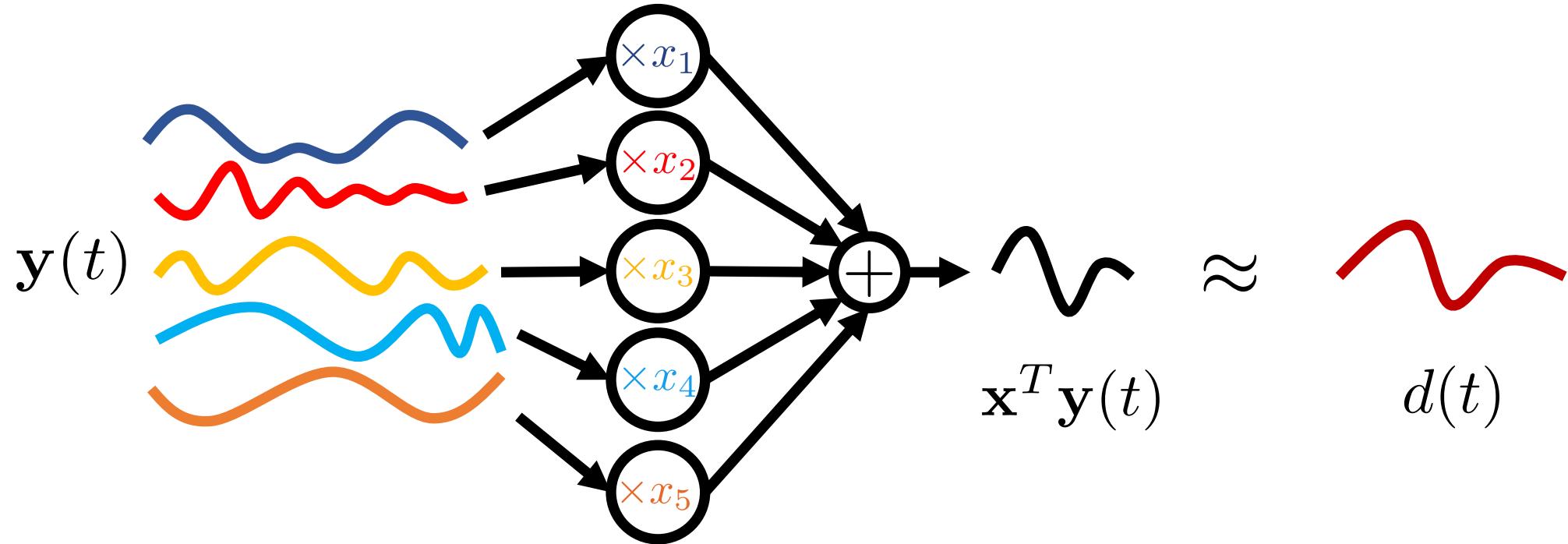
Covariance matrix

$$R_{\mathbf{y}\mathbf{y}} = \mathbb{E}[\mathbf{y}(t)\mathbf{y}^T(t)]$$

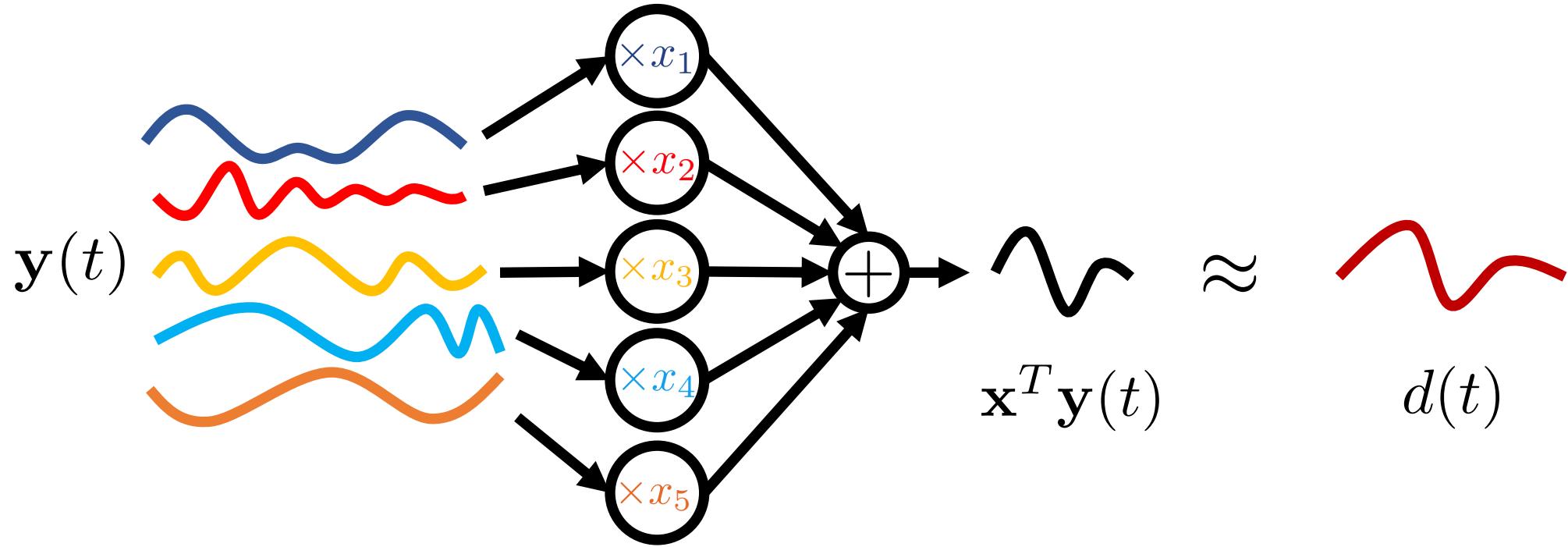
$$\mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \\ y_5(t) \end{bmatrix}$$



MMSE: “ $\mathbf{x}^T \mathbf{y}(t)$ closest to $d(t)$ on average”



MMSE: “ $\mathbf{x}^T \mathbf{y}(t)$ closest to $d(t)$ on average”



$$\min_{\mathbf{x}} \mathbb{E}[|d(t) - \mathbf{x}^T \mathbf{y}(t)|^2]$$

$$\mathbf{x}^* = R_{\mathbf{yy}}^{-1} \mathbf{r}_{yd}$$

Cannot compute $\mathbb{E}[\cdot]$ in practice

Solution: Use sample averages

Collect: $\{\mathbf{y}(0), \mathbf{y}(1), \dots, \mathbf{y}(N - 1)\}$

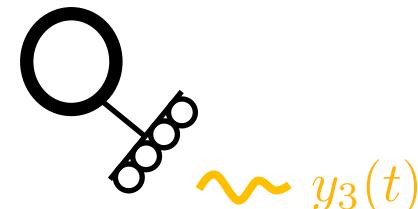
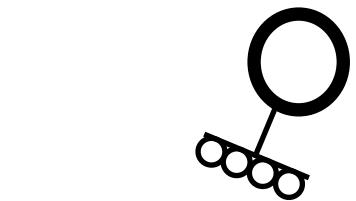
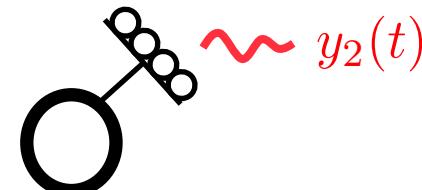
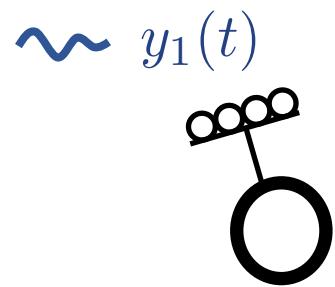
Replace: $R_{\mathbf{y}\mathbf{y}} = \mathbb{E}[\mathbf{y}(t)\mathbf{y}^T(t)] \approx \frac{1}{N} \sum_{\tau=0}^{N-1} \mathbf{y}(\tau)\mathbf{y}^T(\tau)$

Spatial filtering tasks

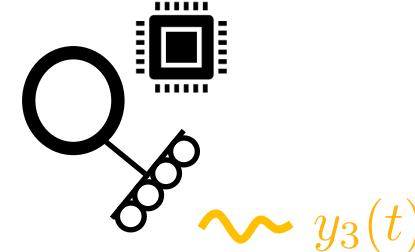
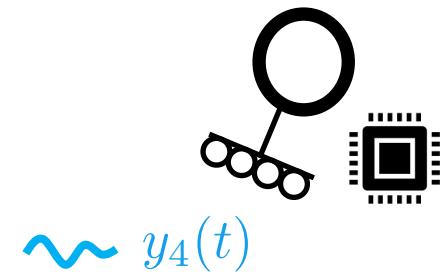
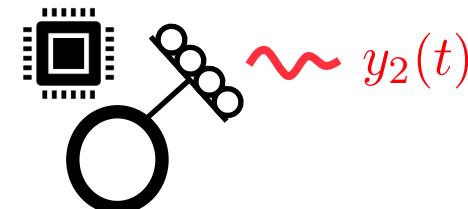
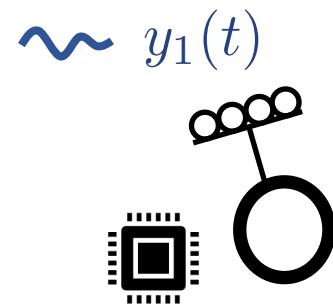
$$\underset{\mathbf{x}}{\text{minimize}} \ f(\mathbf{x}^T \mathbf{y}(t))$$

subject to $\mathbf{x} \in \mathcal{S}$

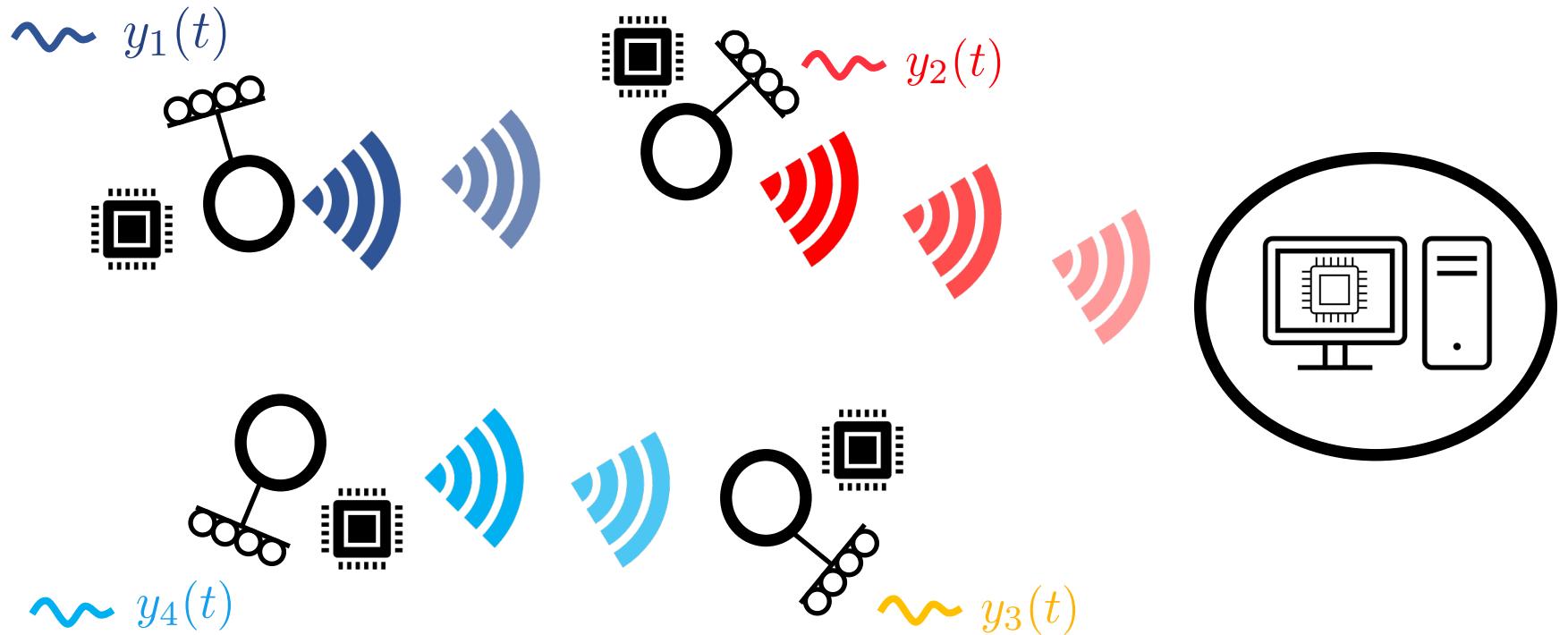
Wireless sensor networks (WSNs)



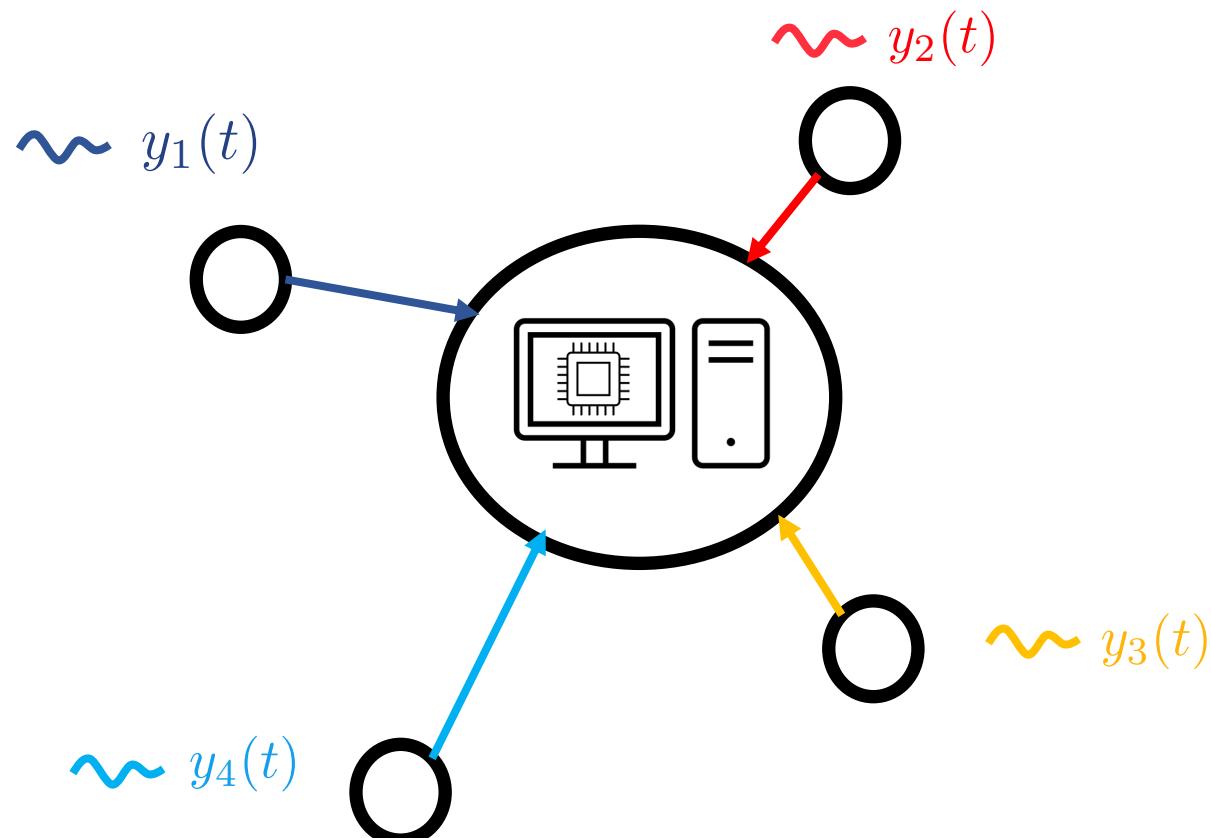
Wireless sensor networks (WSNs)



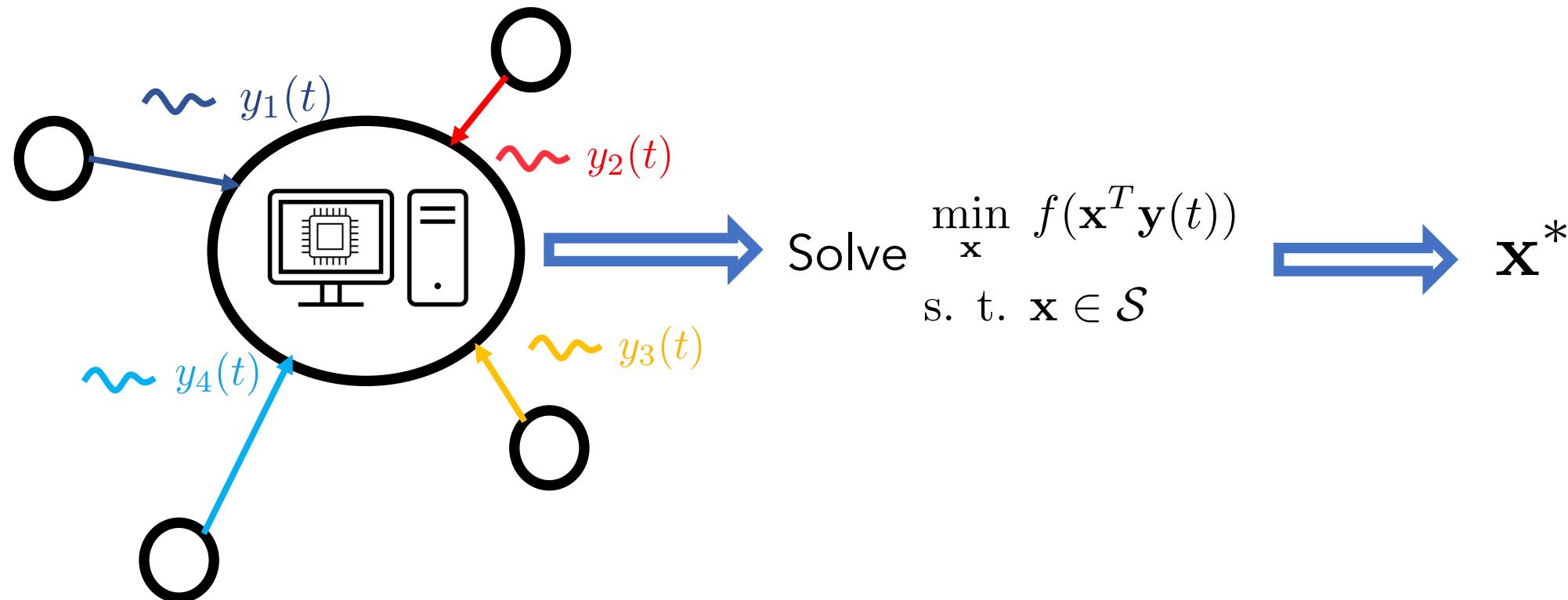
Wireless sensor networks (WSNs)

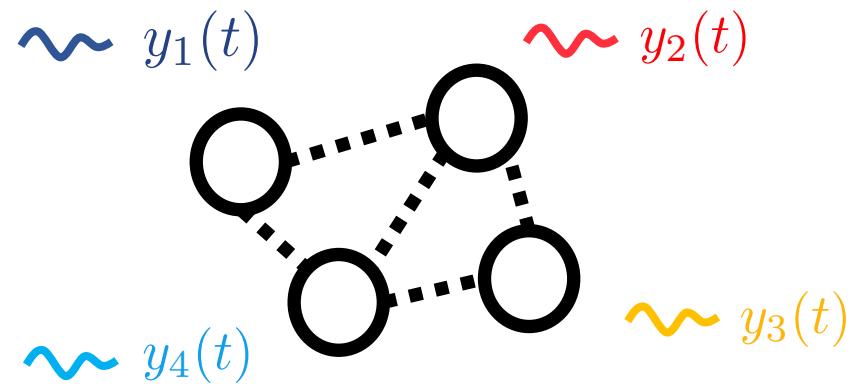
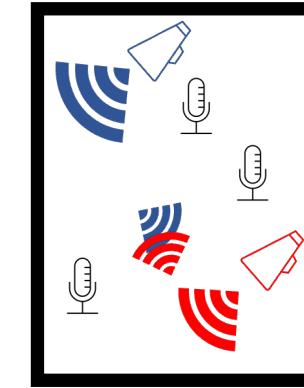
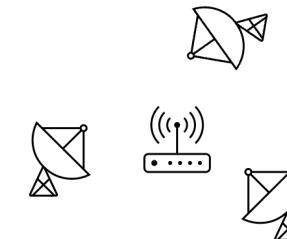
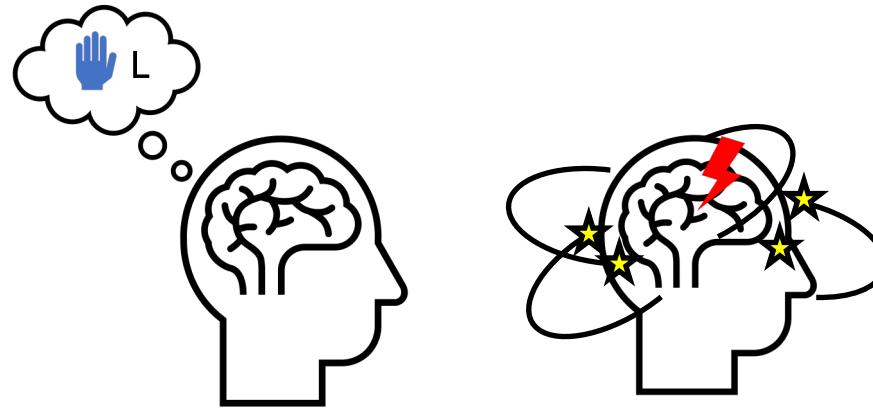


Centralized solution



Centralized solution





Limited bandwidth



Limited energy
resources

Issues with centralized solution



Bandwidth

$\sim y_1(t)$



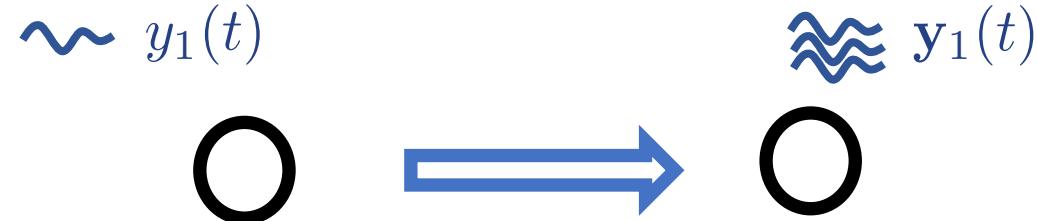
$\approx y_1(t)$



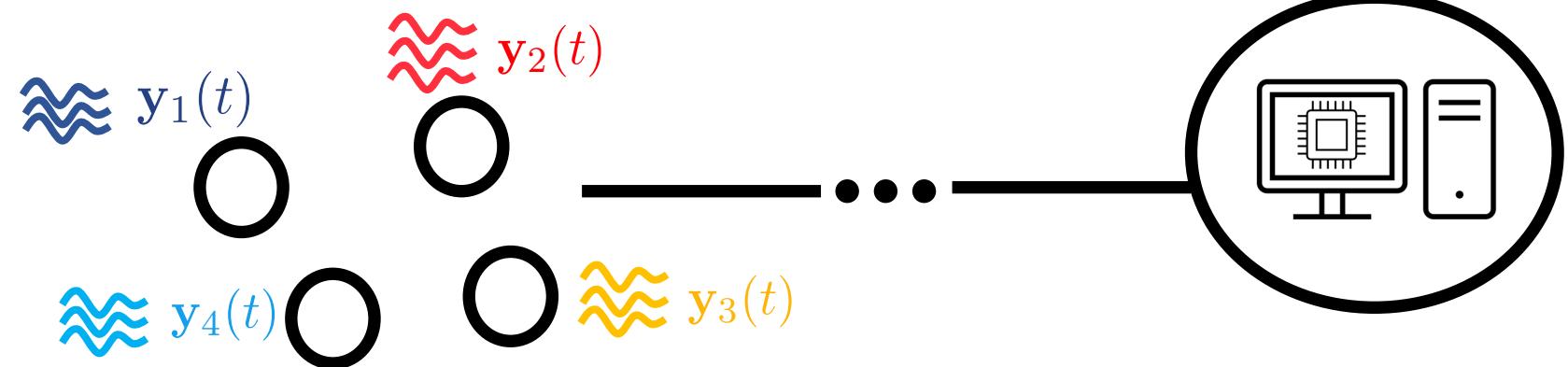
Issues with centralized solution



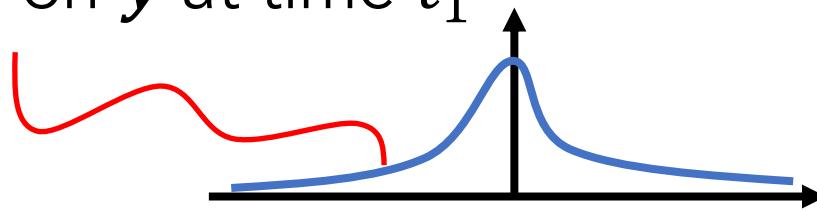
Bandwidth



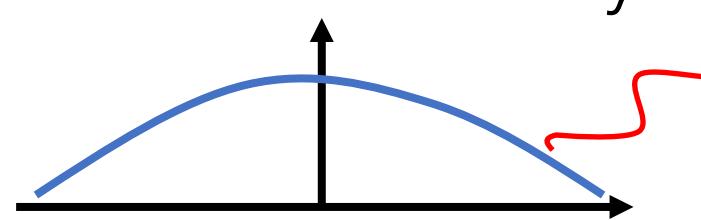
Energy



Uncertainty on \mathbf{y} at time t_1



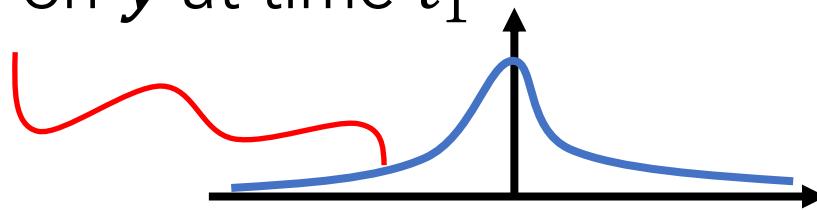
Uncertainty on \mathbf{y} at time t_2



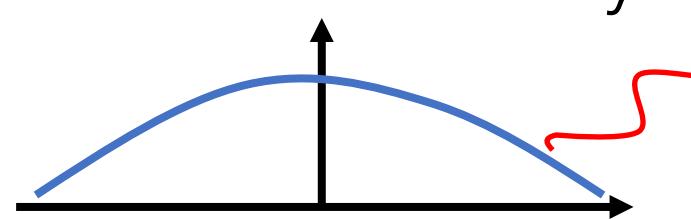
$$R_{\mathbf{yy}}(t_1) \neq R_{\mathbf{yy}}(t_2)$$

$$\mathbf{x}^*(t_1) \neq \mathbf{x}^*(t_2)$$

Uncertainty on \mathbf{y} at time t_1



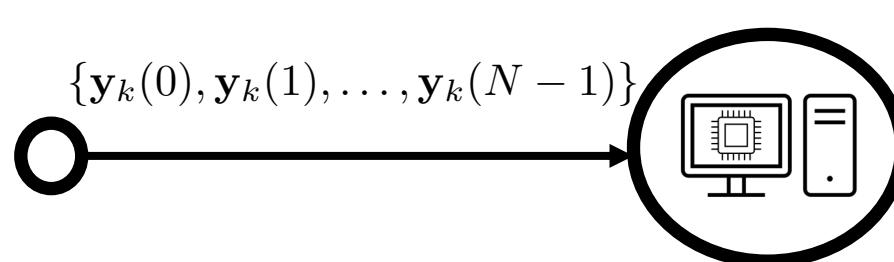
Uncertainty on \mathbf{y} at time t_2



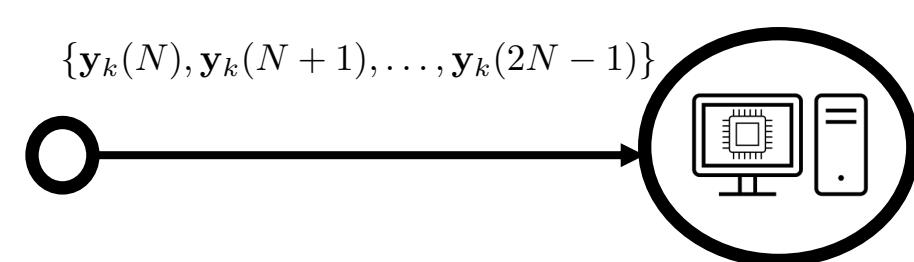
$$R_{\mathbf{yy}}(t_1) \neq R_{\mathbf{yy}}(t_2)$$

$$\mathbf{x}^*(t_1) \neq \mathbf{x}^*(t_2)$$

Not a 1-time operation



Collect new data

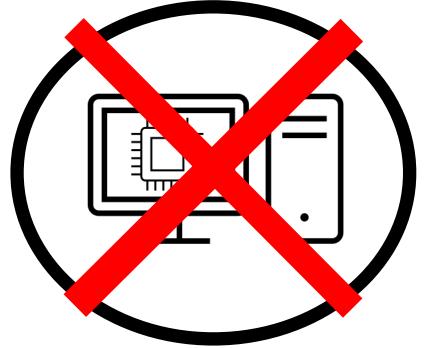


Collect new data

...

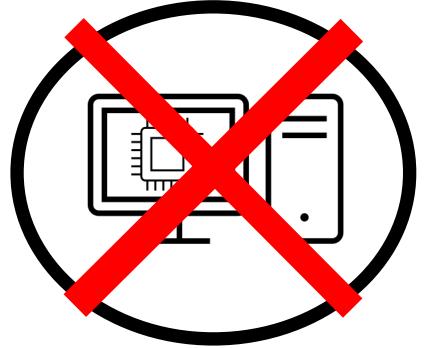
Distributed solutions for WSNs

Existing methods: Consensus, diffusion, ADMM...



Distributed solutions for WSNs

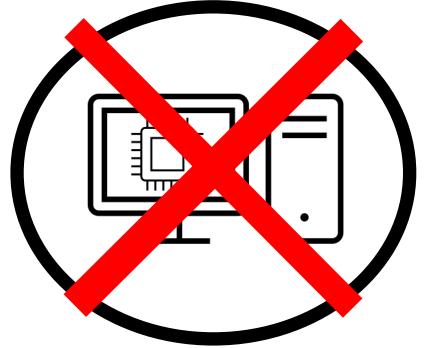
Existing methods: Consensus, diffusion, ADMM...



Node-separable objective

$$f(\mathbf{x}) = \sum_k f_k(\mathbf{x})$$

Distributed solutions for WSNs



Existing methods: Consensus, diffusion, ADMM...

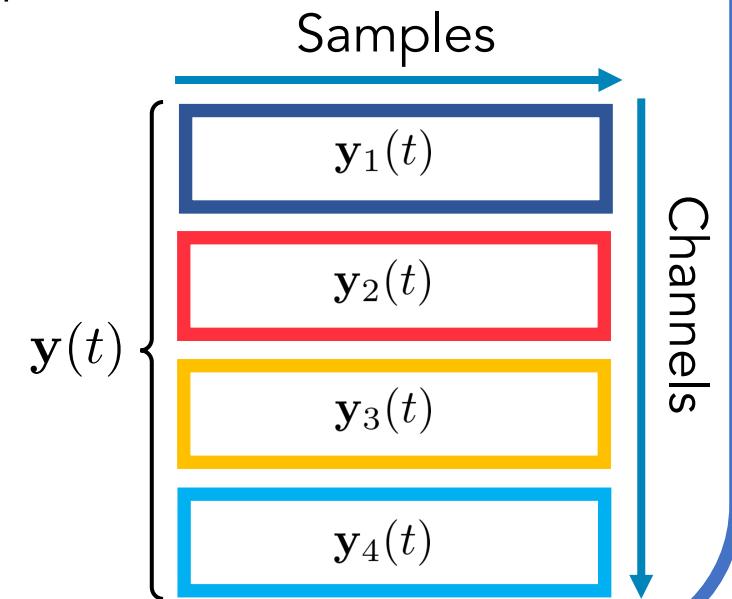
Node-separable objective

$$f(\mathbf{x}) = \sum_k f_k(\mathbf{x}) \neq$$

Spatial filtering

Objective is *not* node-separable

$$f(\mathbf{x}^T \mathbf{y}(t))$$



Distributed spatial filtering in WSNs

Task 1: MMSE

Design task-specific distributed algorithm

Task 2: LCMV

Design task-specific
distributed algorithm

Task 3: PCA

Design task-specific distributed algorithm

Distributed spatial filtering in WSNs

Task 1: MMSE

Design task-specific

distributed algorithm

Task 2: LCMV

Design task-specific

distributed algorithm

Task 3: PCA

Design task-specific

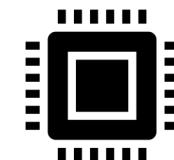
distributed algorithm

Distributed spatial filtering in WSNs

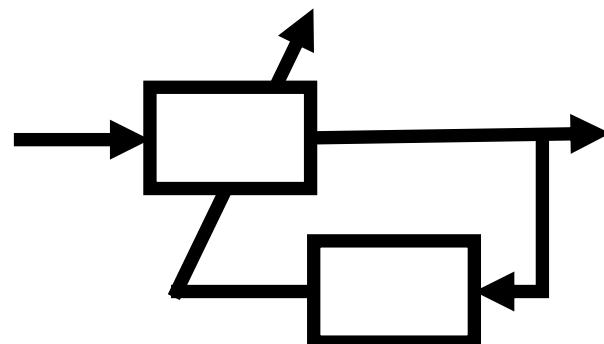
Unified and Generalizable
1 method for spatial filtering tasks



Efficient communication



Computational efficiency

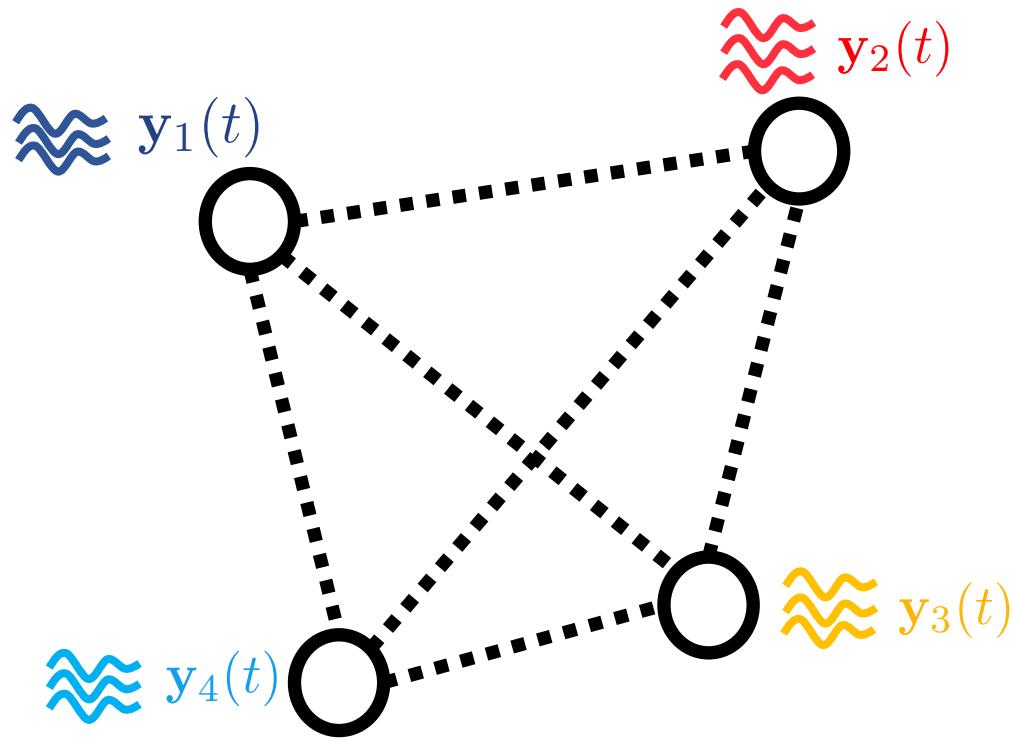


Adaptive

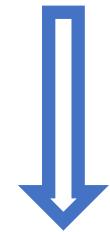
I. The Distributed Adaptive Signal Fusion (DASF) framework

C. A. Musluoglu and A. Bertrand, "A Unified Algorithmic Framework for Distributed Adaptive Signal and Feature Fusion Problems—Part I: Algorithm Derivation," in *IEEE Transactions on Signal Processing*, vol. 71, pp. 1863-1878, 2023, doi: 10.1109/TSP.2023.3275272.

C. A. Musluoglu, C. Hovine and A. Bertrand, "A Unified Algorithmic Framework for Distributed Adaptive Signal and Feature Fusion Problems — Part II: Convergence Properties," in *IEEE Transactions on Signal Processing*, vol. 71, pp. 1879-1894, 2023, doi: 10.1109/TSP.2023.3275273. 44

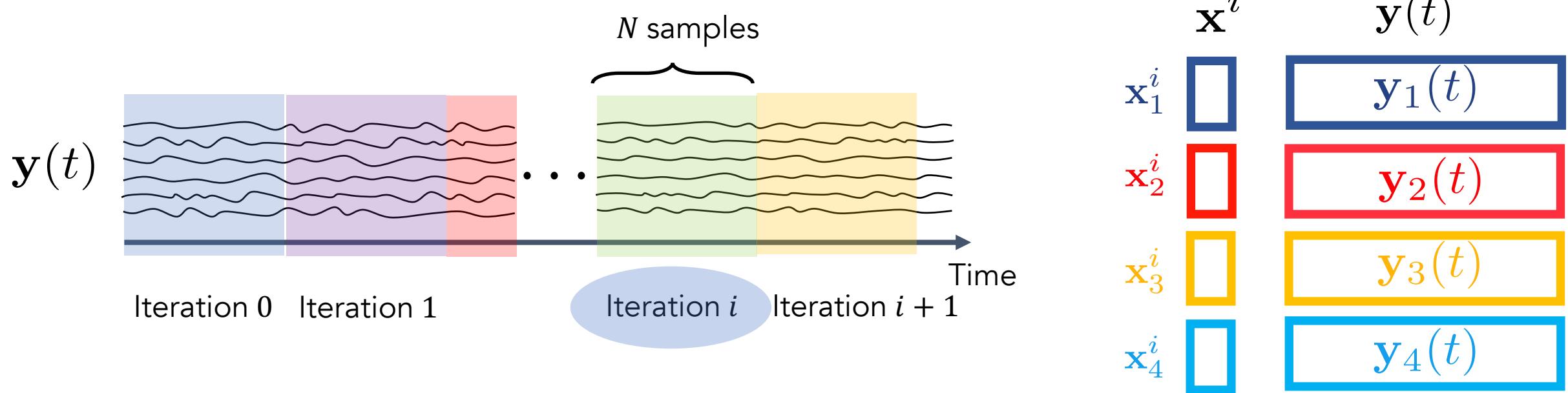


$\underset{\mathbf{x}}{\text{minimize}} \ f(\mathbf{x}^T \mathbf{y}(t))$
 subject to $\mathbf{x} \in \mathcal{S}$

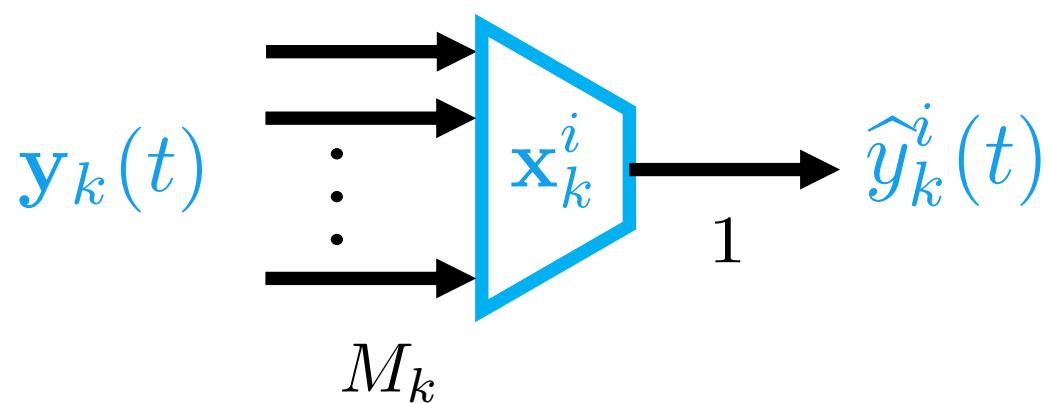


$$\mathbf{y}(t) = \begin{bmatrix} \mathbf{y}_1(t) \\ \mathbf{y}_2(t) \\ \mathbf{y}_3(t) \\ \mathbf{y}_4(t) \end{bmatrix}$$

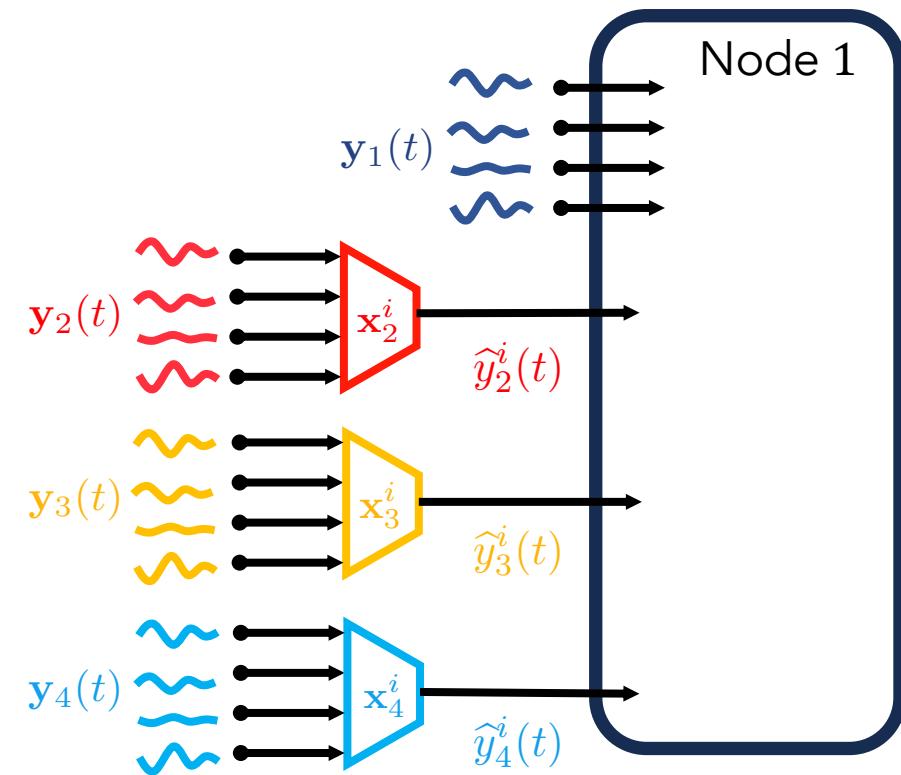
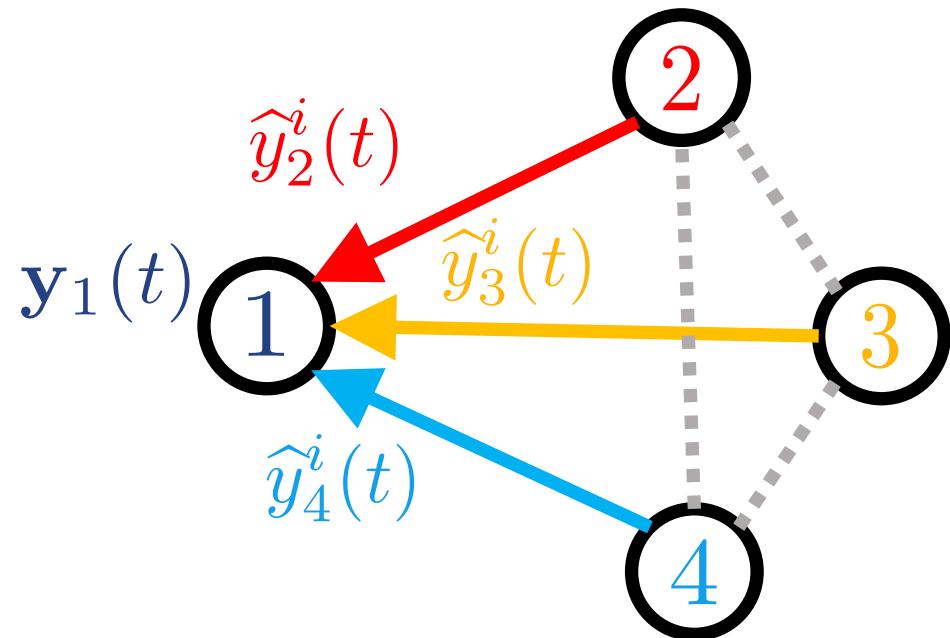
$\underset{\mathbf{x}}{\text{minimize}} \ f(\mathbf{x}^T \mathbf{y}(t))$
 subject to $g(\mathbf{x}^T \mathbf{y}(t)) = 0$
 $h(\mathbf{x}^T \mathbf{y}(t)) \leq 0$



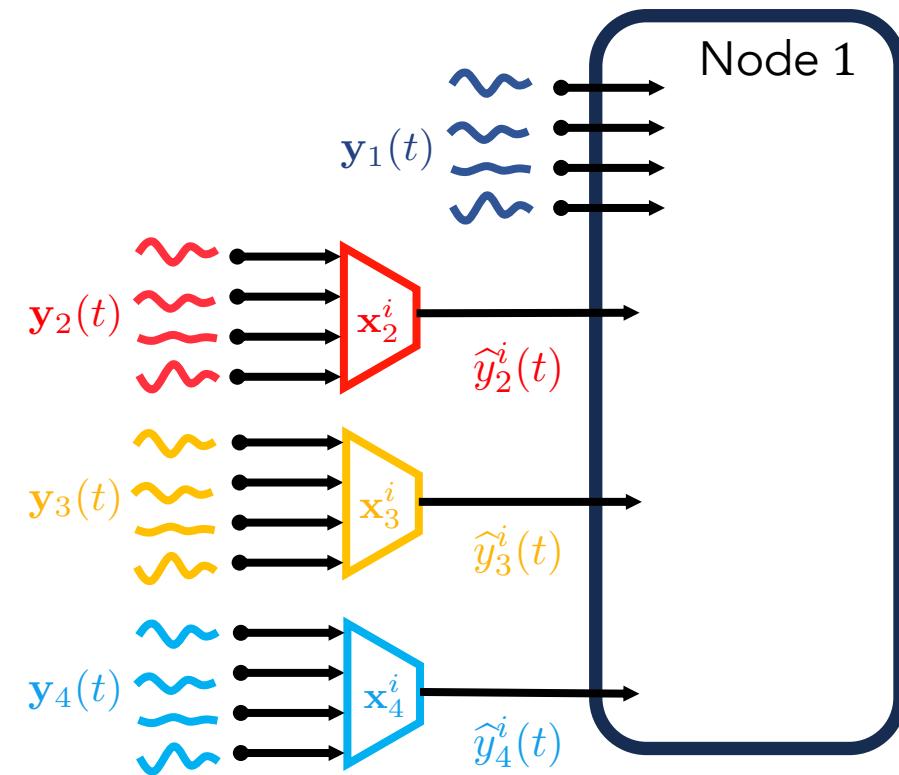
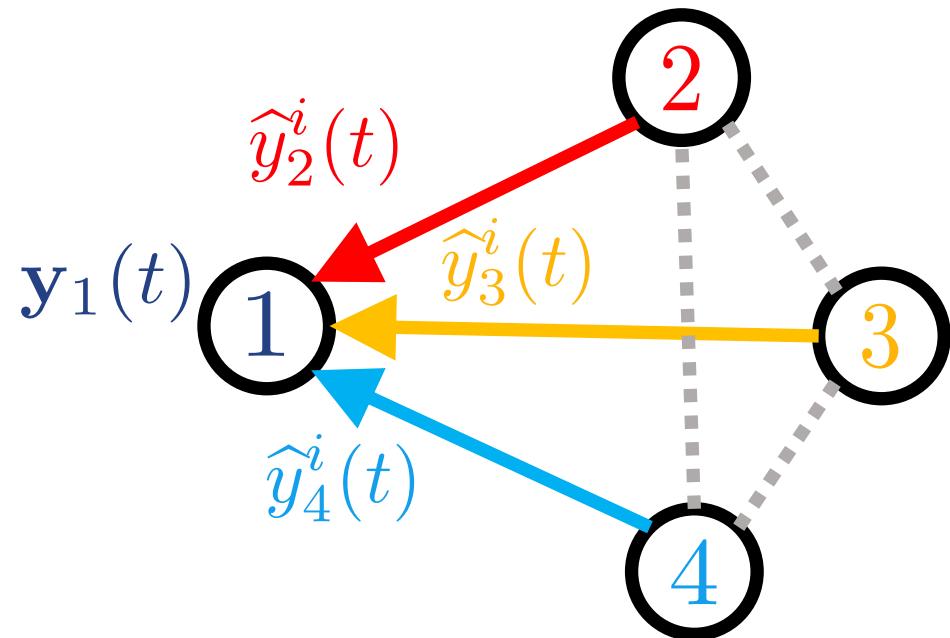
1. Collect and compress signals



2. Select updating node and transmit compressed signals



2. Select updating node and transmit compressed signals



Efficient communication

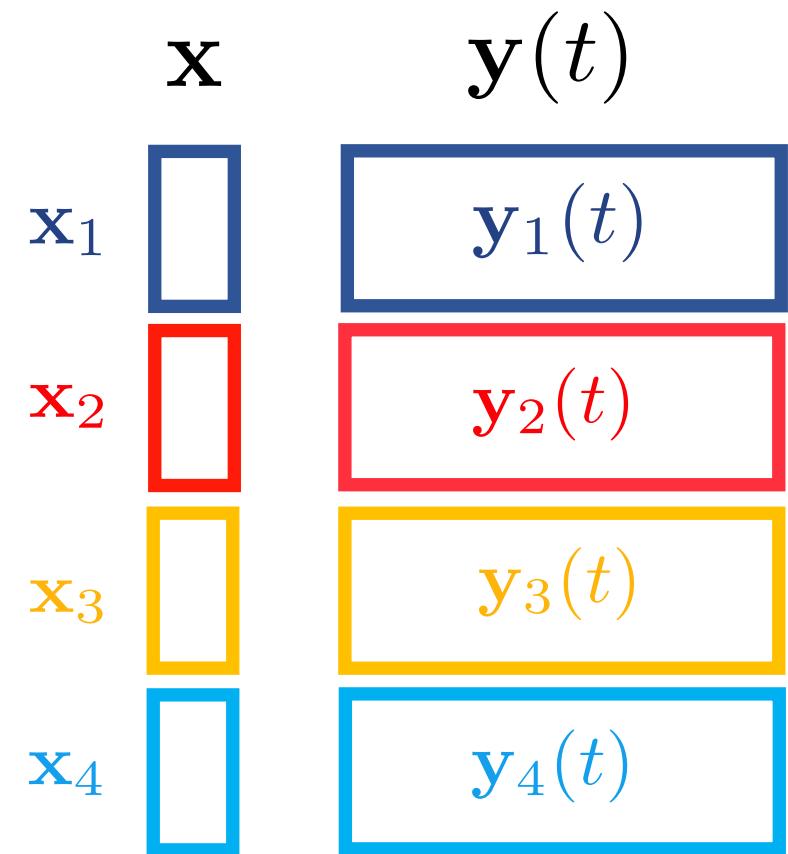
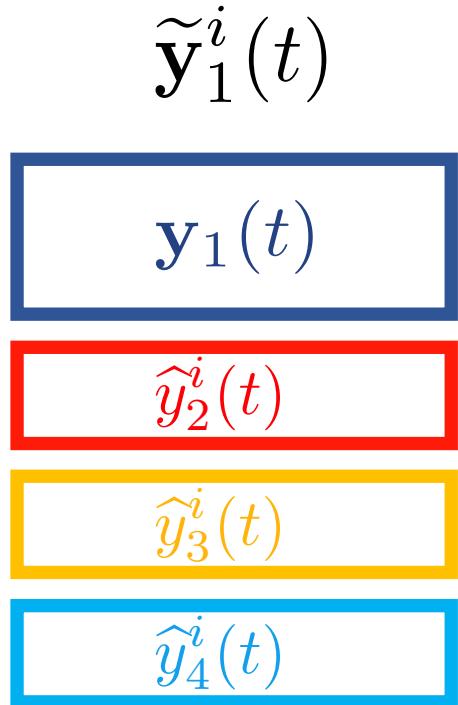
$$\tilde{\mathbf{y}}_1^i(t)$$

$$\mathbf{y}_1(t)$$

$$\widehat{y}_2^i(t)$$

$$\widehat{y}_3^i(t)$$

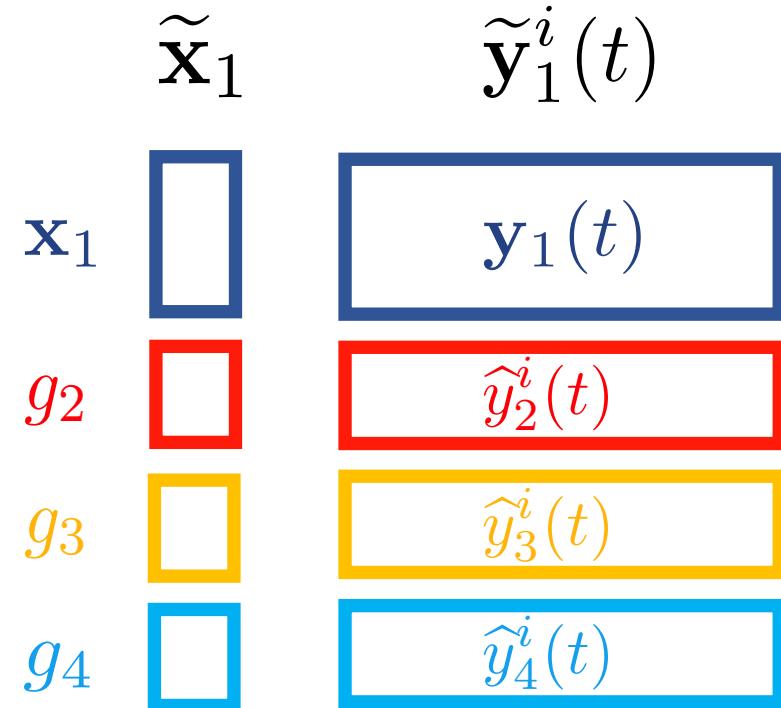
$$\widehat{y}_4^i(t)$$

$$\underset{\mathbf{x}}{\text{minimize}} \ f(\mathbf{x}^T \mathbf{y}(t))$$
$$\text{subject to } g(\mathbf{x}^T \mathbf{y}(t)) = 0$$
$$h(\mathbf{x}^T \mathbf{y}(t)) \leq 0$$


$$\underset{\tilde{\mathbf{x}}_1}{\text{minimize}} \ f(\tilde{\mathbf{x}}_1^T \tilde{\mathbf{y}}_1^i(t))$$

$$\text{subject to } g(\tilde{\mathbf{x}}_1^T \tilde{\mathbf{y}}_1^i(t)) = 0$$

$$h(\tilde{\mathbf{x}}_1^T \tilde{\mathbf{y}}_1^i(t)) \leq 0$$



| | | | |

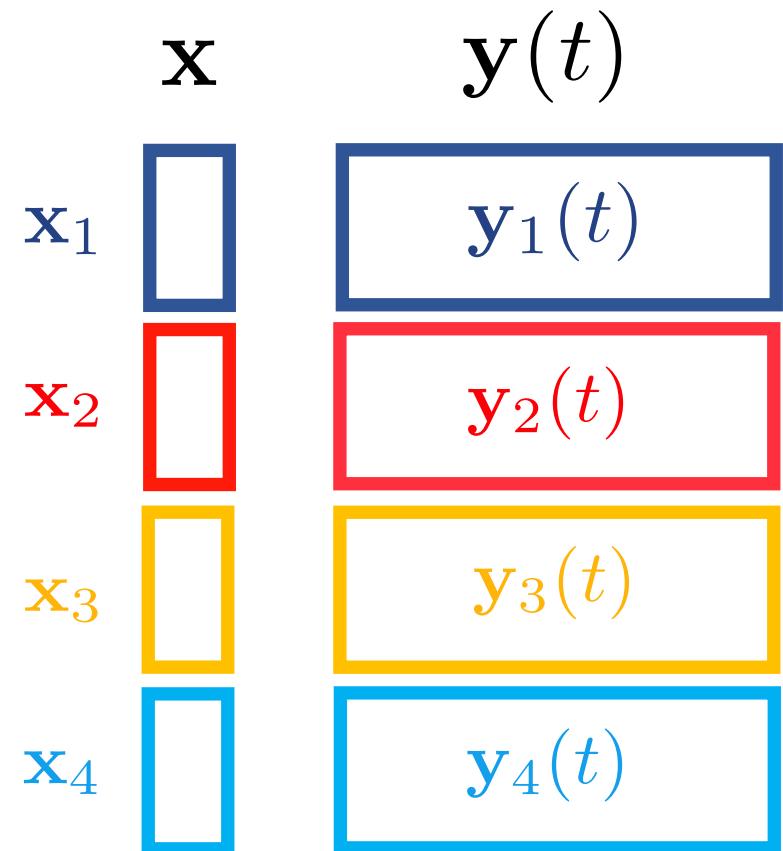
vs

| | | | |

$$\underset{\mathbf{x}}{\text{minimize}} \ f(\mathbf{x}^T \mathbf{y}(t))$$

$$\text{subject to } g(\mathbf{x}^T \mathbf{y}(t)) = 0$$

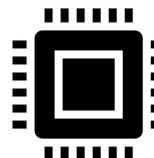
$$h(\mathbf{x}^T \mathbf{y}(t)) \leq 0$$



3. Build and solve local problem at updating node

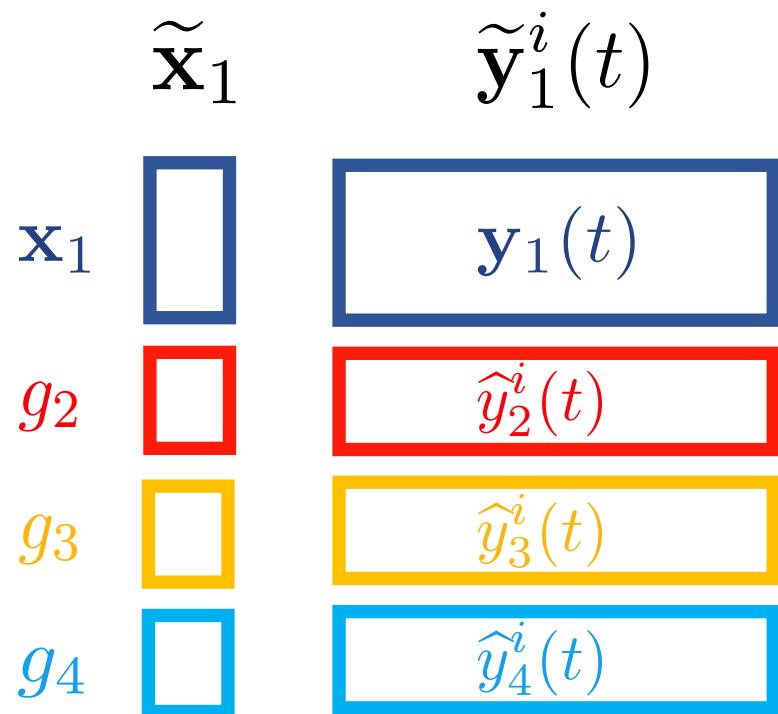
Mimic the global problem locally

$$\begin{aligned}\tilde{\mathbf{x}}_1^* &\xleftarrow{\text{CPU}} \underset{\tilde{\mathbf{x}}_1}{\text{minimize}} f(\tilde{\mathbf{x}}_1^T \tilde{\mathbf{y}}_1^i(t)) \\ &\text{subject to } g(\tilde{\mathbf{x}}_1^T \tilde{\mathbf{y}}_1^i(t)) = 0 \\ &\quad h(\tilde{\mathbf{x}}_1^T \tilde{\mathbf{y}}_1^i(t)) \leq 0\end{aligned}$$

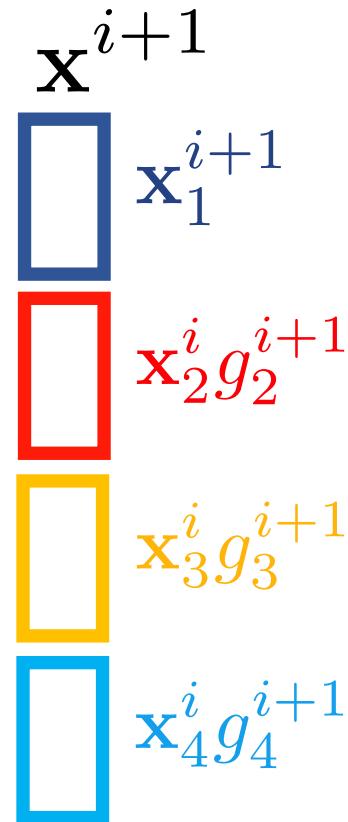
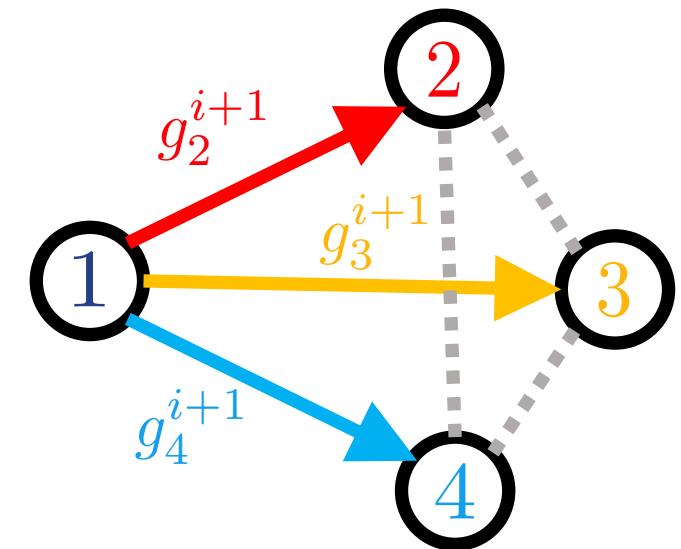
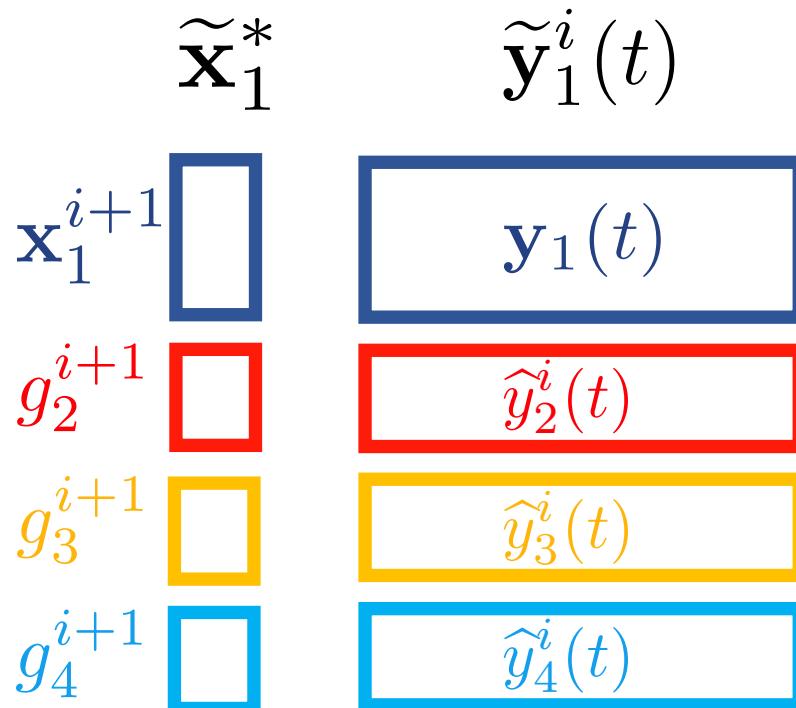


Computational efficiency

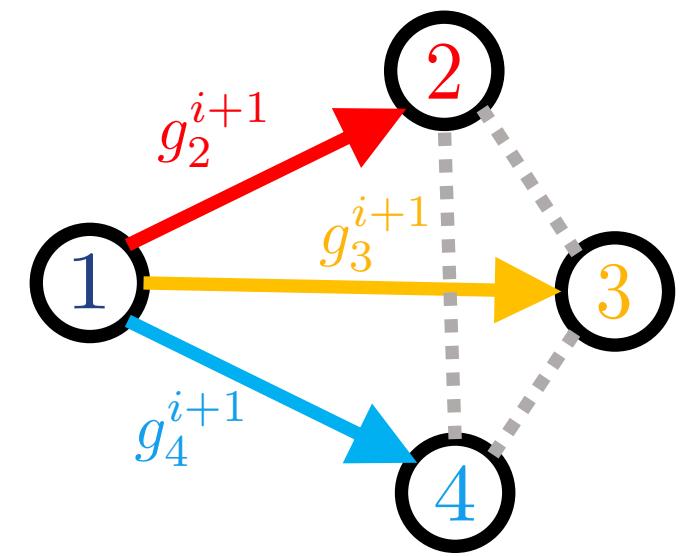
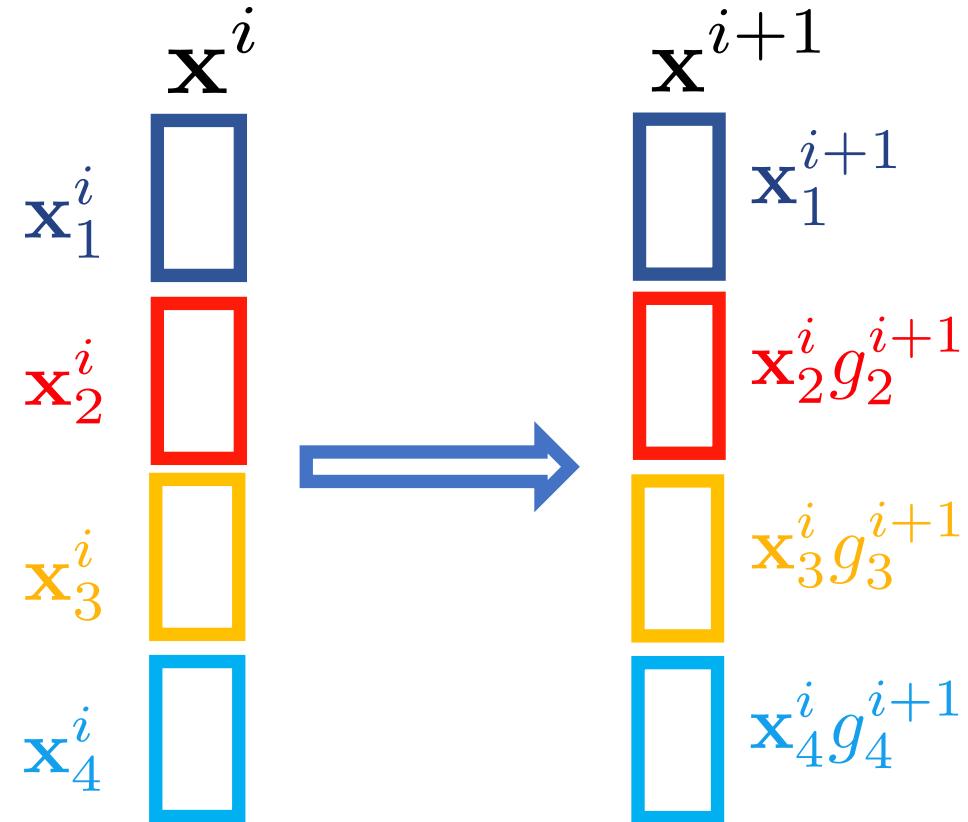
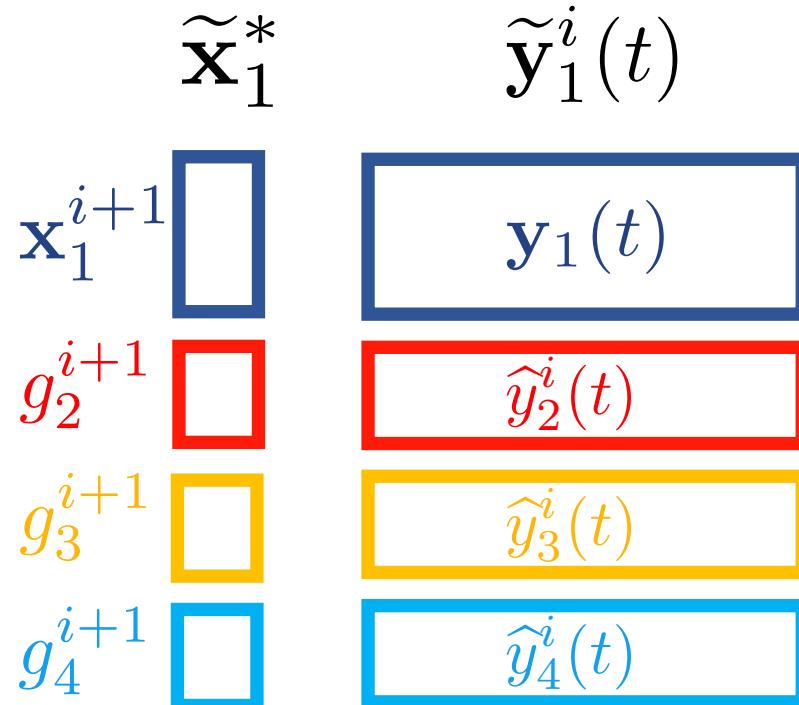
4. Make other nodes aware of this

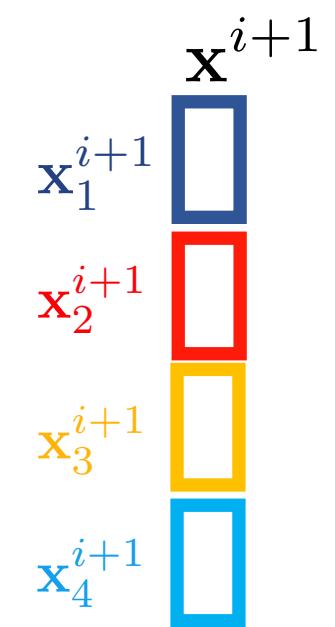
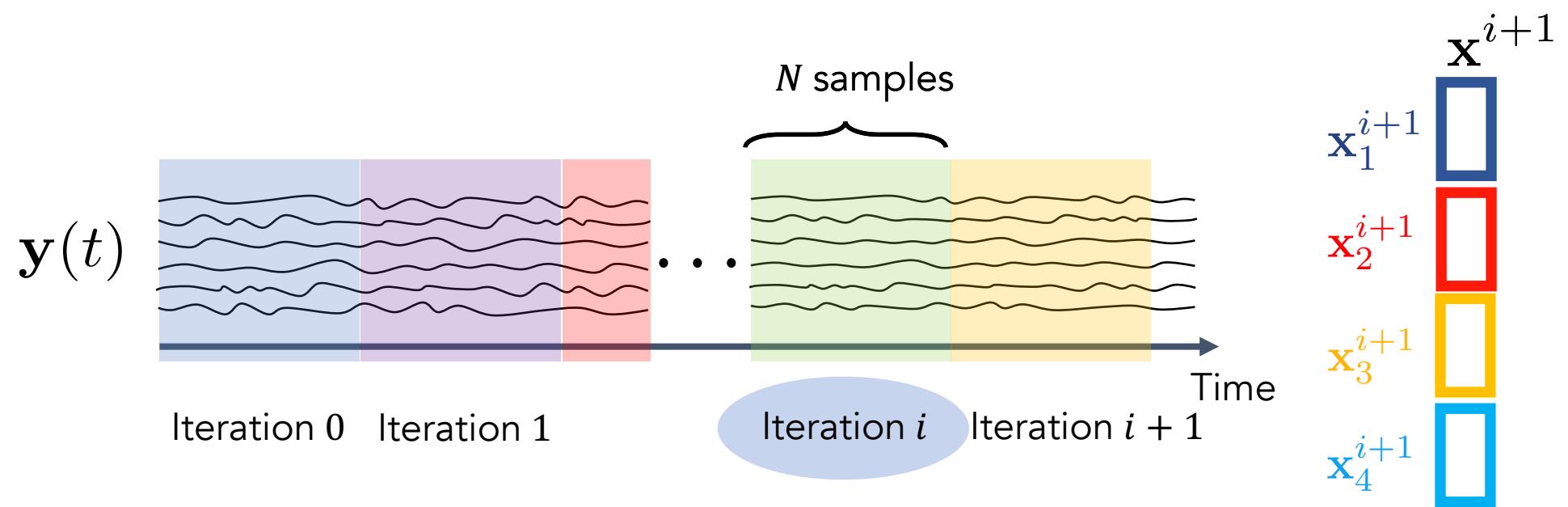


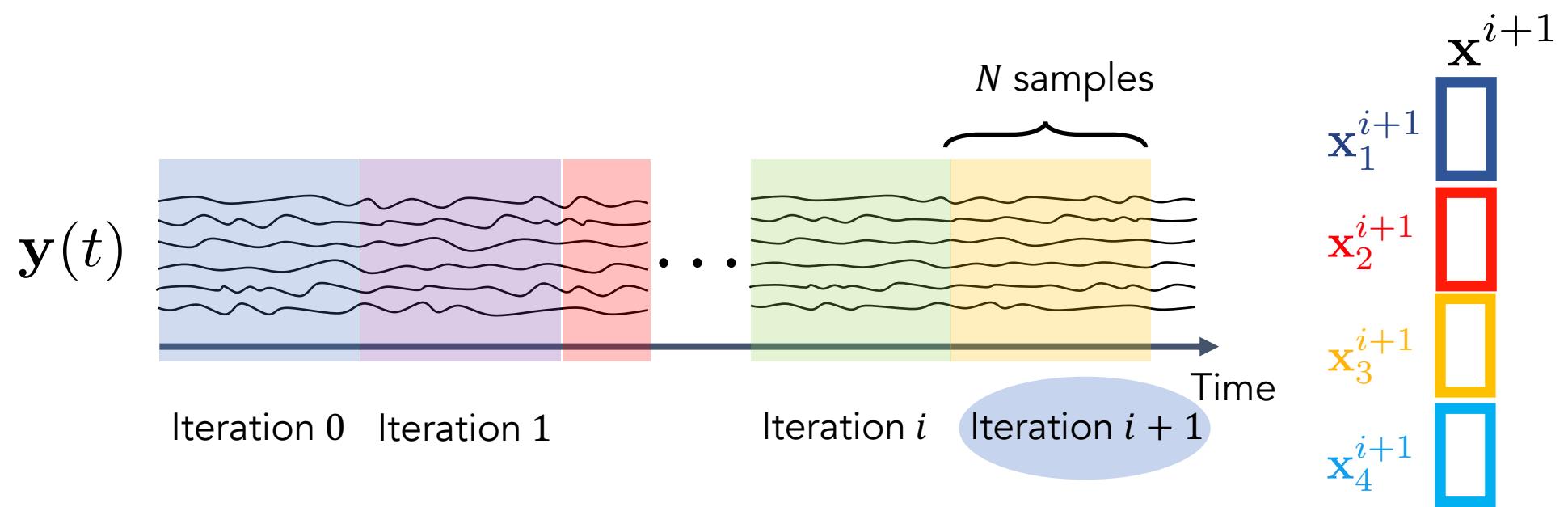
4. Make other nodes aware of this

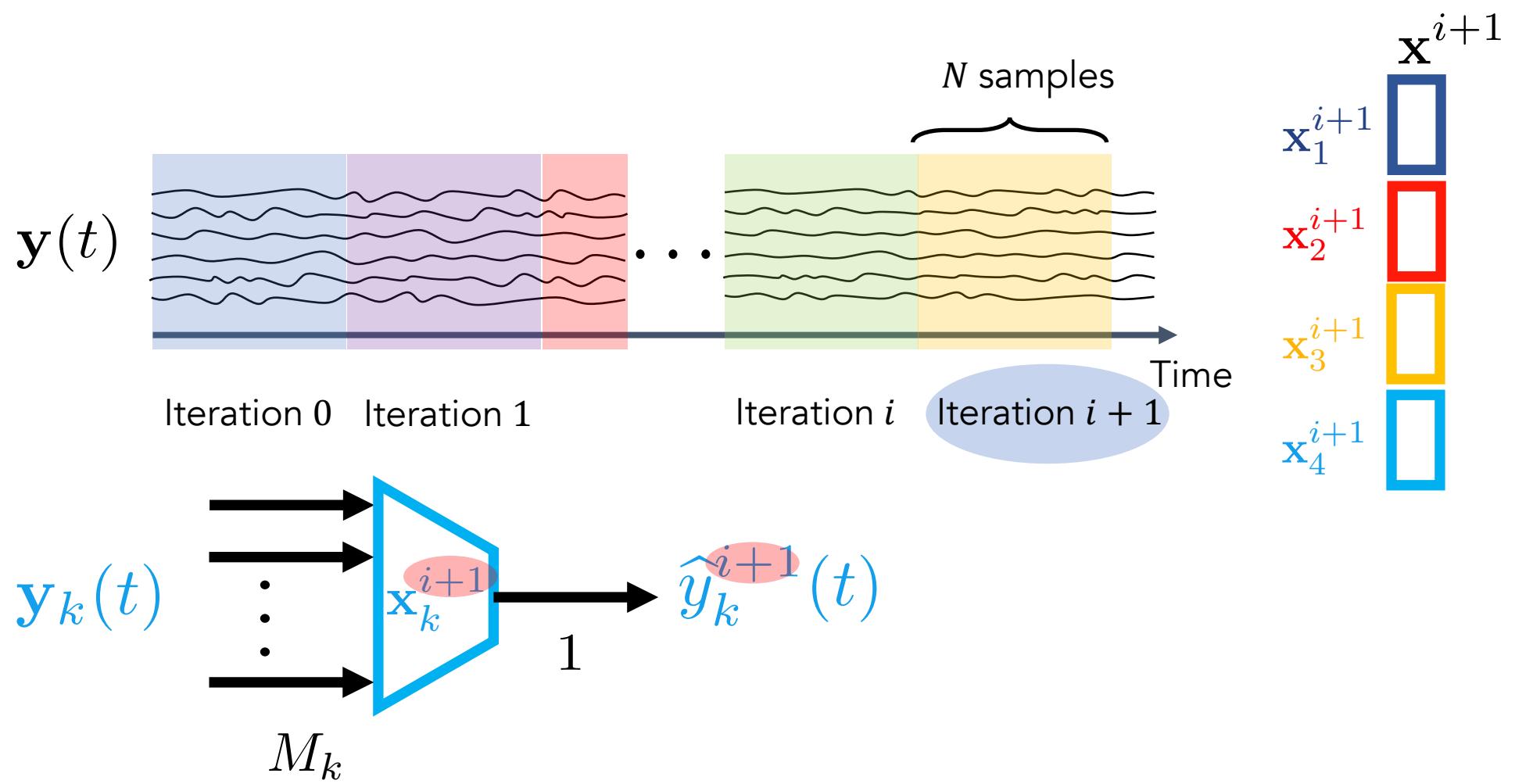


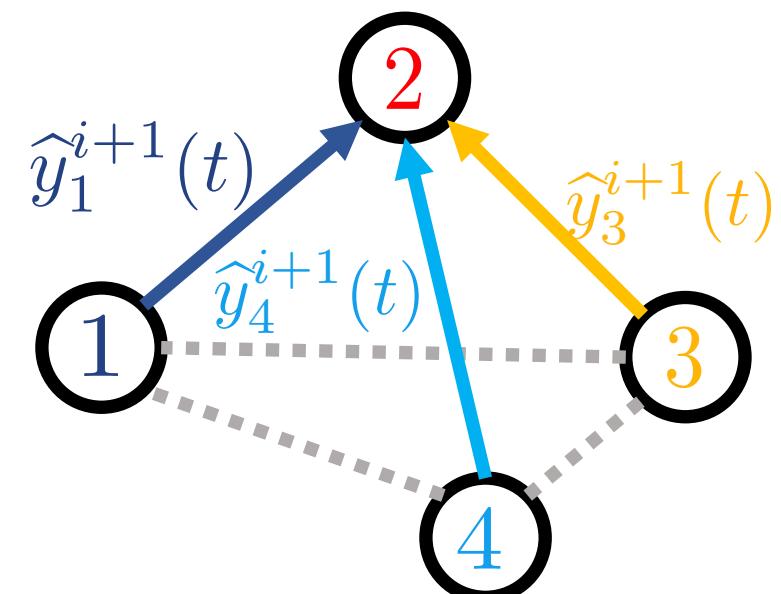
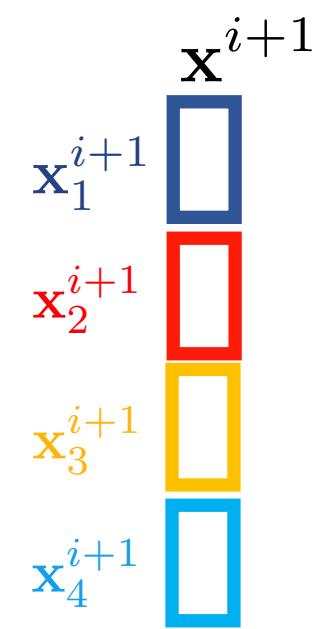
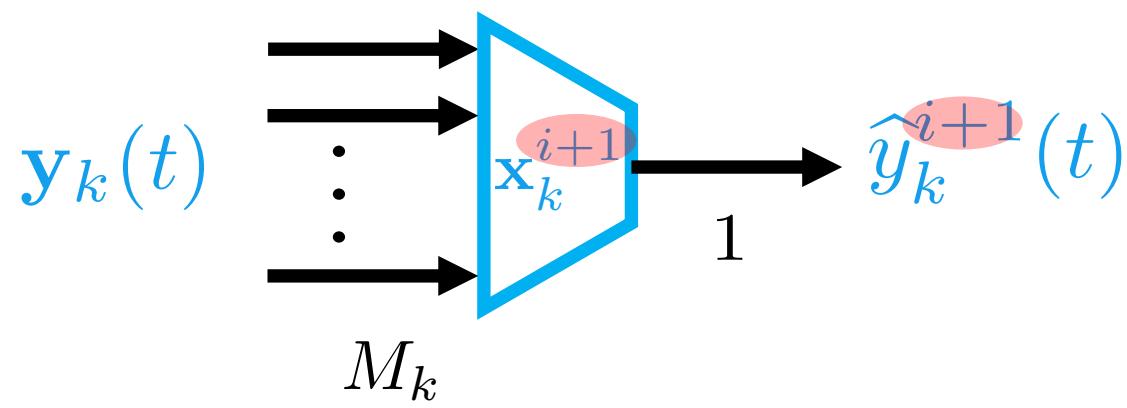
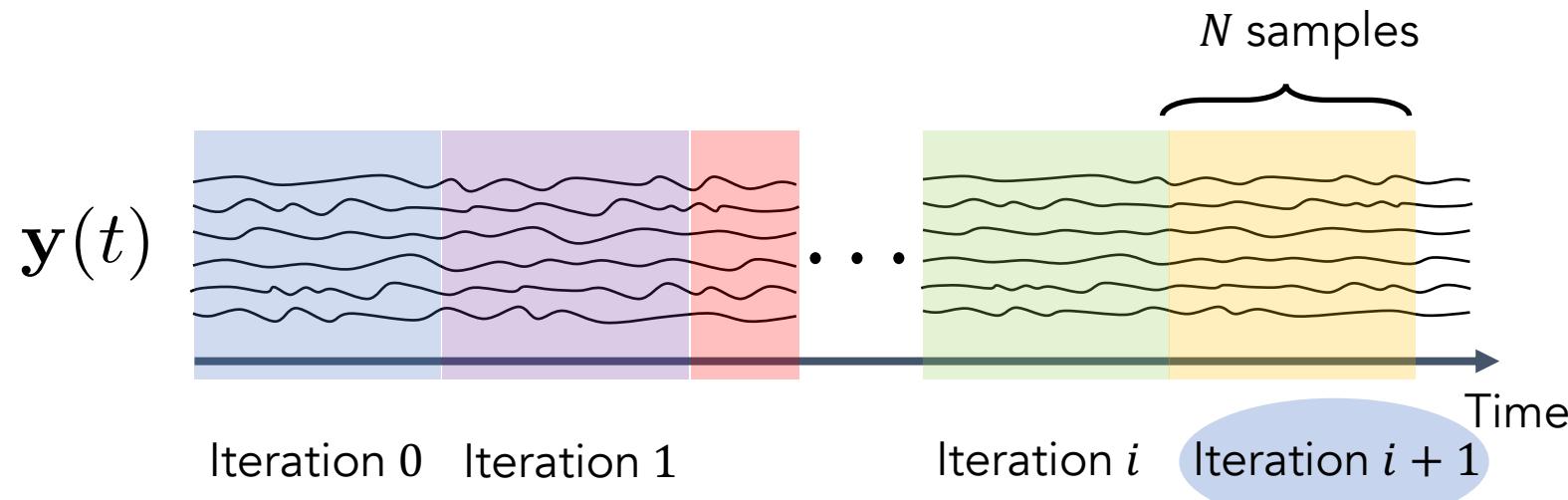
4. Make other nodes aware of this

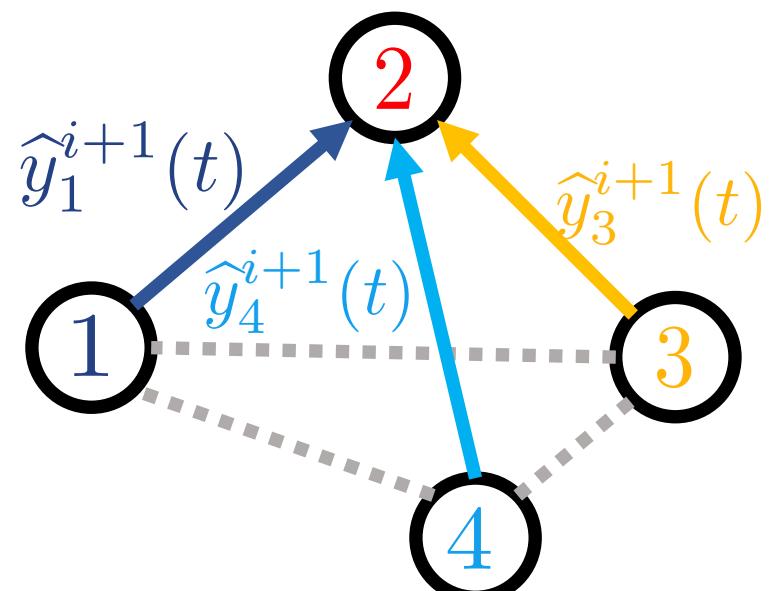
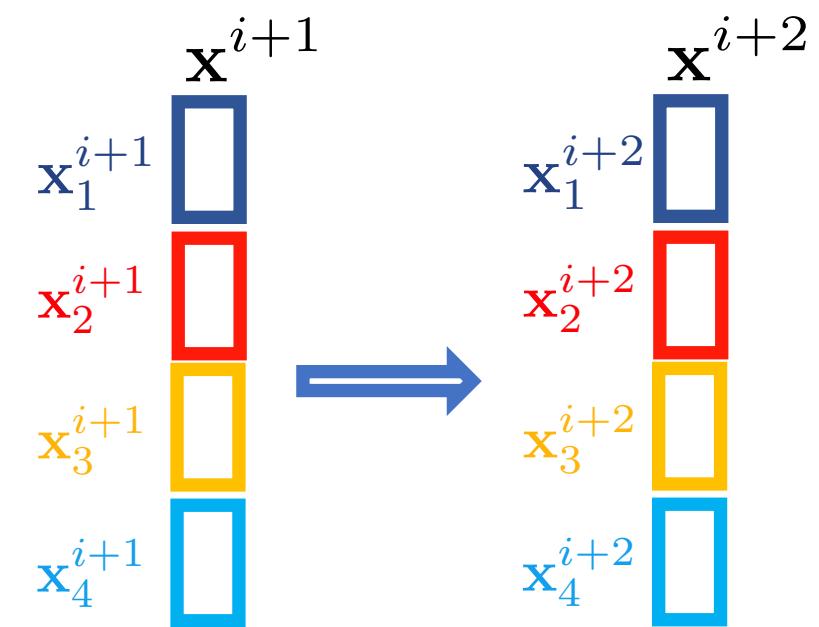
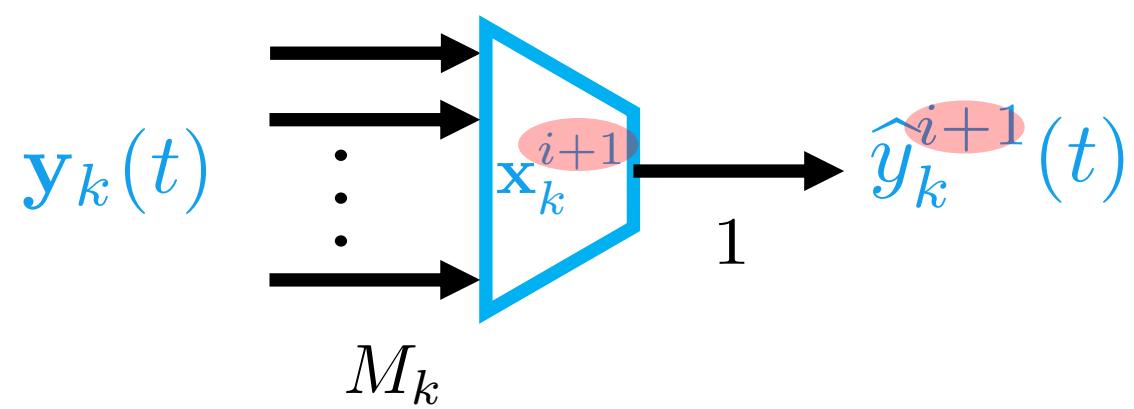
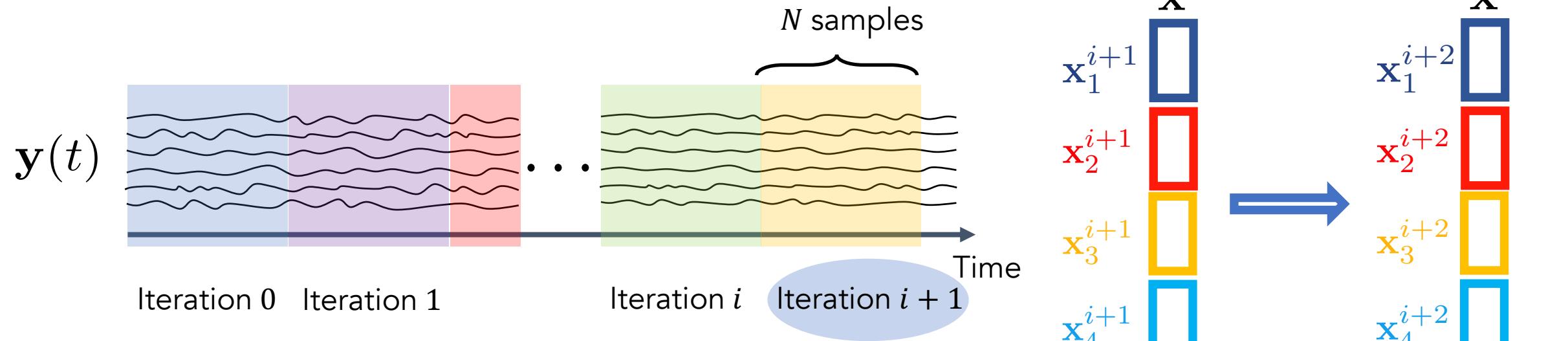


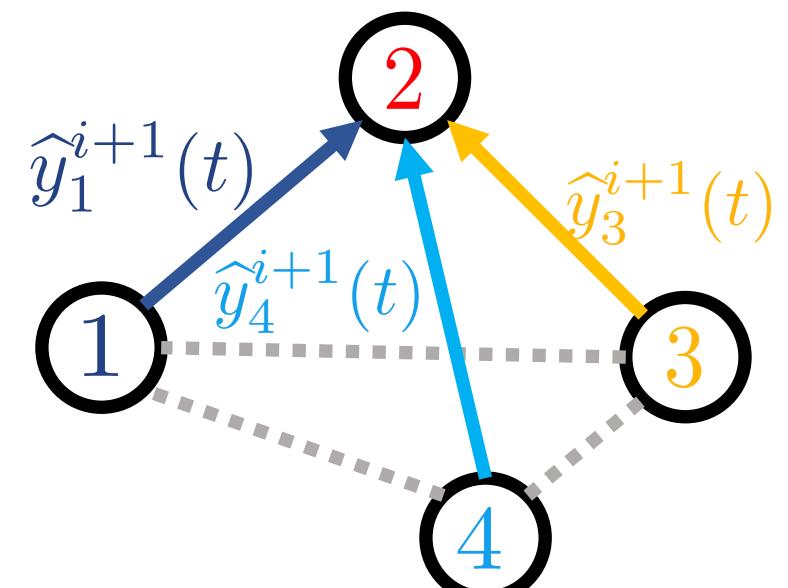
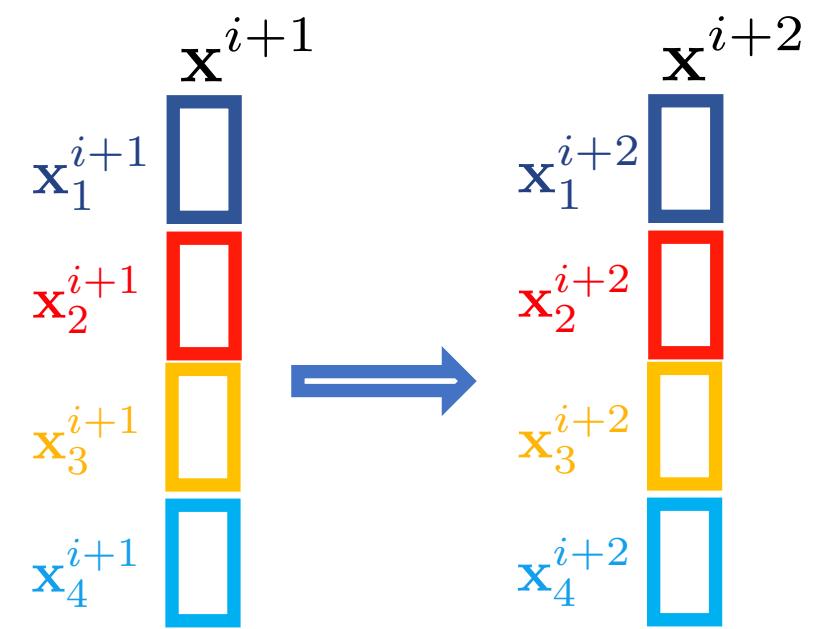
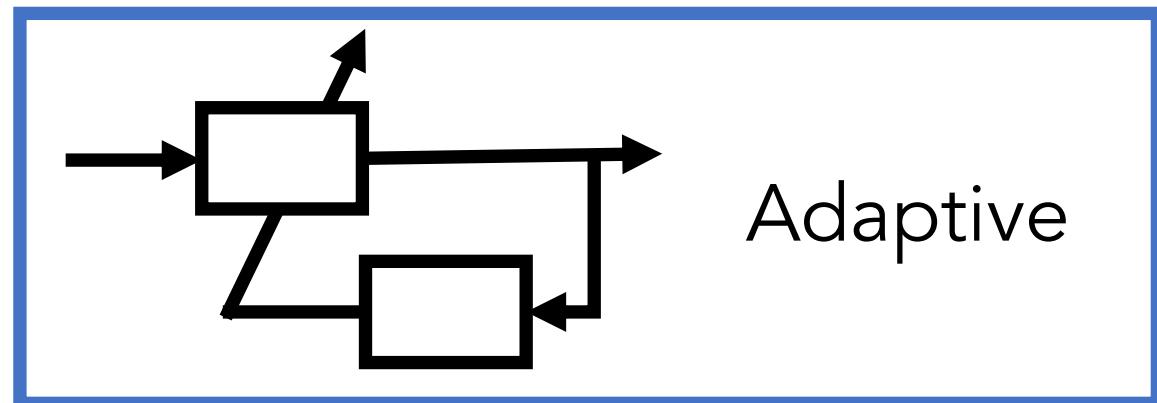
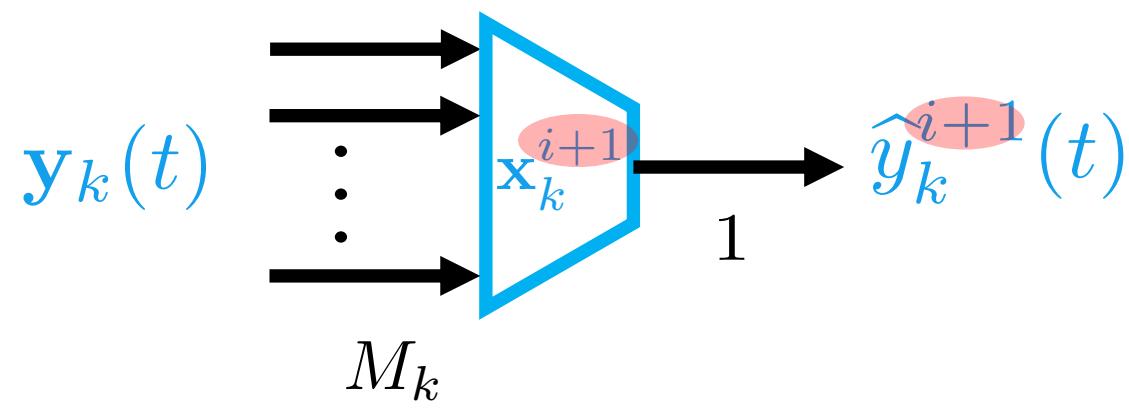
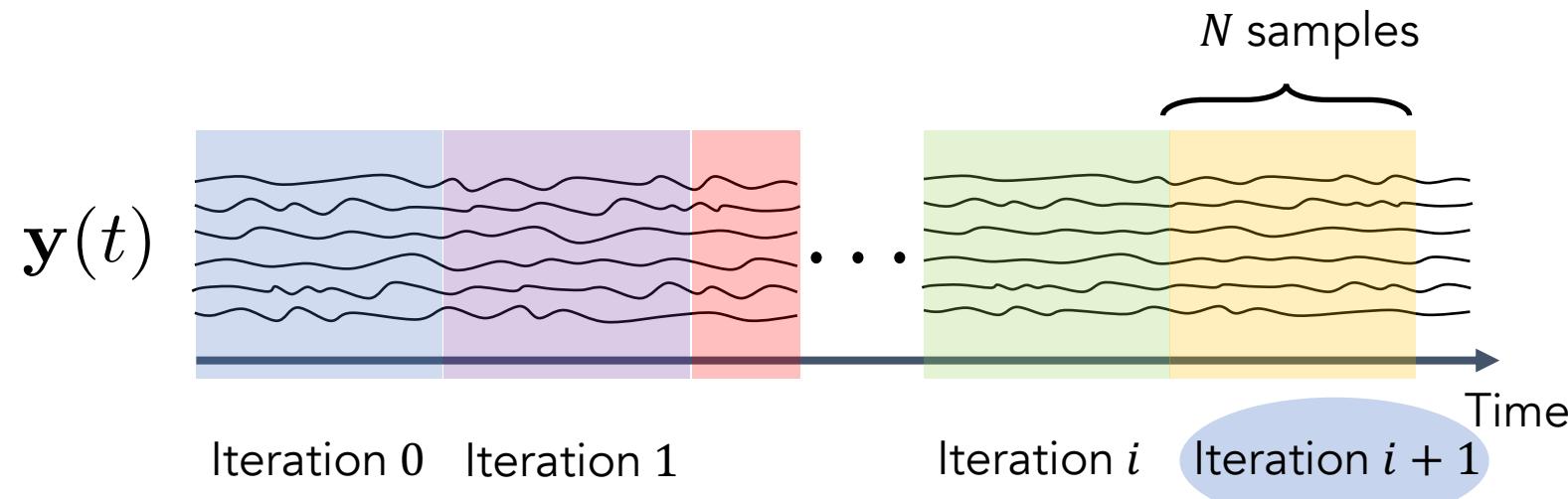












$$\min_{\mathbf{x}} \ f(\mathbf{x}^T \mathbf{y}(t))$$

$$\text{s. t. } g(\mathbf{x}^T \mathbf{y}(t)) = 0$$

$$h(\mathbf{x}^T \mathbf{y}(t)) \leq 0$$

Multi-channel outputs

$$\mathbf{x} \longrightarrow X = \begin{bmatrix} | & & | \\ \mathbf{x}(1) & \dots & \mathbf{x}(Q) \\ | & & | \end{bmatrix}$$

Multiple signals

$$X^T \mathbf{y}(t), X^T \mathbf{v}(t), \dots$$

GEVD

$$\begin{aligned} & \min_{\mathbf{x}} f(\mathbf{x}^T \mathbf{y}(t)) \\ & \text{s. t. } g(\mathbf{x}^T \mathbf{y}(t)) = 0 \\ & \quad h(\mathbf{x}^T \mathbf{y}(t)) \leq 0 \end{aligned}$$

Parameters

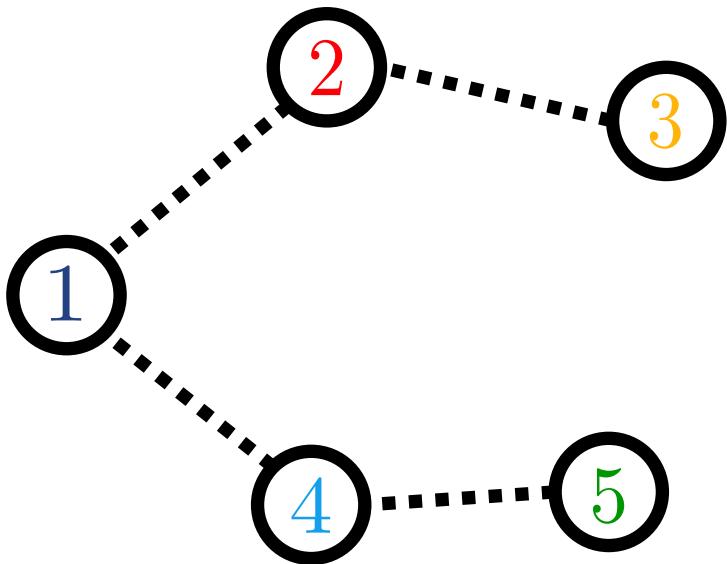
$$X^T B, X^T X, \dots$$

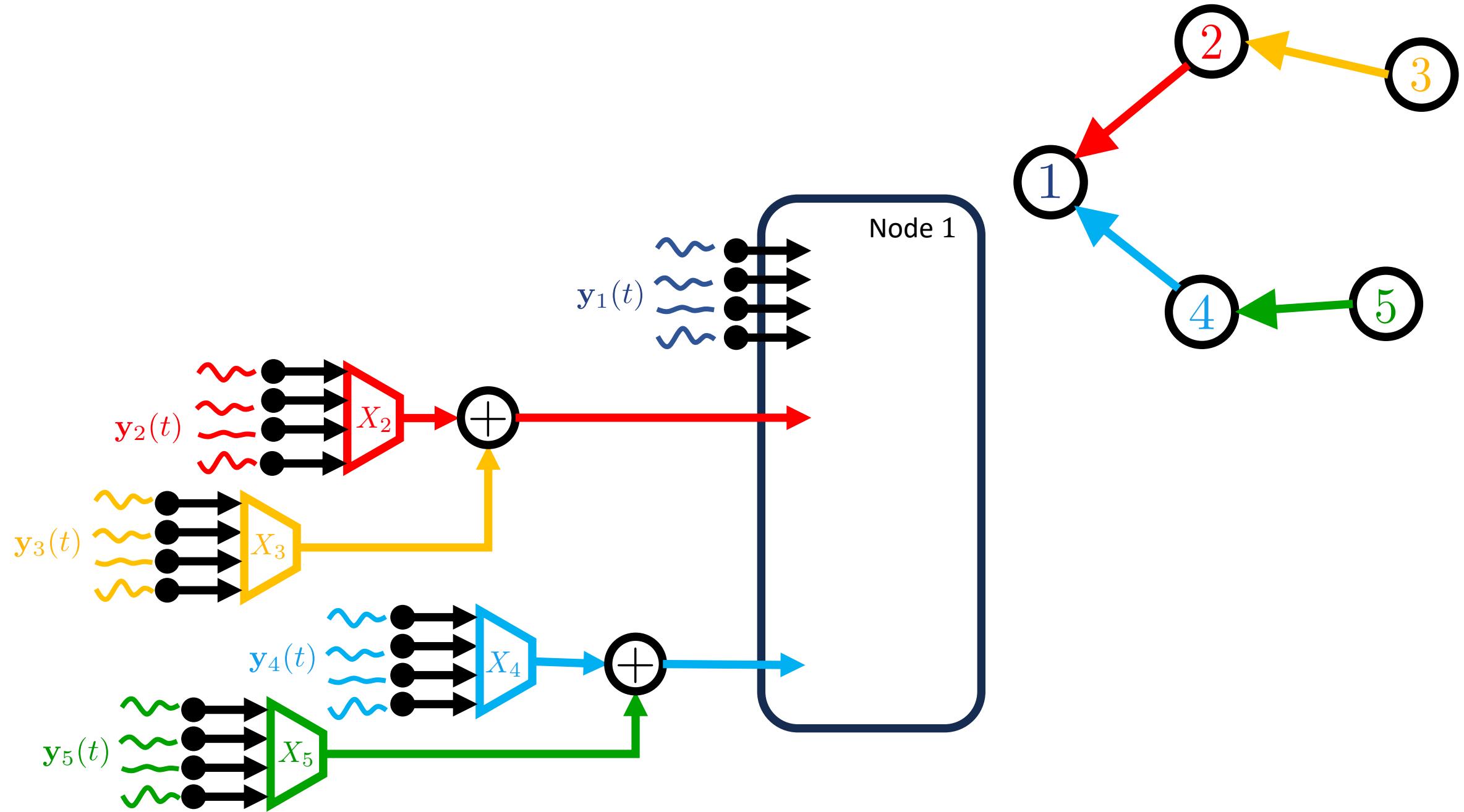
LCMV, PCA, TRO

Multiple filters

$$X^T \mathbf{y}(t), W^T \mathbf{v}(t), \dots$$

CCA





$$X^i \xrightarrow{\quad ? \quad} X^*$$

$$X^i \xrightarrow{\quad} X^*$$

Mild technical conditions

↳ Satisfied in practice

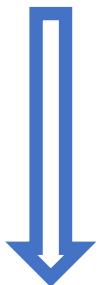
Measuring performance of DASF

Select task

$$\min_X f(X^T \mathbf{y}(t))$$

s. t. $X \in \mathcal{S}$

Solve



X^*

Measuring performance of DASF

Select task

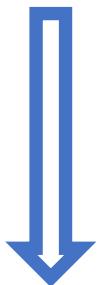
$$\min_X f(X^T \mathbf{y}(t))$$

s. t. $X \in \mathcal{S}$

Run DASF on task

$$\{X^0, X^1, X^2, \dots\}$$

Solve



$$X^*$$

Measuring performance of DASF

Select task

$$\begin{aligned} \min_X \quad & f(X^T \mathbf{y}(t)) \\ \text{s. t. } X \in \mathcal{S} \end{aligned}$$

Run DASF on task

$$\{X^0, X^1, X^2, \dots\}$$

Solve

$$X^*$$

$$\frac{\|X^i - X^*\|_F^2}{\|X^*\|_F^2}$$

Measuring performance of DASF

Select task

$$\begin{aligned} \min_X \quad & f(X^T \mathbf{y}(t)) \\ \text{s. t. } X \in \mathcal{S} \end{aligned}$$

Run DASF on task

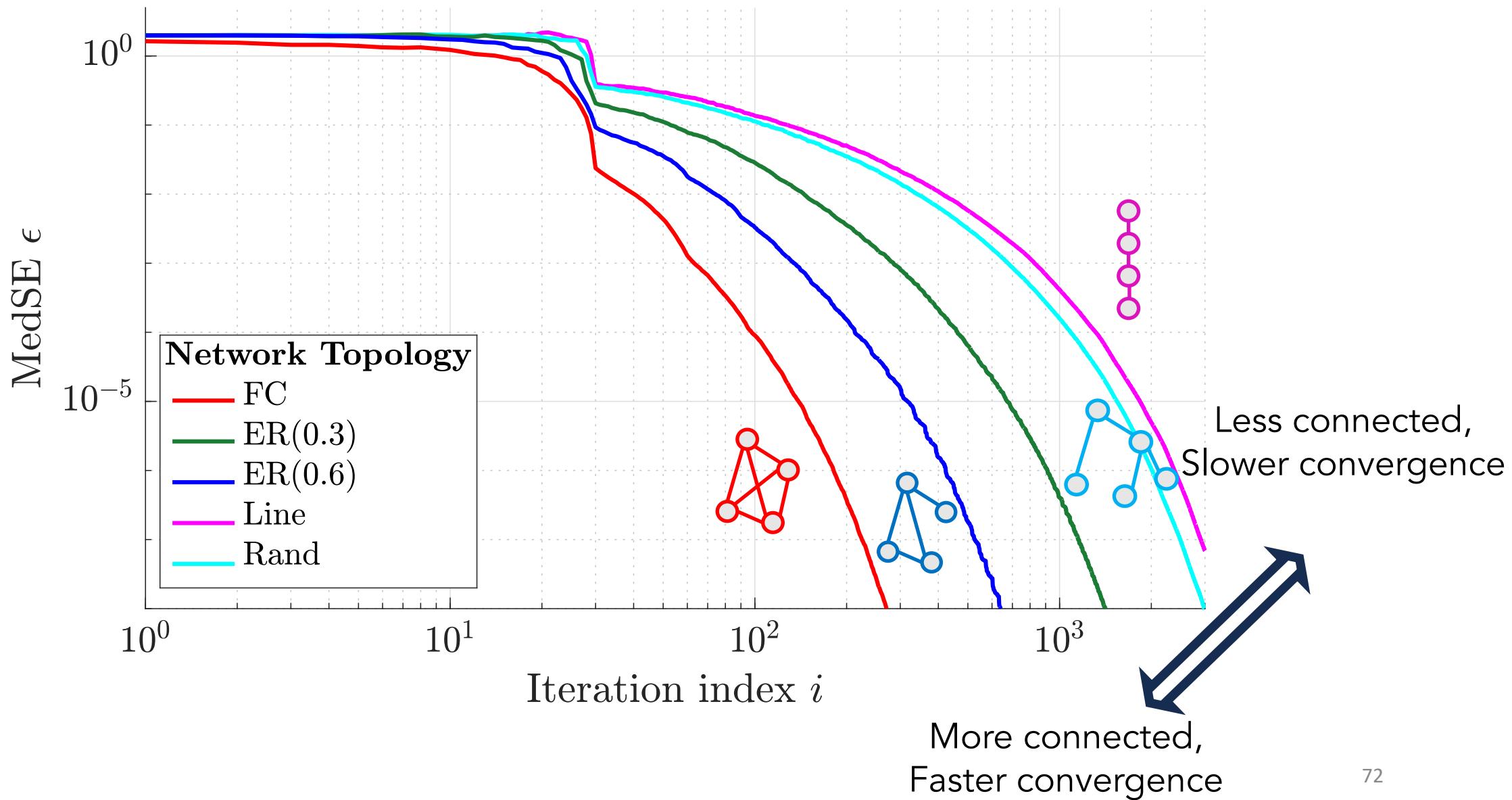
$$\{X^0, X^1, X^2, \dots\}$$

Solve

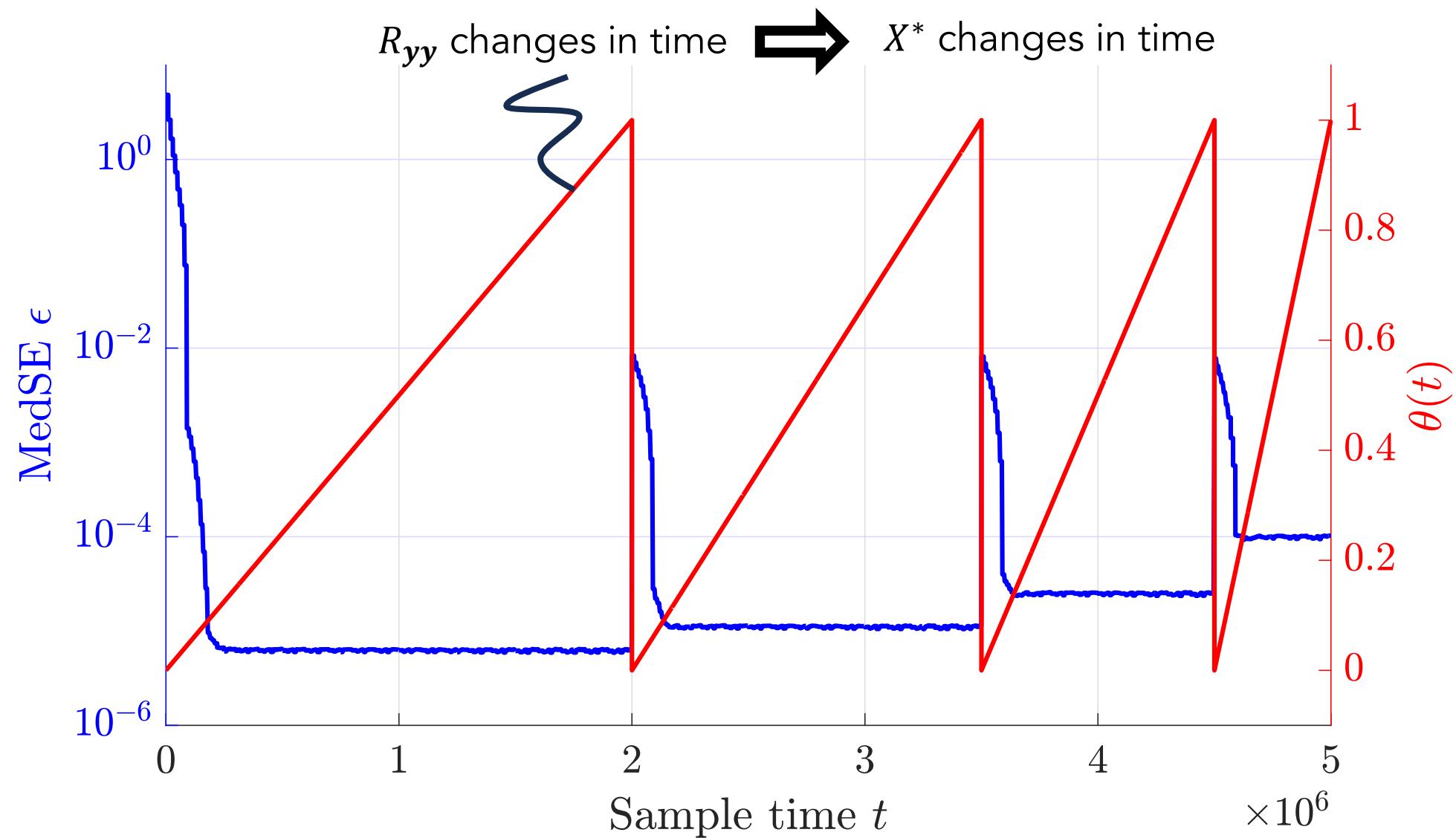
$$X^*$$

$$\epsilon(i) = \text{median} \left(\frac{\|X^i - X^*\|_F^2}{\|X^*\|_F^2} \right)$$

MedSE



Stationary vs Adaptive

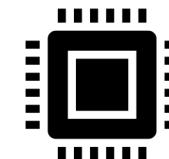


DASF framework

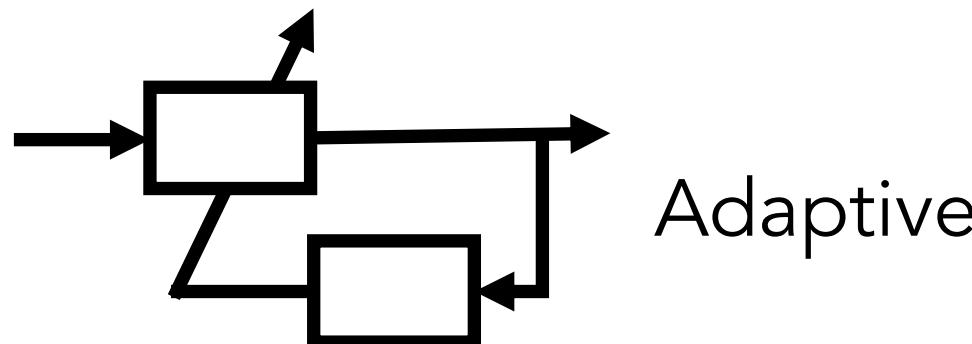
Unified and Generalizable
1 method for spatial filtering tasks



Efficient communication

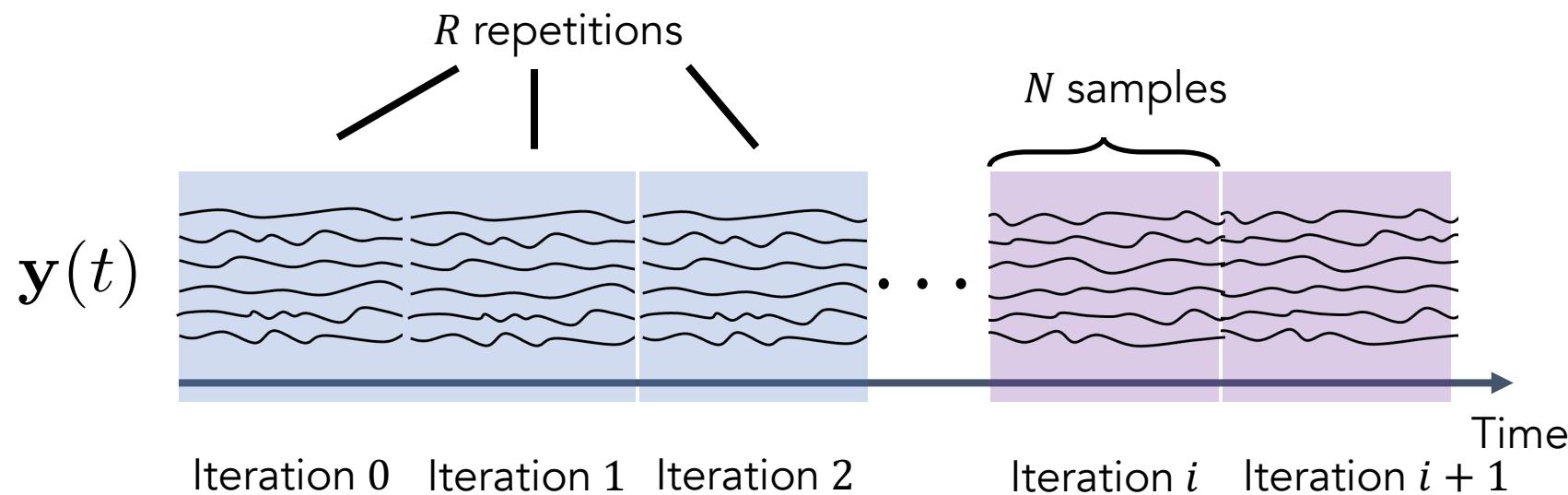
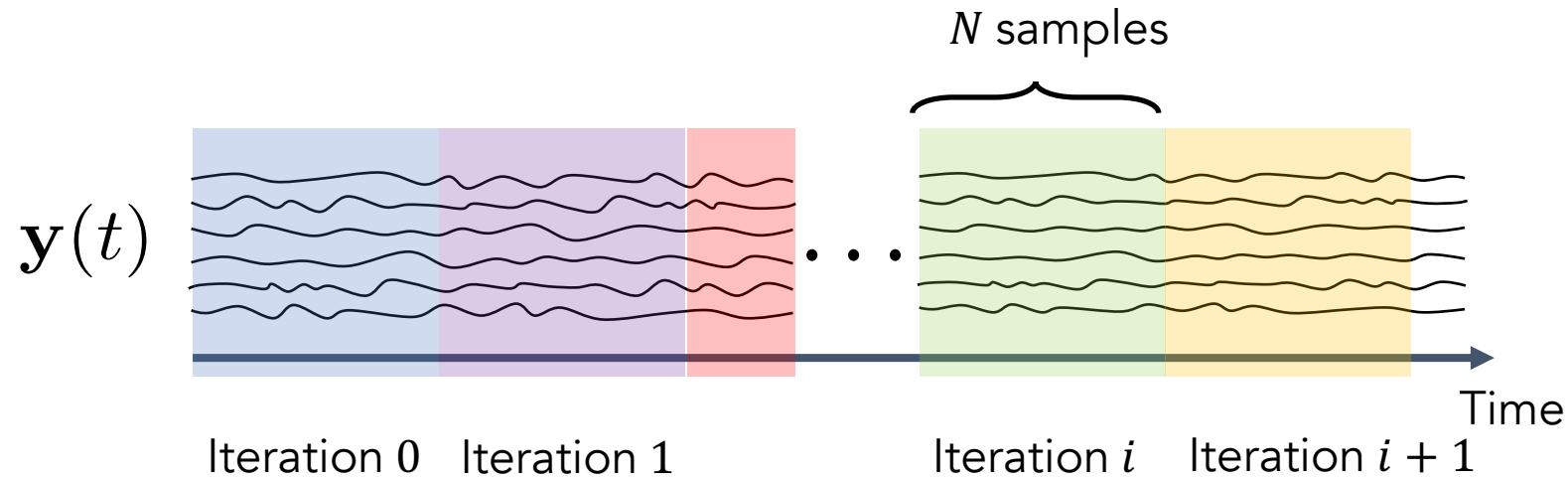


Computational efficiency

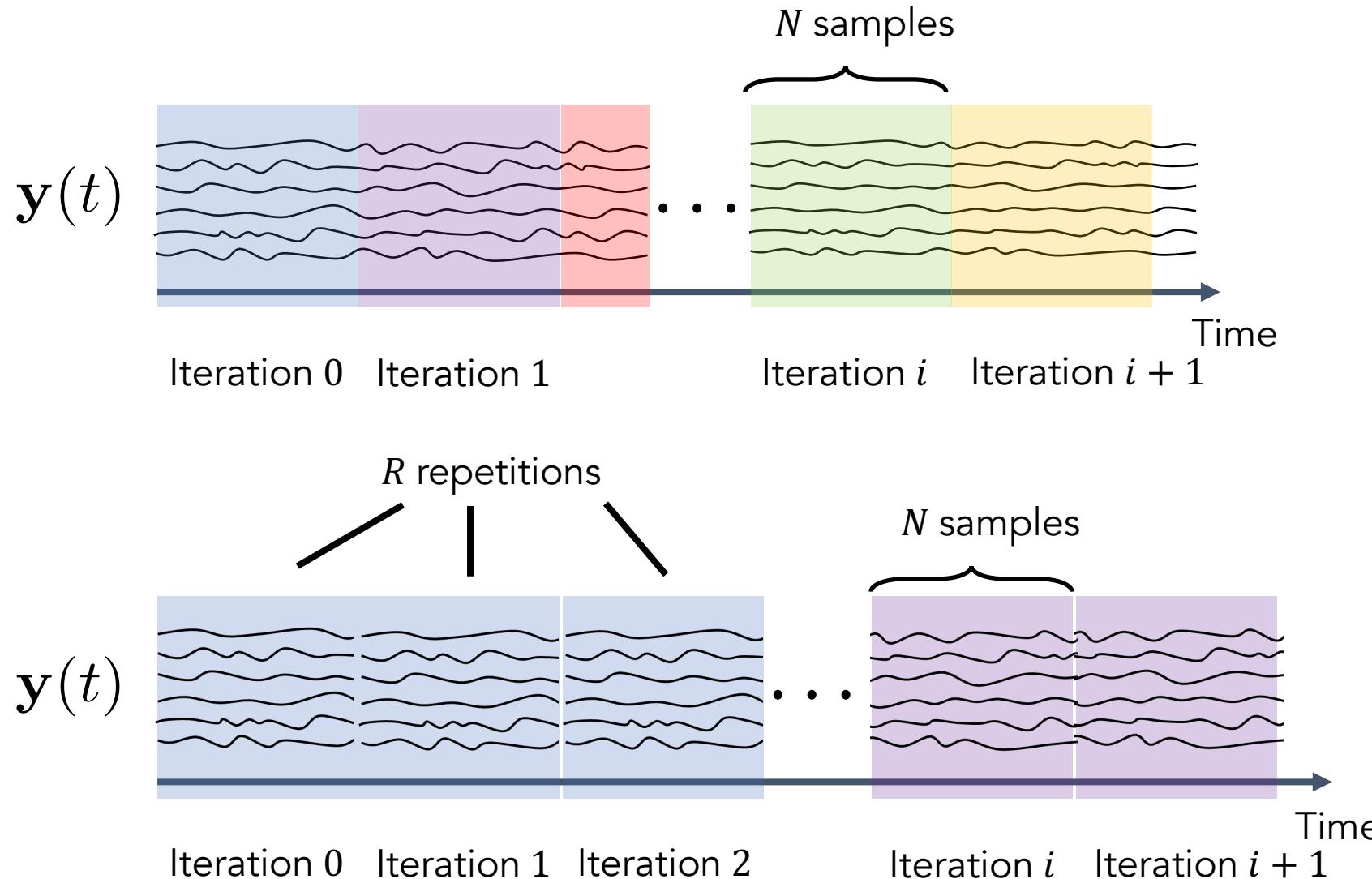


II. Extensions of the DASF framework

Improving adaptivity



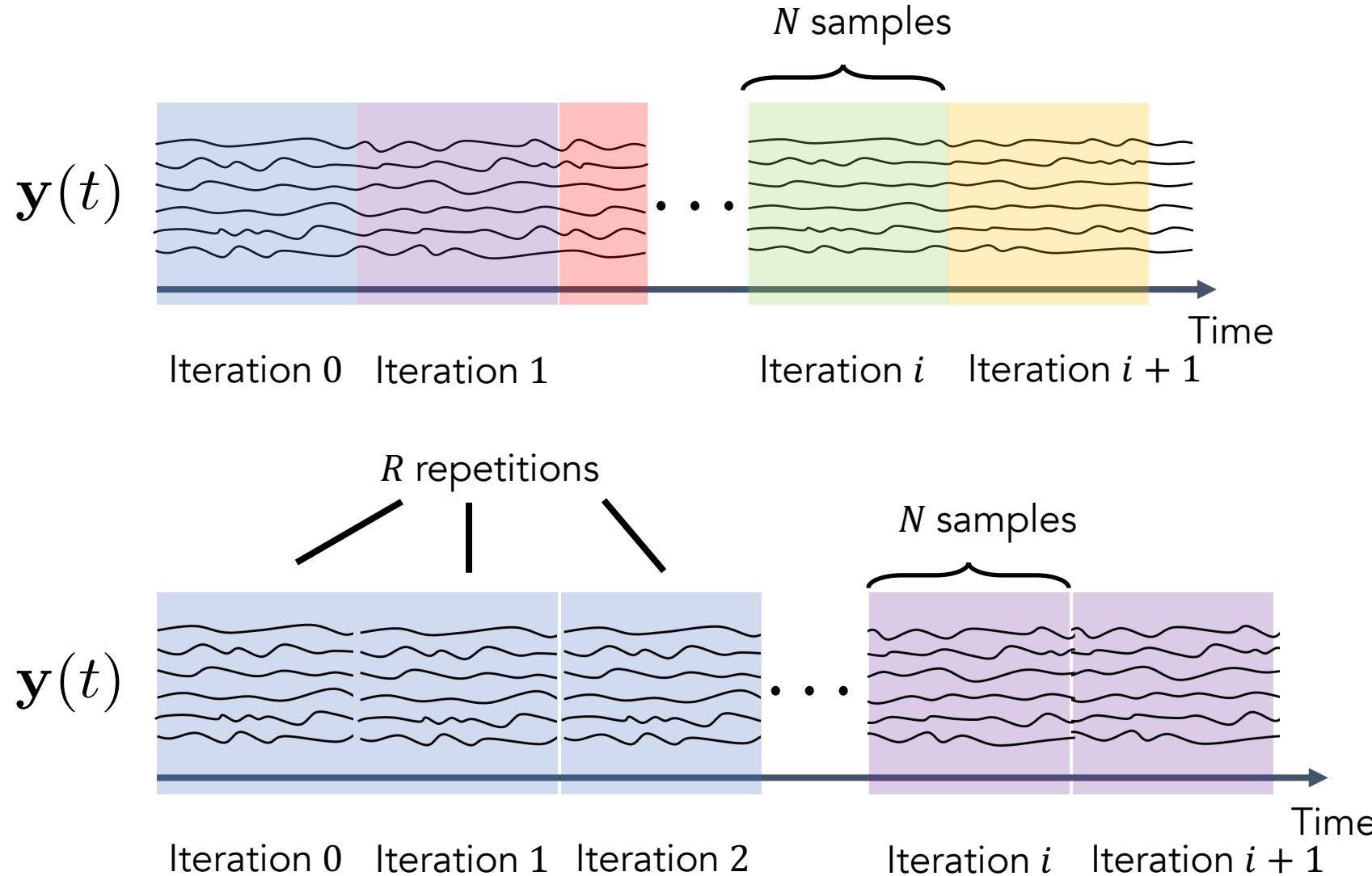
Improving adaptivity



$R \times$ increase in adaptivity

$R \times$ increase in communication

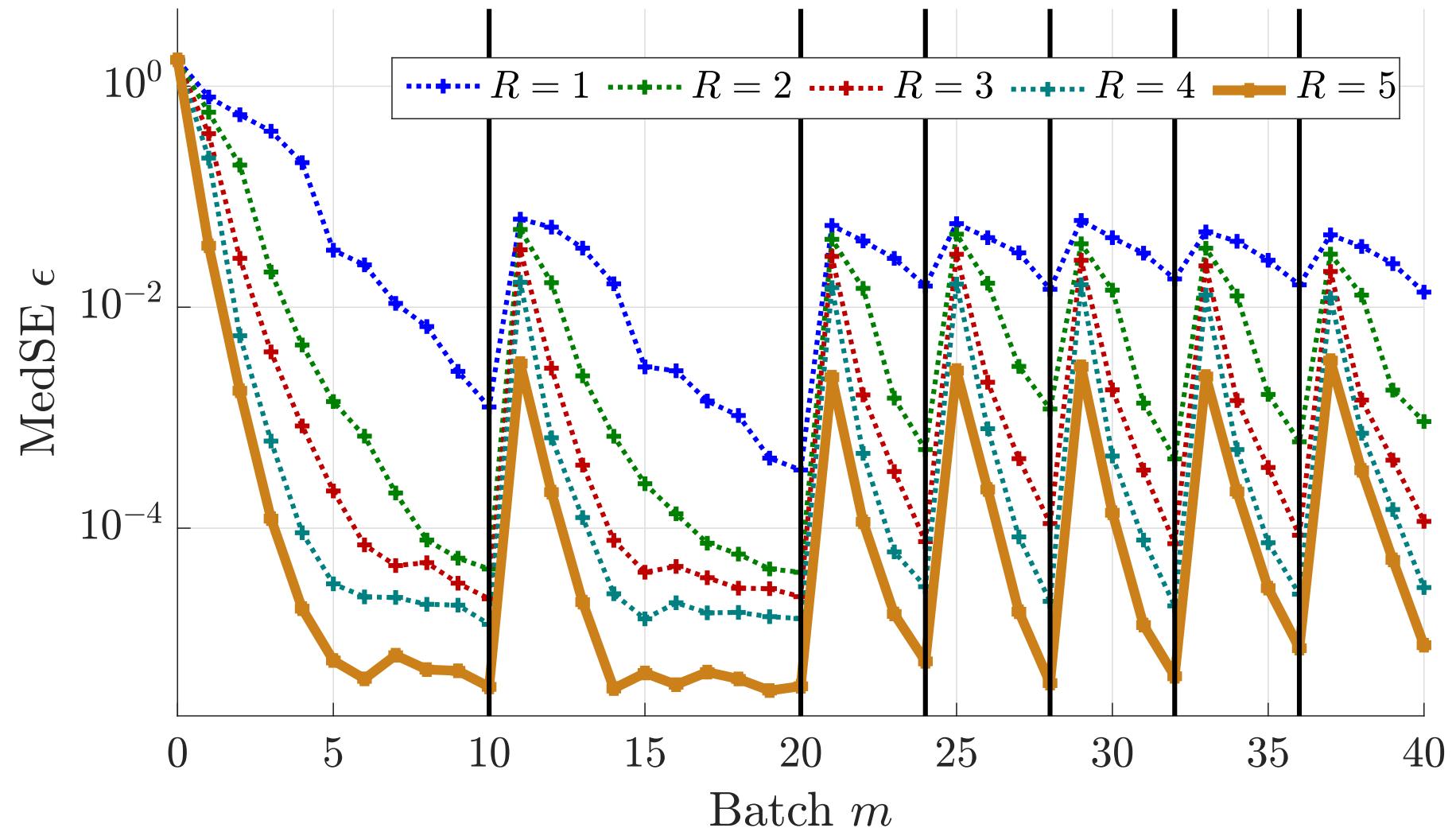
Improving adaptivity



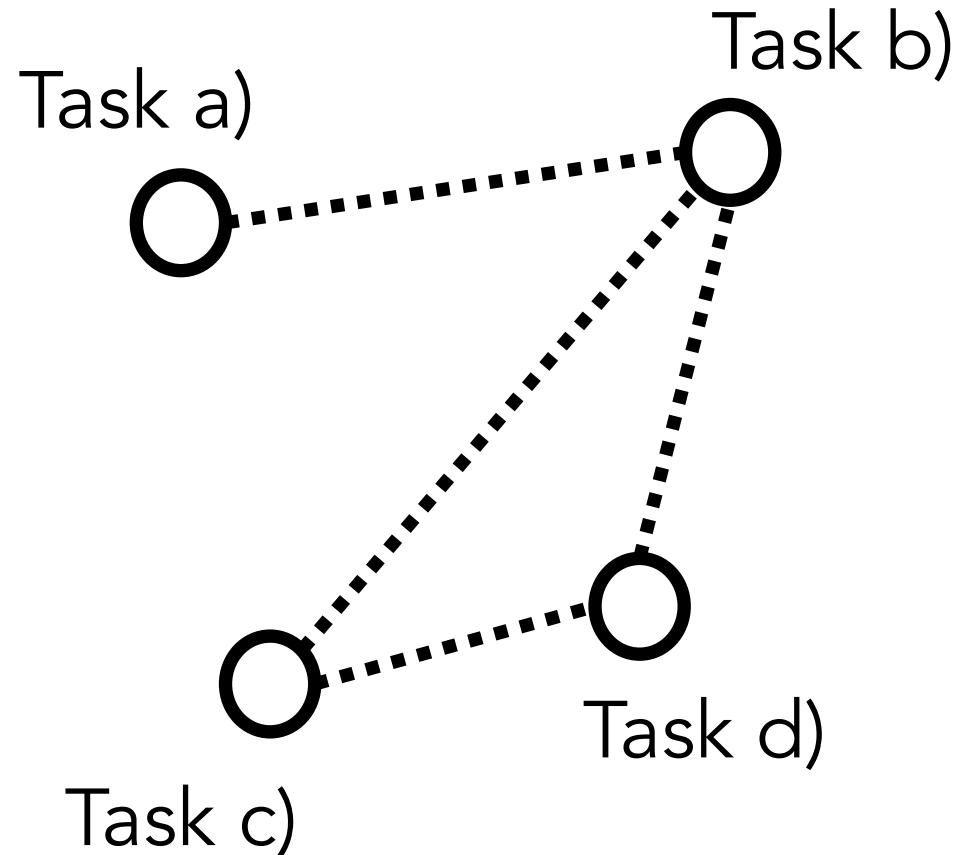
$R \times$ increase in adaptivity

$R \times$ increase in communication

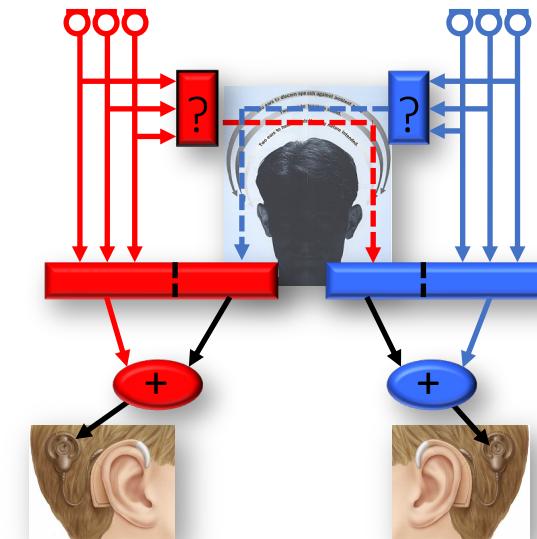
$< R \times$ increase in communication

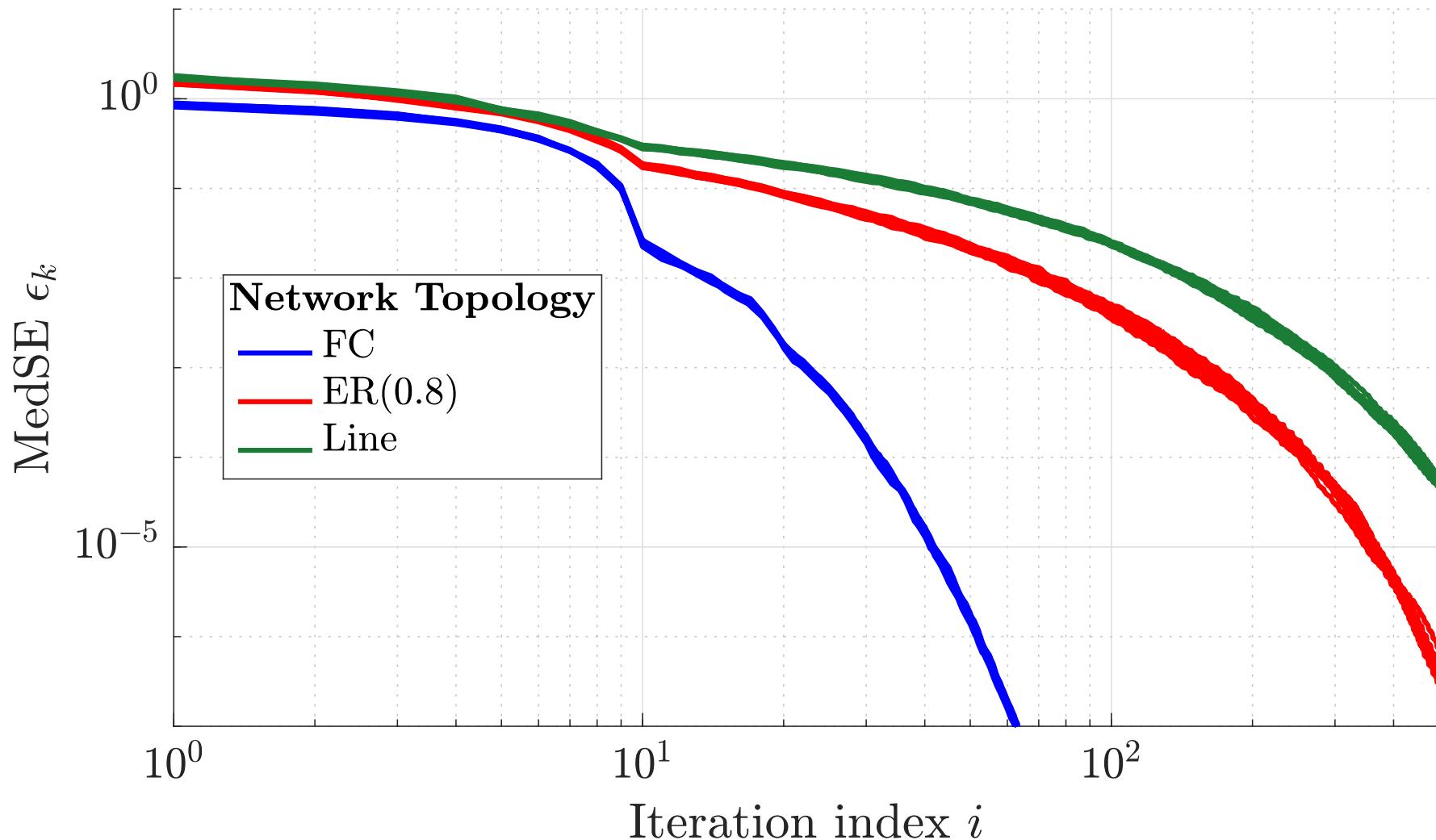


DASF for node-specific tasks (DANSF)



Task a) ~ Task b) ~ Task c) ~ Task d)





Fractional DASF (F-DASF)

$$\underset{X}{\text{minimize}} \ f(X^T \mathbf{y}(t)) = \frac{f_1(X^T \mathbf{y}(t))}{f_2(X^T \mathbf{y}(t))} \rightarrow$$

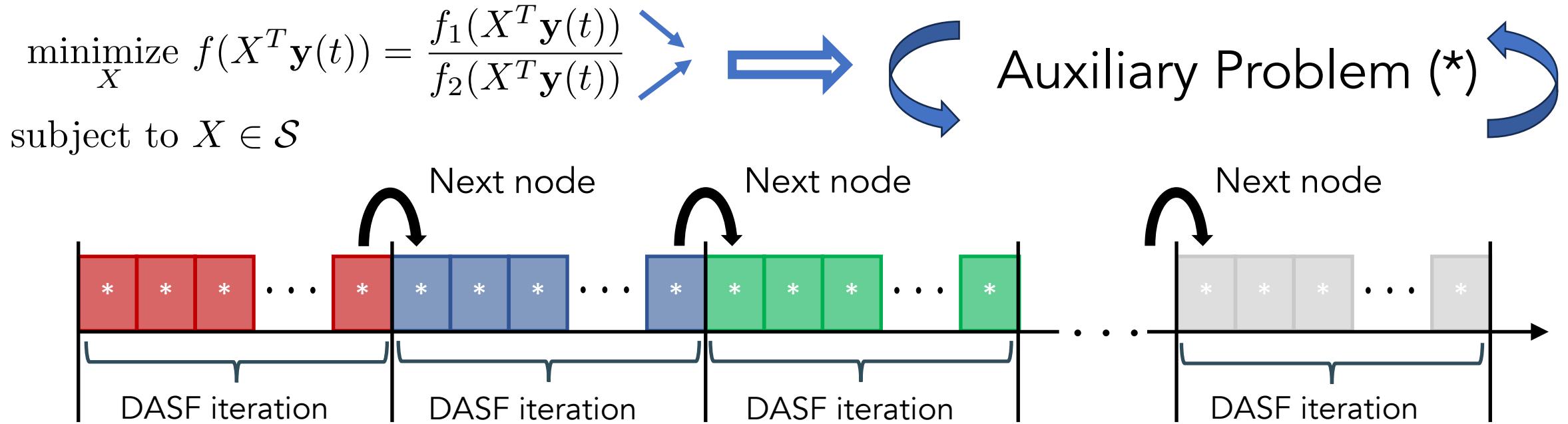
subject to $X \in \mathcal{S}$

Fractional DASF (F-DASF)

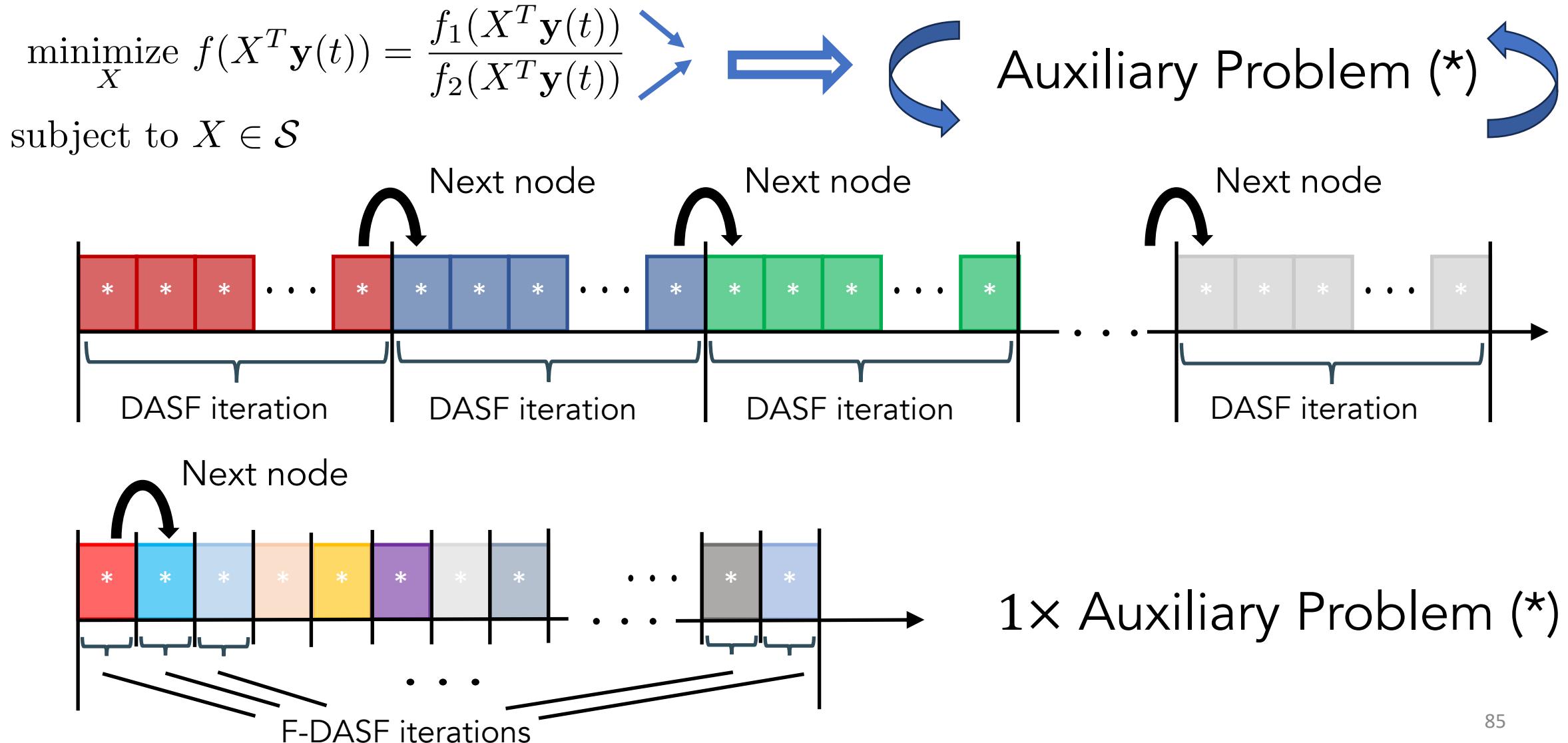
$$\underset{X}{\text{minimize}} \ f(X^T \mathbf{y}(t)) = \frac{f_1(X^T \mathbf{y}(t))}{f_2(X^T \mathbf{y}(t))} \rightarrow \rightarrow \text{Auxiliary Problem (*)} \curvearrowleft$$

subject to $X \in \mathcal{S}$

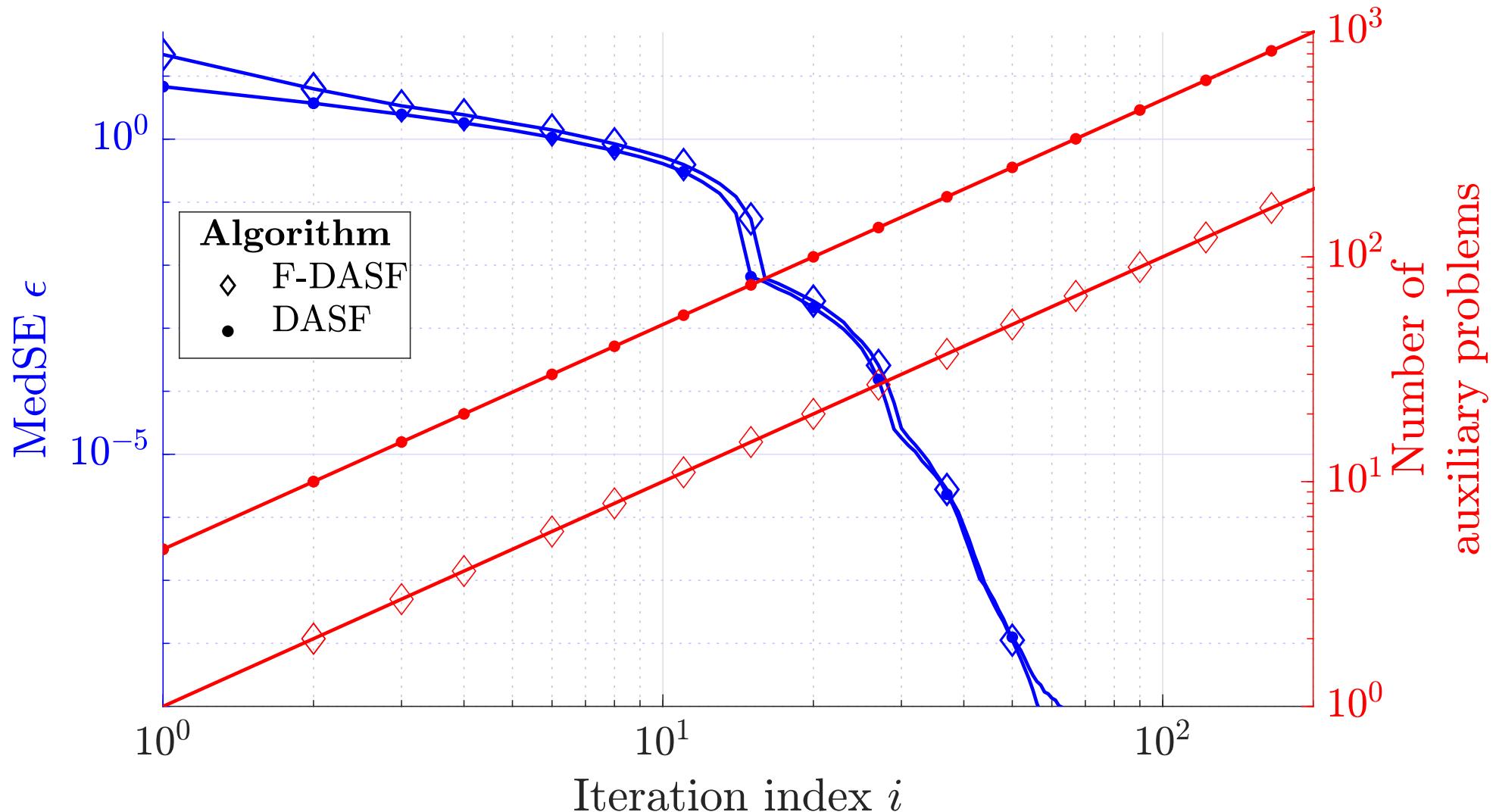
Fractional DASF (F-DASF)



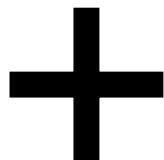
Fractional DASF (F-DASF)



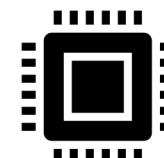
1× Auxiliary Problem (*)



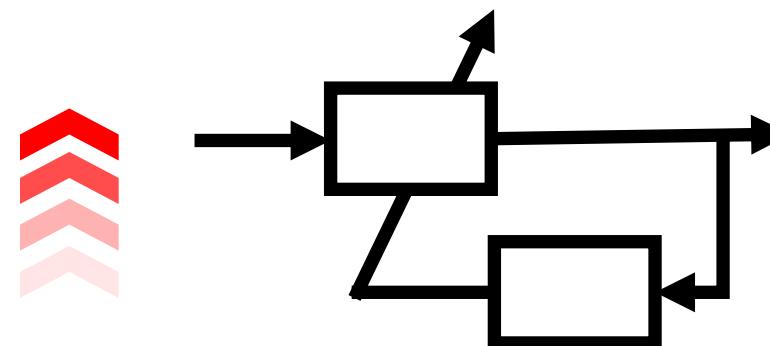
Extensions DASF framework



Efficient communication



Computational efficiency



Adaptive

