A Distributed Adaptive Algorithm for Node-Specific Signal Fusion Problems in Wireless Sensor Networks

Cem Ates Musluoglu, Alexander Bertrand {cemates.musluoglu, alexander.bertrand}
@esat.kuleuven.be

EUSIPCO2023

1. Node-specific problems for spatial filtering...

Examples:

- Minimum mean square error (MMSE):

$$\underset{X_k\in\mathbb{R}^{M\times Q}}{\text{minimize}}\ \mathbb{E}[||\mathbf{d}_k(t)-X_k^T\mathbf{y}(t)||^2]$$

- Linearly constrained minimum variance (LCMV):

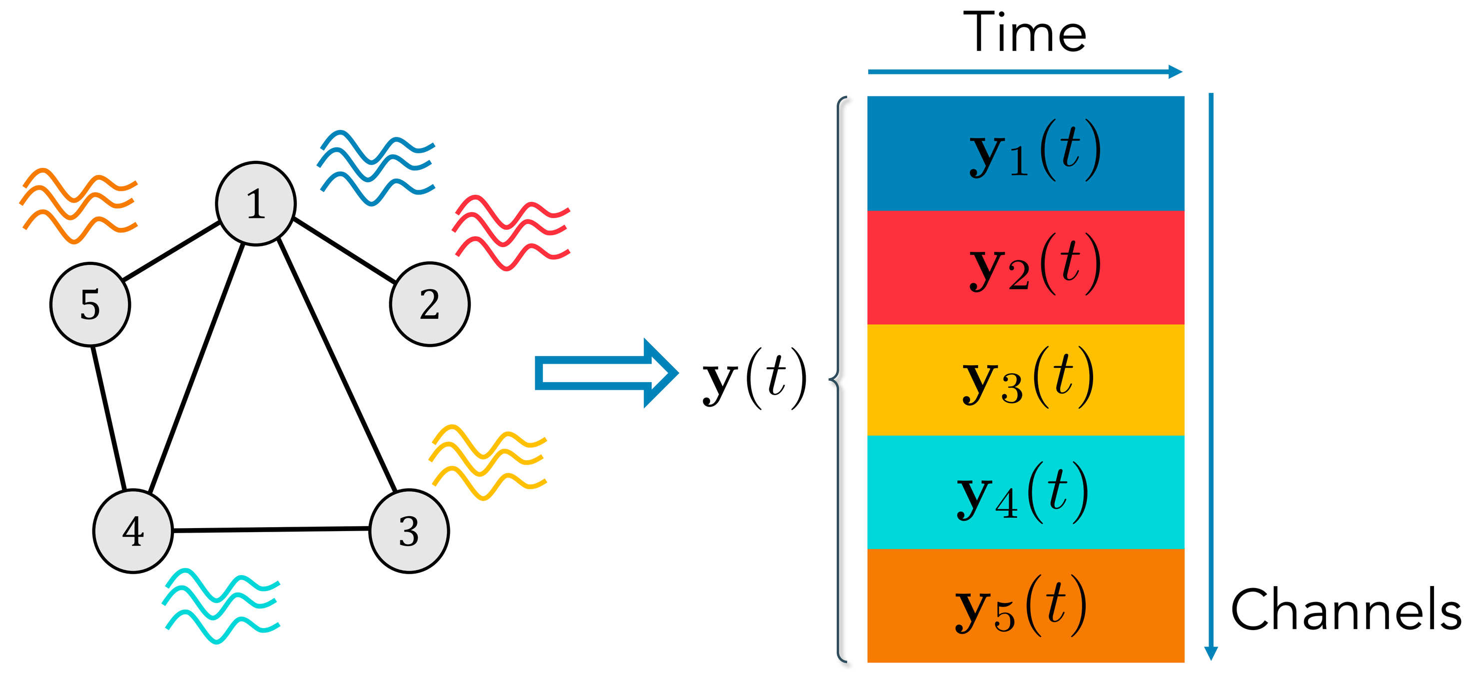
$$\underset{X_k\in\mathbb{R}^{M\times Q}}{\text{minimize}}\quad \mathbb{E}[||X_k^T\mathbf{y}(t)||^2]$$
$$\text{subject to}\quad X_k^T B = H_k$$

General form of spatial filtering problems with node-specific objectives:

$$\mathbb{P}_k:\ \underset{X_k\in\mathbb{R}^{M\times Q}}{\text{minimize}}\quad f_k(X_k^T\mathbf{y}(t), X_k^T B)$$
$$\text{subject to}\quad X_k\in\mathcal{S}_k$$

Assumption: $X_k^*=X_l^*\cdot D_{k,l}$

... in a distributed setting



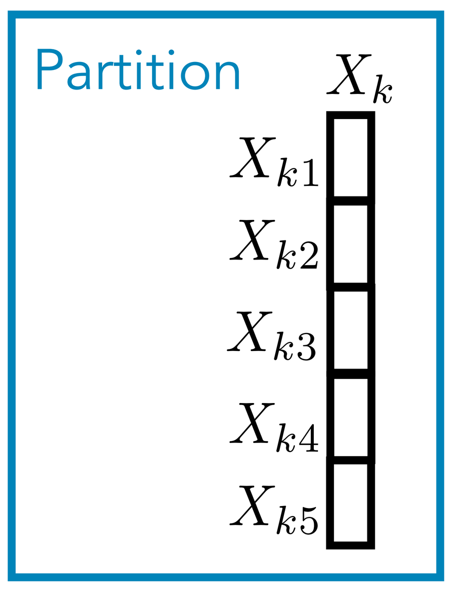
Need to exploit the correlation between all channels of $\mathbf{y}$ but centralization is too costly

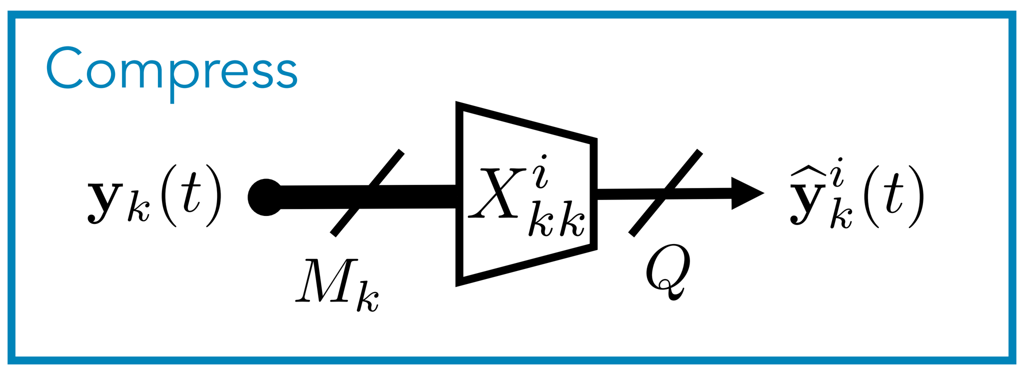
Goal

Solve node-specific spatial filtering problems without centralizing the data

2. Proposed method: DANSF data flow

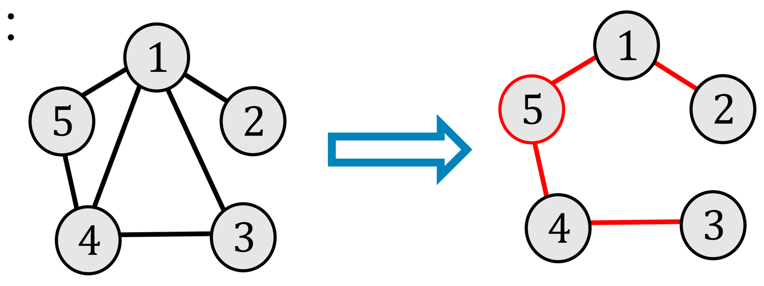
- Compress signals measured at nodes k using current estimate X_{kk}^i of the filter: $\widehat{\mathbf{y}}_k^i(t)=X_{kk}^{iT}\mathbf{y}_k(t)$

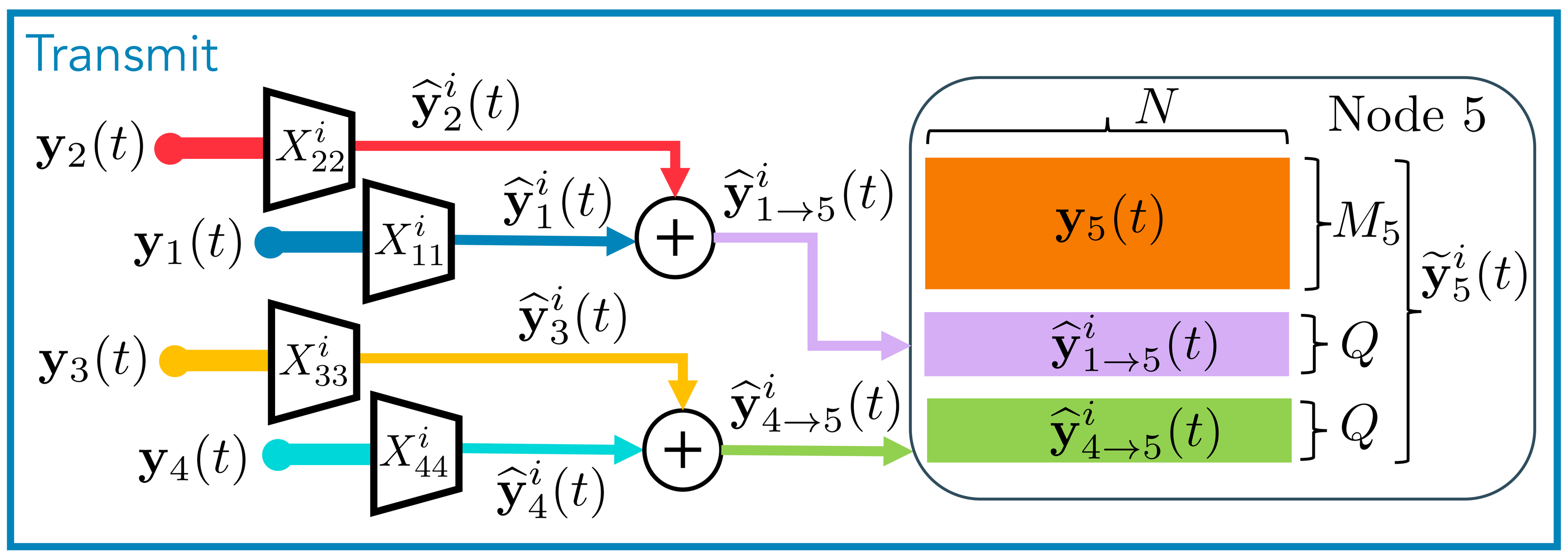




- Send N samples of compressed signals towards the updating node q after pruning the graph:

Example with $q=5$:





- Same procedure for the matrix B

3. Proposed method: DANSF updating

- At node q build and solve local version of the network-wide problem using $\widetilde{\mathbf{y}}_q^i$ instead of $\mathbf{y}$ and $\widetilde{B}_q^i$ instead of B

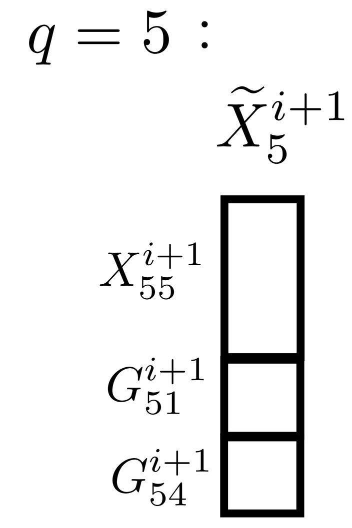
Solve

$$\widetilde{X}_q^{i+1}\leftarrow\underset{\widetilde{X}_q\in\widetilde{\mathcal{S}}_q^i}{\text{argmin}}\ f_k(\widetilde{X}_q^T\widetilde{\mathbf{y}}_q^i(t),\widetilde{X}_q^T\widetilde{B}_q^i)$$

- Node q updates X_q as

Update

$$X_{qk}^{i+1}=\begin{cases}X_{qq}^{i+1} & \text{if } k=q\\ X_{kk}^i G_{qn}^{i+1} & \text{if } k\neq q\end{cases}$$



- At node q, the filtered optimal signal is estimated as

$$X_q^{*T}\mathbf{y}(t)\approx\widetilde{X}_q^{(i+1)T}\widetilde{\mathbf{y}}_q^i(t)$$

- Node q transmits $\widetilde{X}_q^{(i+1)T}\widetilde{\mathbf{y}}_q^i(t)$ and $\widetilde{X}_q^{(i+1)T}\widetilde{B}_q^i$ to the other nodes for them to update X_k
- Repeat process for other updating nodes and new signal samples

4. Results

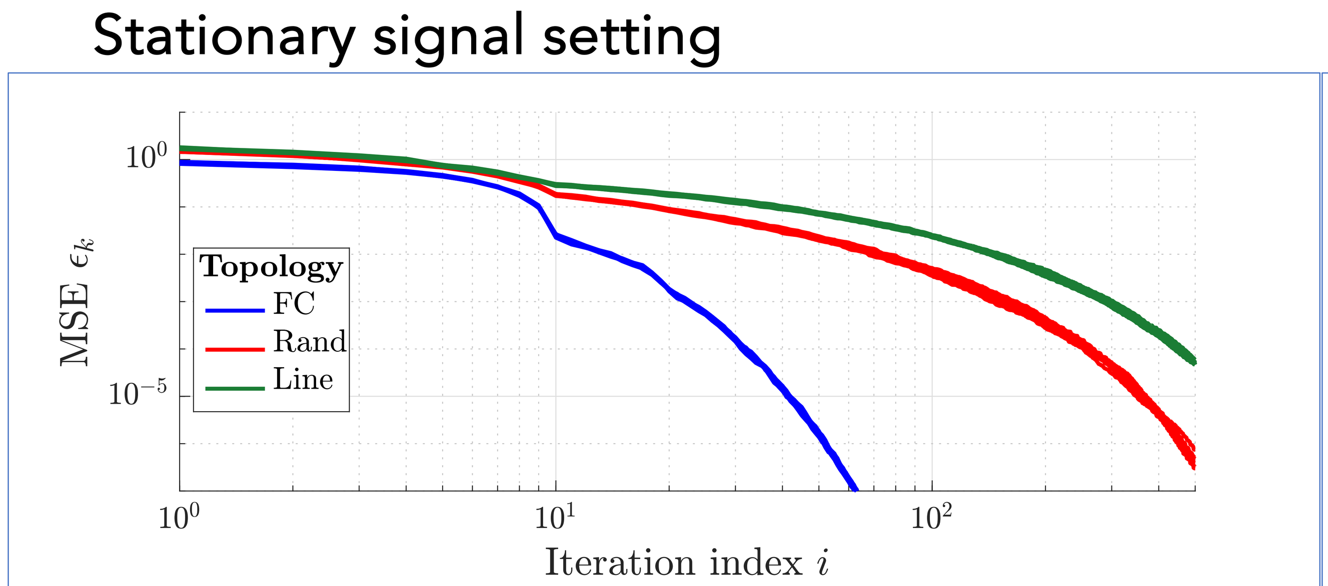
DANSF converges to a solution X_k^* of $\mathbb{P}_k$ for each node k under similar conditions as the original DASF algorithm[1,2]

Example problem:

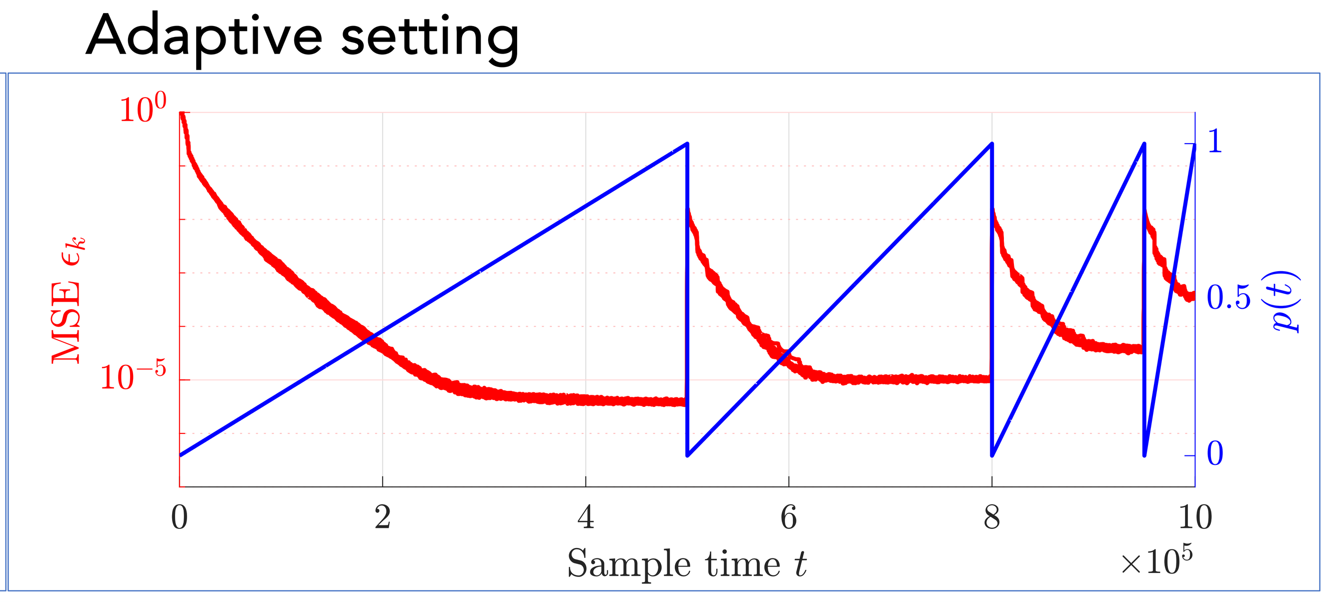
$$\underset{X_k\in\mathbb{R}^{M\times Q}}{\text{minimize}}\quad \text{trace}(X_k^T B_k)$$
$$\text{subject to}\quad \text{trace}(X_k^T R_{\mathbf{yy}}X_k)\leq 1$$

Stationary signal setting

Convergence for each node without large deviations



Adaptive setting



- Statistics of $\mathbf{y}$ change with p
- Slow changes tracked, abrupt ones corrected

KU LEUVEN

STADIUS Center for Dynamical Systems, Signal Processing and Data Analytics

erc European Research Council

[1] C. A. Musluoglu and A. Bertrand, "A Unified Algorithmic Framework for Distributed Adaptive Signal and Feature Fusion Problems—Part I: Algorithm Derivation," in *IEEE Transactions on Signal Processing*, vol. 71, pp. 1863-1878, 2023, doi: 10.1109/TSP.2023.3275272.

[2] C. A. Musluoglu, C. Hovine and A. Bertrand, "A Unified Algorithmic Framework for Distributed Adaptive Signal and Feature Fusion Problems — Part II: Convergence Properties," in *IEEE Transactions on Signal Processing*, vol. 71, pp. 1879-1894, 2023, doi: 10.1109/TSP.2023.3275273.