A Distributed Adaptive Algorithm for Node-Specific Signal Fusion Problems in Wireless Sensor Networks

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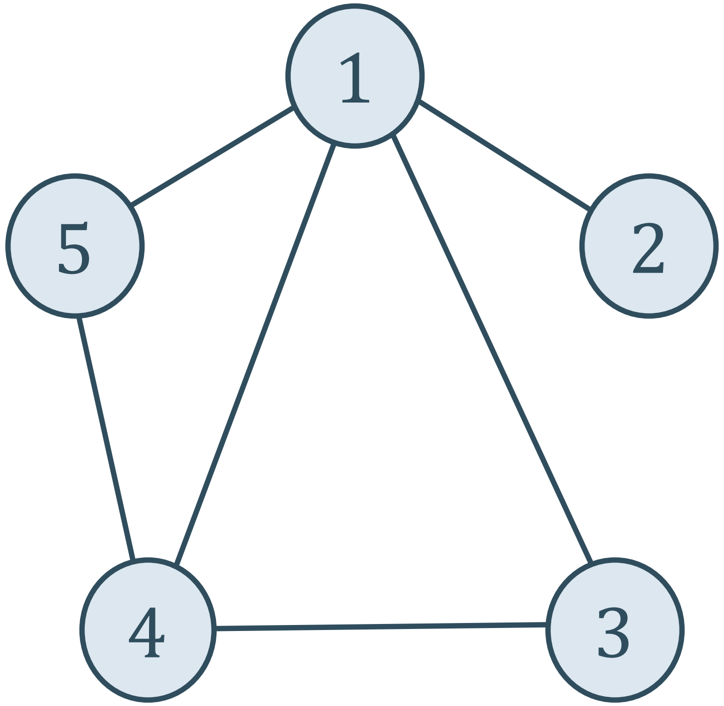
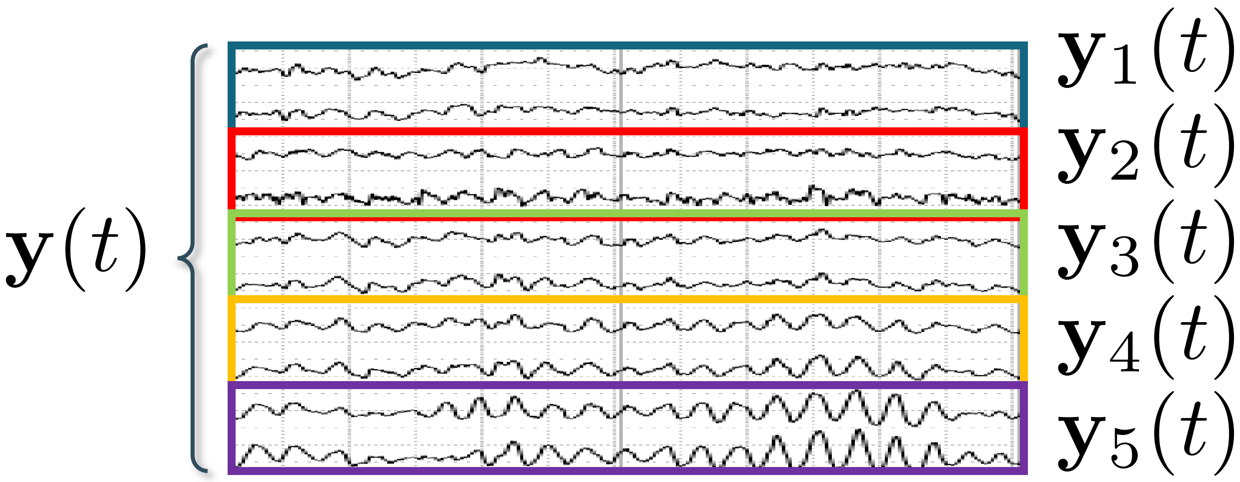
Distributed spatial filtering setting

Each node k is a local sensor array measuring <u>multi-channel</u> signal $\boldsymbol{y}_k(t)$





Exploiting the spatial coherence

Goal: Exploit spatial coherence between channels of $\boldsymbol{y}$ by optimally combining them

using a linear filter X applied to $\boldsymbol{y}$: $X^T\boldsymbol{y}$

Without data centralization

$\Downarrow$

Optimization problem to find X

Spatial filtering examples

MMSE:

$$\underset{X}{\text{minimize}}\ \mathbb{E}[||\mathbf{d} - X^T\mathbf{y}||^2]$$

PCA:

$$\underset{X}{\text{maximize}}\ \mathbb{E}[||X^T\mathbf{y}||^2]$$

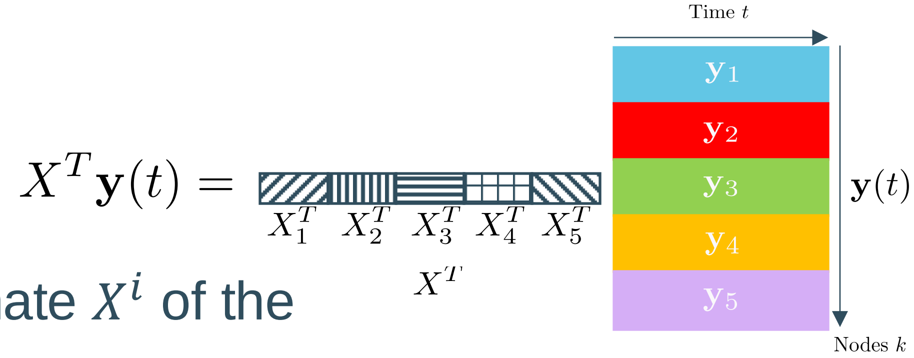
$$\text{subject to } X^T X = I_Q$$

LCMV:

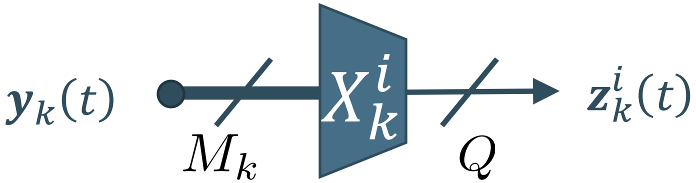
$$\underset{X}{\text{minimize}}\ \mathbb{E}[||X^T\mathbf{y}(t)||]^2$$

$$\text{subject to } X^T B = H$$

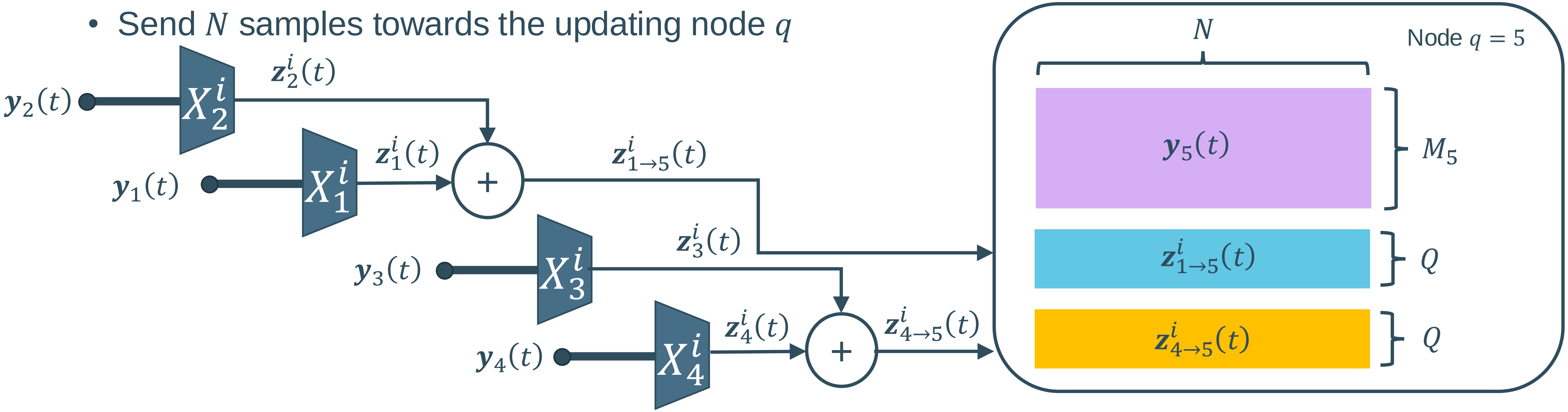
The DASF framework [1] (1/2)



- Compress signals measured at nodes k using current estimate X^i of the filter: $\boldsymbol{z}_k^i(t) = X_k^{iT}\boldsymbol{y}_k(t)$.



- Send N samples towards the updating node q



[1] C. A. Musluoglu and A. Bertrand, "A Unified Algorithmic Framework for Distributed Adaptive Signal and Feature Fusion Problems—Part I: Algorithm Derivation," in *IEEE Transactions on Signal Processing*, vol. 71, pp. 1863-1878, 2023, doi: 10.1109/TSP.2023.3275272.

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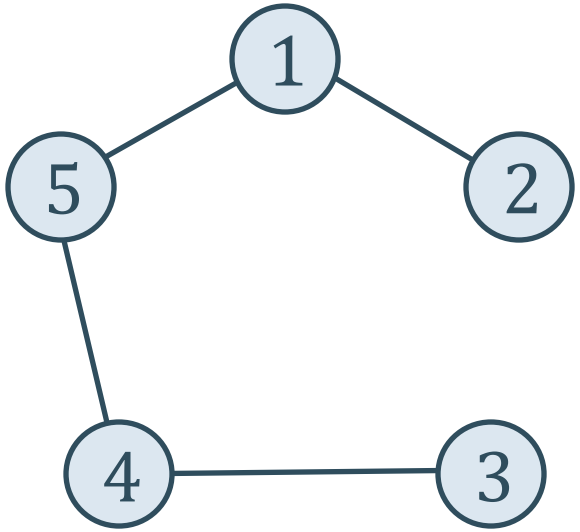

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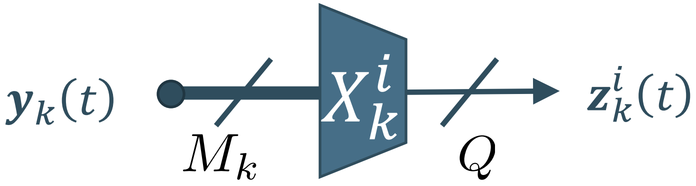

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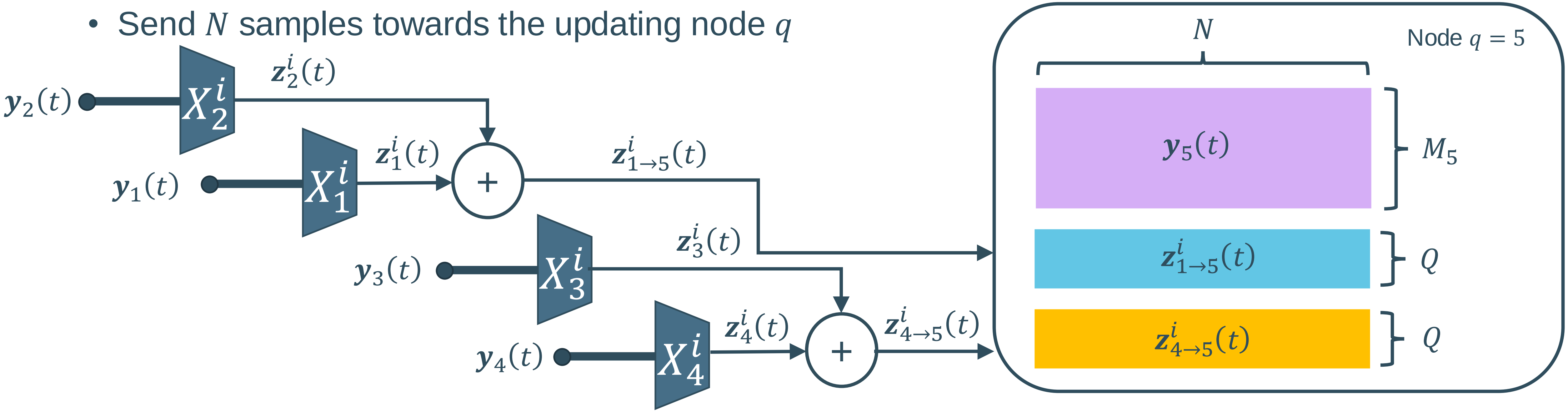

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The DASF framework [1] (2/2)

- At node q, build a **compressed version of the network-wide problem** using the available local data and **solve it** to obtain new estimate X^{i+1}.

$$\begin{bmatrix} \boldsymbol{y}_5(t) \\ \boldsymbol{z}^i_{1\rightarrow 5}(t) \\ \boldsymbol{z}^i_{4\rightarrow 5}(t) \end{bmatrix} = \widetilde{\boldsymbol{y}}_5(t)$$

Local problem

$$\underset{\widetilde{X}}{\text{minimize}}\ \mathbb{E}[||\mathbf{d} - \widetilde{X}^T\widetilde{\mathbf{y}}||^2]$$

Network-wide problem

$$\underset{X}{\text{minimize}}\ \mathbb{E}[||\mathbf{d} - X^T\mathbf{y}||^2]$$

- Repeat for other nodes with a new batch of samples.

- Convergence to X^*, the global solution of the centralized problem (for any network topology) [1].

[1] C. A. Musluoglu and A. Bertrand, "A Unified Algorithmic Framework for Distributed Adaptive Signal and Feature Fusion Problems—Part I: Algorithm Derivation," in *IEEE Transactions on Signal Processing*, vol. 71, pp. 1863-1878, 2023, doi: 10.1109/TSP.2023.3275272

Node-specific spatial filtering problems

- Different optimization problem at each node

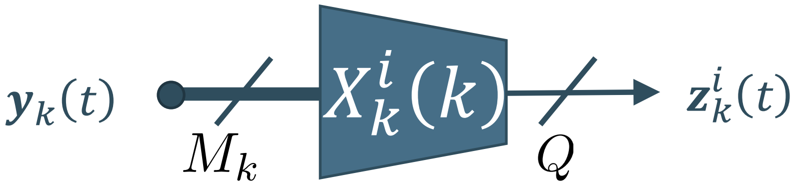
MMSE: $$\min_{X(k)} \mathbb{E}[||\mathbf{d}_k(t) - X(k)^T \mathbf{y}(t)||^2]$$

LCMV: $$\min_{X(k):X(k)^T B = H(k)} \mathbb{E}[||X(k)^T \mathbf{y}(t)||^2]$$

- **Assumption:** There exists a set of invertible matrices $D_{k,l}$ such that for any pair (k, l) of nodes, the solutions $X(k)^*$ and $X(l)^*$ satisfy $X(k)^* = X(l)^* \cdot D_{k,l}$.

The DANSF algorithm (1/2)

- Compress signals measured at nodes k using current estimate of the block k of $X^i(k)$ of the filters: $\mathbf{z}_k^i(t) = \left(X_k^i(k)\right)^T \mathbf{y}_k(t)$.



- Send N samples towards the updating node q using the same data flow as the DASF algorithm.

The DANSF algorithm (2/2)

- The local problem (compressed versions of global problem) is solved at the updating node q.

Local problem

$$\min_{\widetilde{X}(k)} \mathbb{E}[||\mathbf{d}_k(t) - \widetilde{X}(k)^T \widetilde{\mathbf{y}}(t)||^2]$$

Network-wide problem

$$\min_{X(k)} \mathbb{E}[||\mathbf{d}_k(t) - X(k)^T \mathbf{y}(t)||^2]$$

- From the linear relationship between the solutions, the solution of a local problem at node q can also be used by other nodes to update their own filter.

- Convergence to the optimal filter can then be achieved at each node.

Simulations

Linear problem with a quadratic constraint:

$$\begin{aligned} \underset{X(k) \in \mathbb{R}^{M \times Q}}{\text{minimize}} \quad & \text{trace}(X(k)^T B(k)) \\ \text{subject to} \quad & \text{trace}(X(k)^T R_{\mathbf{yy}} X(k)) \leq 1 \end{aligned}$$

Solution for node k:

$$X(k)^* = -\sqrt{\text{trace}(B(k)^T R_{\mathbf{yy}}^{-1} B(k))^{-1}} \cdot R_{\mathbf{yy}}^{-1} B(k)$$

Assumption on linear relationship between solutions satisfied if:

$$B(k) = B \cdot D(k)$$

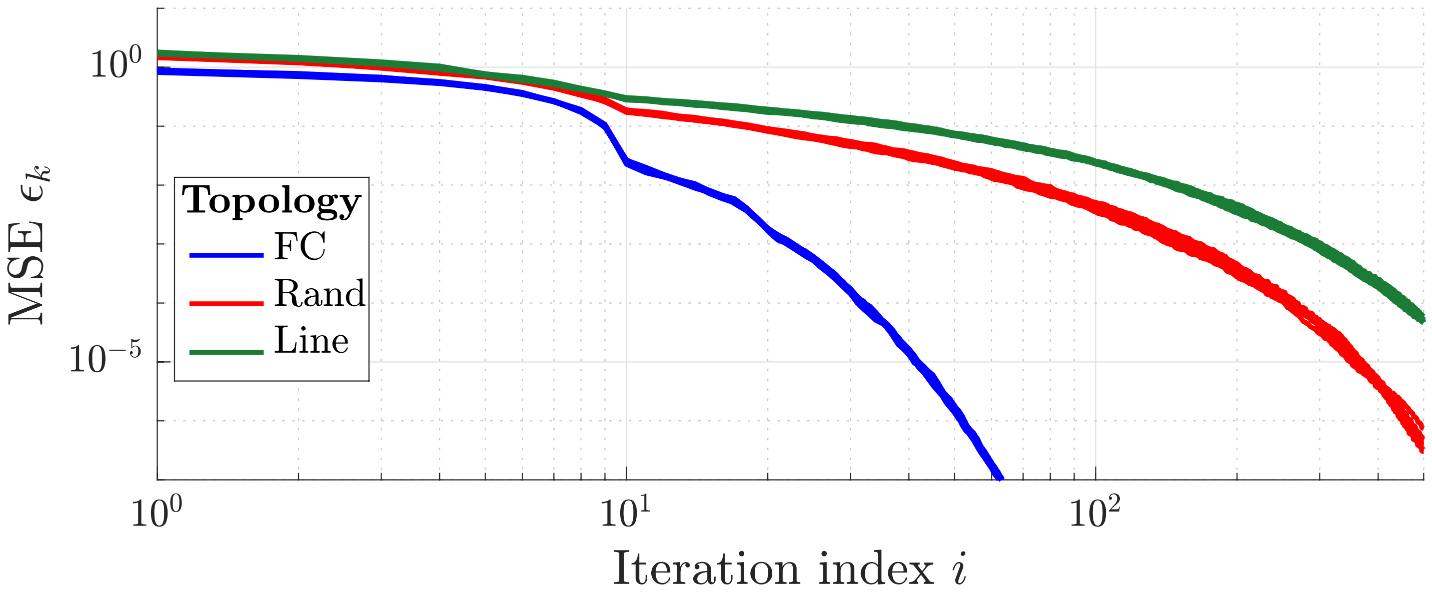
Results – Stationary setting

Linear problem with a quadratic constraint:

$$\begin{aligned} \underset{X(k)\in\mathbb{R}^{M\times Q}}{\text{minimize}} \quad & \text{trace}(X(k)^T B(k)) \\ \text{subject to} \quad & \text{trace}(X(k)^T R_{\mathbf{yy}} X(k)) \leq 1 \end{aligned}$$

$$\epsilon_k(X^i(k)) = \frac{||X^i(k) - X(k)^*||_F^2}{||X(k)^*||_F^2}$$

Convergence for each node k



Results – Adaptive setting

Linear problem with a quadratic constraint:

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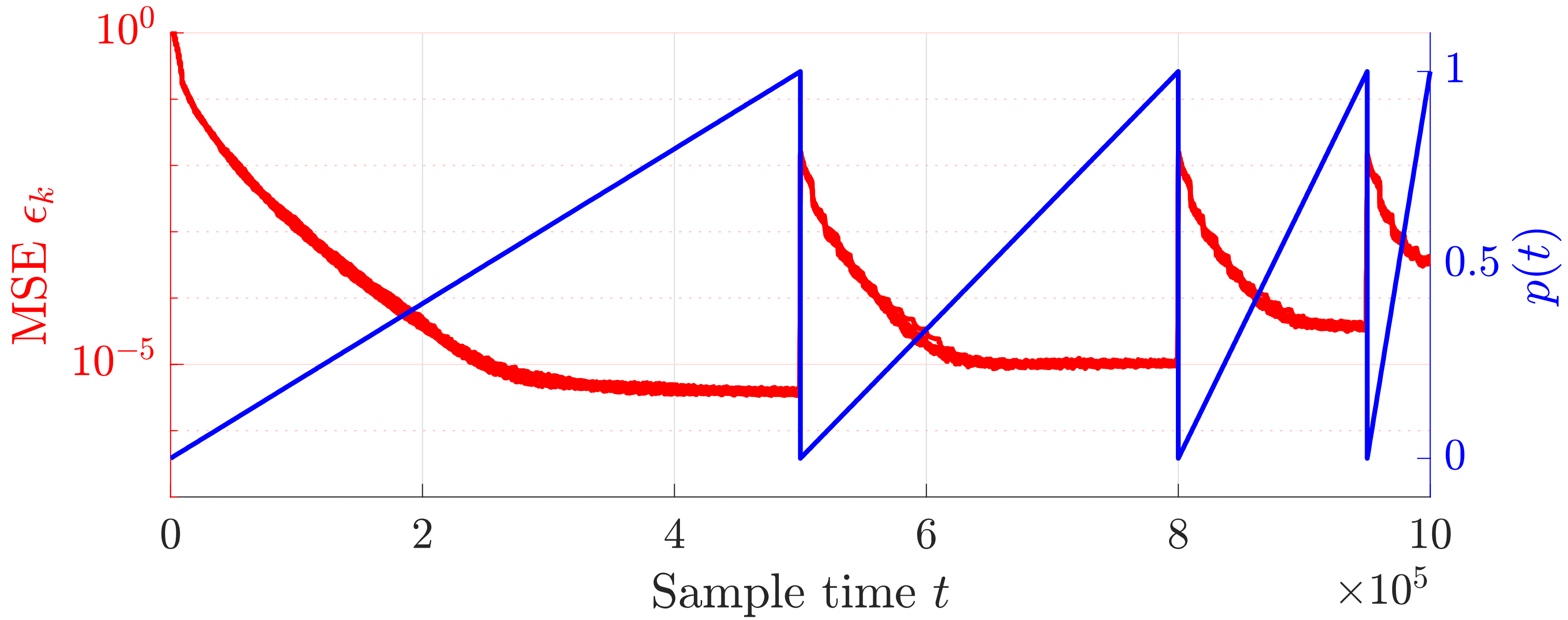
Able to track changes in statistics of signals (statistics of $\boldsymbol{y}$ dependent on p)



Conclusion

- The DANSF algorithm provides an extension of the DASF framework to spatial filtering problems with node-specific objectives/constraints

- Under the assumption that the solutions of the problems at the different nodes have a linear relationship, the DANSF algorithm converges for each node to the respective optimal solution

Future work:

- Study node-specific problems with different relationships between them

Thank you