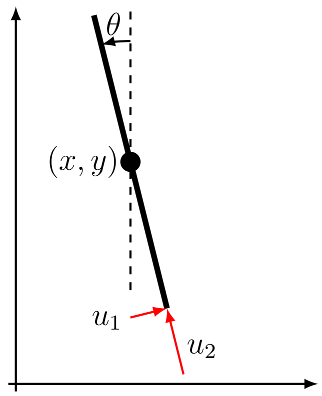

Figure 1: Rocket landing

Rocket Landing - Dynamics

The 70 meter long Falcone 9 rocket is about to land. Your task is to design a control trajectory that steers the rocket to land on the platform. There is a thruster system with two control inputs controlling the lateral ($u_1 = F_1$) and longitudinal ($u_2 = F_2$) forces.

The rocket is described by a rod with length l, mass M and moment of intertia J. To derive the equations we introduce $L = T - V$, where kinetic and potential energies are given by

$$T = \frac{1}{2}M(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}J\dot{\theta}^2 \tag{1}$$

$$V = Mgy \tag{2}$$

The Euler-Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F_x$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = F_y$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = M_\theta$$

give

$$M\ddot{x} = F_x = \cos(\theta)u_1 - \sin(\theta)u_2 \tag{3}$$

$$M\ddot{y} + Mg = F_y = \sin(\theta)u_1 + \cos(\theta)u_2 \tag{4}$$

$$J\ddot{\theta} = M_\theta = \frac{l}{2}u_1. \tag{5}$$

If we normalize $M = 1$ and put $\alpha = \frac{l}{2J}$ we get

$$\ddot{x} = \cos(\theta)u_1 - \sin(\theta)u_2 \tag{6}$$
$$\ddot{y} = \sin(\theta)u_1 + \cos(\theta)u_2 - g \tag{7}$$
$$\ddot{\theta} = \alpha u_1. \tag{8}$$

We now linearize the system around a trajectory where $(u_1, u_2, \theta) \approx (0, u_2^0, 0)$ where u_2^0 is a constant describing the average control signal $u_2(t)$ during the landing. This gives the approximative linear system

$$\ddot{x}(t) = u_1(t) - u_2^0\,\theta(t) \tag{9}$$
$$\ddot{y}(t) = u_2(t) - g \tag{10}$$
$$\ddot{\theta}(t) = \alpha u_1(t). \tag{11}$$

The units for u_1 and u_2 are m/s^2. For a uniform rod one has $J = Ml^2/12$, which with $M = 1$ gives $\alpha = 6/l$. A state space description is given by

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ \theta \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -u_2^0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 - g \end{bmatrix}.$$

This contintuous time system $\dot{x}(t) = Ax(t) + Bu(t)$ can then be transformed to the discrete time system $x(k+1) = \Phi x(k) + \Gamma u(k)$ by the command `c2d`.