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EEE-473 Homework-1

1)

Figure1: Torso X Ray Medical Imaging [1]

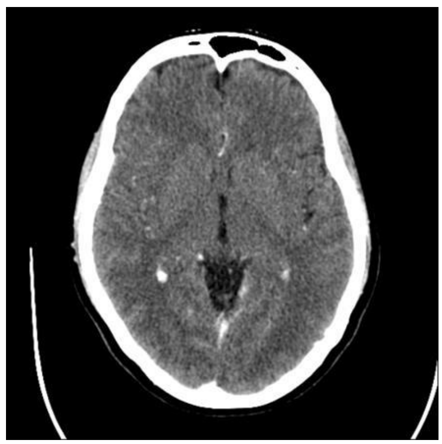

Figure 2: Brain CT Medical Imaging [2]

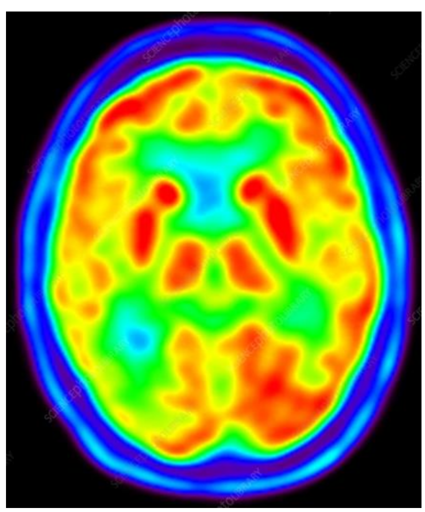

Figure 3: Brain PET Medical Imaging [3]

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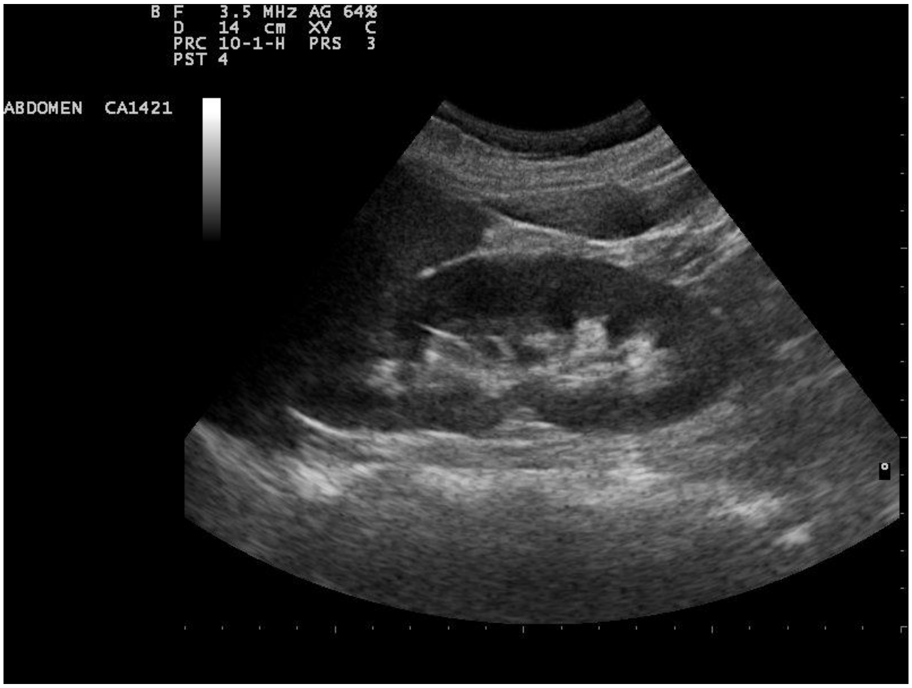

Figure 4: Kidney Ultrasound Medical Imaging [4]

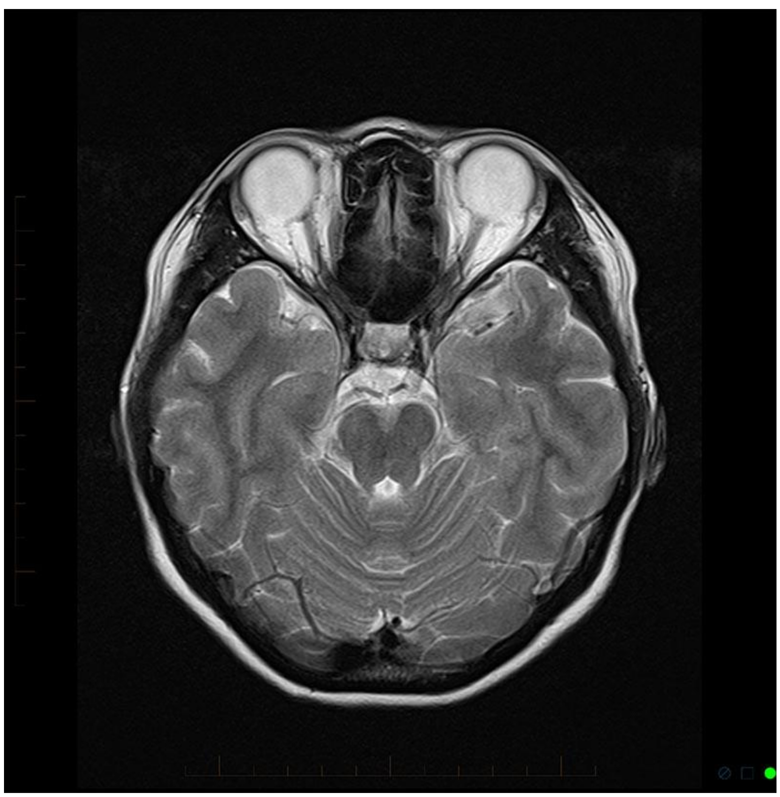

Figure 5: Brain MRI Medical Imaging [5]

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2-3-4)

Question 2

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a) $g(x,y) = f(x,-1) + f(0,y)$

Linearity

New input: $f'(x,-1) + f'(0,y) = \sum_{k=1}^{K} w_k \left(f_k(x,-1) + f_k(0,y)\right)$

$g'(x,y) = \sum_{k=1}^{K} w_k \left(\underbrace{f_k(x,-1) + f_k(0,y)}_{g_k(x,y)}\right) = \sum w_k \left(g_k(x,y)\right)$

System is Linear

Shift Invariance

New input: $f(x-x_0, -1-y_0) + f(-x_0, y-y_0)$

Shifting the output $\Rightarrow g(x-x_0, y-y_0) = f(x-x_0,-1) + f(0, y-y_0)$

Not equal

They are not equal thus system is Not Shift invariant

b) $g(x,y) = \max(f(x,y), 0)$

New input: $\sum_{k=1}^{K} w_k f_k(x,y)$

$g'(x,y) = \max\left(\sum_{k=1}^{K} w_k f_k(x,y), 0\right)$

Not the Same

Sum of outputs

$\sum g'(x,y) = \sum_{k=1}^{K} \max(f(x,y), 0) \cdot w_k$

System is not linear

CamScanner ile tarandı

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Shift invariance

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New input : $f'(x,y) = f(x-x_0, y-y_0)$

$g'(x,y) = \max(f(x-x_0, y-y_0), 0)$
$g(x-x_0, y-y_0) = \max(f(x-x_0, y-y_0), 0)$ Same

System is Shift Invariant

Question 3

Student number = 21703986, $x_1 = 6$, $y_1 = 8$

a) $f(x,y) \cdot \delta(x-6, y+8) = f(6,-8)\,\delta(x-6, y+8)$

$f(6,-8) \cdot \delta(x-6, y+8) = e^{j2\pi(-2)} \cdot \delta(x-6, y+8)$

$= \underline{e^{-j4\pi}\,\delta(x-6, y+8)}$

b) $f(x,y) * \delta(x-6, y+8) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x-6, y+8)\,dxdy$

$= f(6,-8) = e^{j2\pi(-2)} = \underline{e^{-4j\pi}}$

c) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(x-6, 3y+8)\, f(x,y)\,dxdy = f(6, \frac{-8}{3})$

$f(6, -\frac{8}{3}) = e^{j2\pi(\frac{10}{3})} = \underline{e^{\frac{j20\pi}{3}}}$

d) $f(x+1, -8) * \delta(x-6, y+1) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x+1,-8)\,\delta(x-6, y+8)\,dxdy$

$= f(7,-8) = e^{j2\pi(-1)} = \underline{e^{-2\pi j}}$

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Question 4

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ID number = 21703986, $x_1 = 6$, $y_1 = 8$

a) $g(x,y) = f\left(\frac{x}{6}, 8y-1\right)$

$f\left(\frac{x}{6}, 8\left(y-\frac{1}{8}\right)\right) \underset{\substack{\text{Scaling}\\\text{property}}}{\Longrightarrow} \frac{6}{8} f\left(x, y-\frac{1}{8}\right)$

$\mathcal{F}_{2D}\left(\frac{6}{8} f\left(x, y-\frac{1}{8}\right)\right) = \frac{6}{8} \mathcal{F}_{2D}\left(f(x,y)\right) e^{-j2\pi\left(\frac{v}{8}\right)}$

$= \frac{3}{4} e^{-\frac{\pi j}{4}} \cdot \underbrace{\mathcal{F}_{2D}\left(f(x,y)\right)}_{1} = \frac{3}{4} e^{-\frac{\pi j}{4}}$

b) $g(x,y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{j2\pi(nx+my)}$

Fourier transform is a lined system thus:

$\mathcal{F}_{2D}\left(e^{j2\pi(nx+my)}\right) = f(u-n, v-m)$

$\mathcal{F}_{2D}\left(\sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{j2\pi(nx+my)}\right) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} f(u-n, v-m)$

c) $g(x,y) = \operatorname{sinc}(3x+6, 8y-1)$

$\mathcal{F}_{2D}(\operatorname{sinc}(3x+6, 8y-1)) \underset{\substack{\text{Translation}\\\text{property}}}{\Longrightarrow} \mathcal{F}_{2D}(\operatorname{sinc}(3x, 8y))\, e^{-j2\pi(-6u+v_0)}$

$e^{-j2\pi(-6u+v)} \cdot \mathcal{F}_{2D}(\operatorname{sinc}(3x, 8y)) \underset{\substack{\text{Scaling}\\\text{property}}}{\Longrightarrow} \frac{1}{24} \cdot e^{-j2\pi(-6u+v)} \operatorname{rect}\left(\frac{u}{3}, \frac{v}{8}\right)$

$\mathcal{F}_{2D}(\operatorname{sinc}(x,y)) = \operatorname{rect}(u,v) \rightarrow$ from table

$\mathcal{F}_{2D}(\operatorname{sinc}(3x+6, 8y-1)) = \operatorname{rect}\left(\frac{u}{3}, \frac{v}{8}\right)$

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$$d)\ g(x,y) = rect\left(6x, \frac{8y}{8}\right) e^{j2\pi(u_0 x + 4v_0 y)}$$

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Seperablity

$$g(x,y) = \left(rect(6x) \cdot rect\left(\frac{y}{8}\right)\right) \cdot e^{j2\pi(u_0 x + 4v_0 y)}$$

$\Downarrow$ Product becomes convolution

$$\mathcal{F}_{2D}\{g(x,y)\} = \left(Sinc\left(\frac{u}{6}, 8v\right) \cdot \frac{8}{3}\right) * \frac{1}{4}\delta\left(u-u_0, \frac{v-v_0}{4}\right)$$

$$= Sinc\left(\frac{u_0}{6}, 8v_0\right) \cdot \frac{1}{3}$$

$$e)\ g(x,y) = e^{-2\pi(4x^2+y^2)} * \cos(2\pi x + \pi y)$$

$$\mathcal{F}_{2D}\left(e^{-\pi(8x^2+2y^2)}\right) \xRightarrow[\text{property}]{\text{Scaling}} e^{-\pi\left(\frac{u^2}{8}+v^2\right)} \cdot \frac{1}{8}\ )$$

$$\mathcal{F}_{2D}(g(x,y)) = \mathcal{F}_{2D}\left(e^{-2\pi(4x^2+y^2)} * \cos(2\pi x + \pi y)\right)$$

$\Downarrow$ Convoluton becomes product

$$= \frac{1}{8} e^{-\pi\left(\frac{u^2}{8}+v^2\right)} \cdot \frac{1}{2}\left[\delta\left(u-1, v-\frac{1}{2}\right) + \delta\left(u+1, v+\frac{1}{2}\right)\right]$$

$$= \frac{1}{16}\left(e^{-\pi\left(\frac{1}{8}+\frac{1}{4}\right)} + e^{-\pi\left(\frac{1}{8}+\frac{1}{4}\right)}\right) = \frac{e^{-\frac{3\pi}{8}}}{8}$$

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f) i) $f(x,y) = f(-x,-y)$

$\mathcal{F}_{2D}(f^*(x,y)) = F^*(-u,-v) = F(u,v)$ if $f(x,y)$ is real

$\Rightarrow$ Conjugation Rule

$\mathcal{F}_{2D}(f^*(-x,-y)) \xrightarrow[\text{property}]{\text{Scaling}} F^*(u,v)$

$F^*(u,v) = F(u,v)$ becuse they're both fourier transform of $f^*(x,y)$ and $f^*(-x,-y)$ which is stated to be equal in the question.

ii) $f(x,y) = -f(-x,-y)$

$\mathcal{F}_{2D}(f^*(x,y)) = F^*(-u,-v) = F(u,v)$ if $f(x,y)$ is real $\Rightarrow$ Conjugation rule

$\mathcal{F}_{2D}(-f^*(-x,-y)) \xrightarrow{\text{Scaling rule}} -F^*(u,v)$

$-F^*(u,v) = F(u,v)$ becuse they are fourier transform of respectively $-f(-x,-y)$ and $f(x,y)$ and it is stated that $f(x,y) = -f(-x,-y)$. Thus $-F(u,v) = F^*(u,v)$

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5) Matlab Question

All the matlab code is given in the appendix section of the homework.

a)

Figure 6: Ideal Image

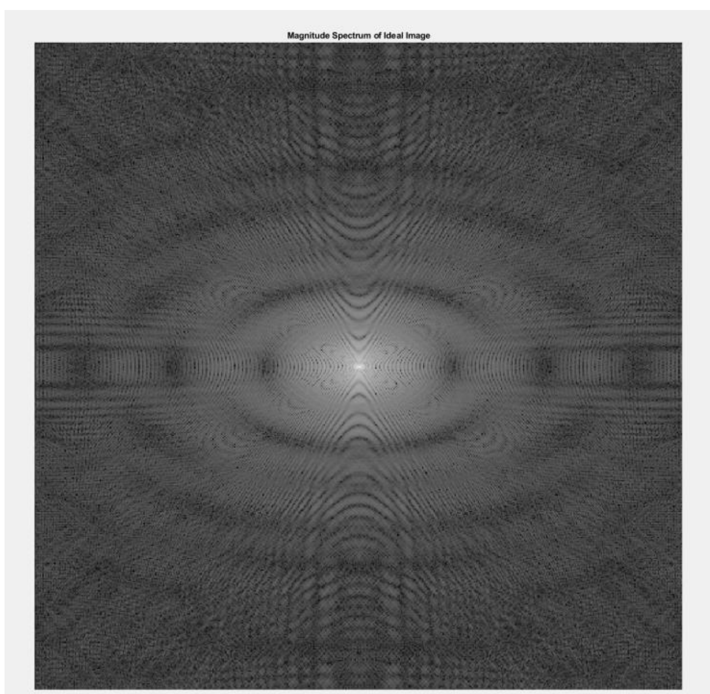

Figure 7: Ideal Image Magintude Spectrum

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b)

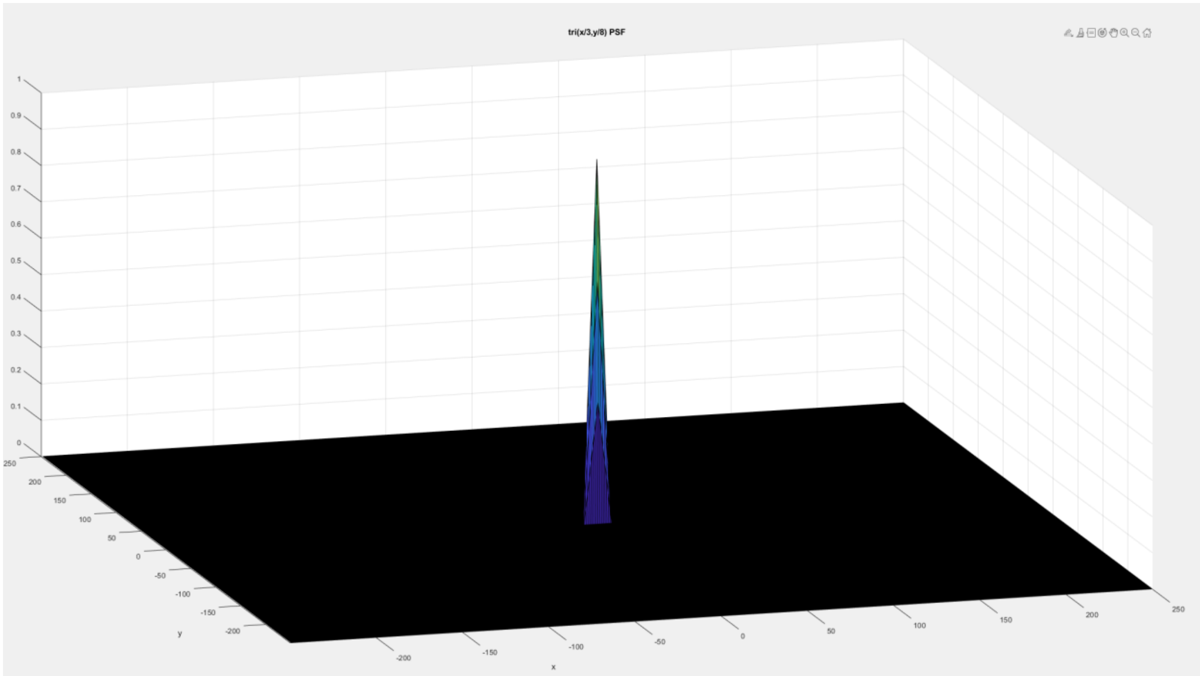

Figure 8: h1tri function PSF 3D graph

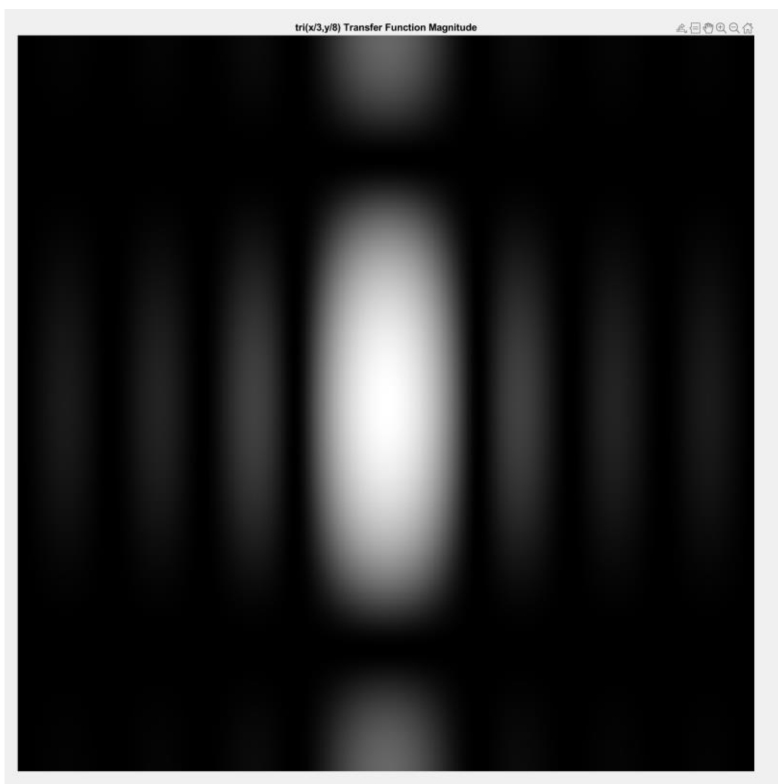

Figure 9: h1 trifunction Magintude Spectrum

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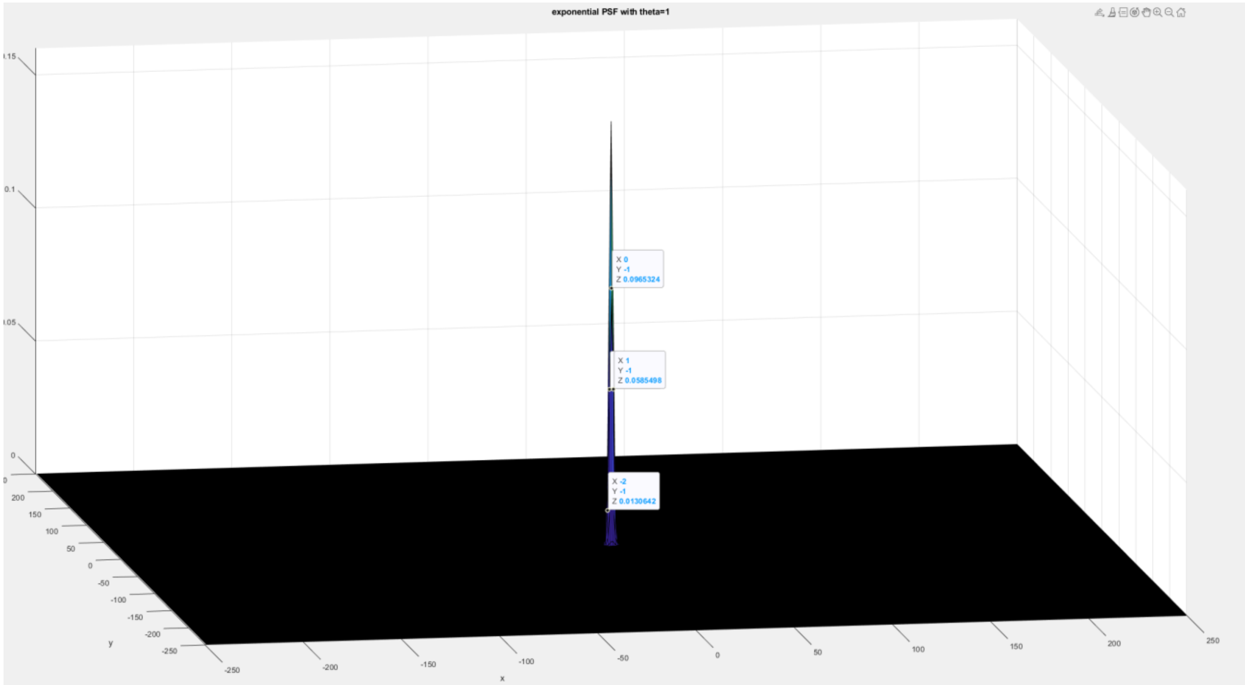

Figure 10: h2 exponential function PSF 3D graph

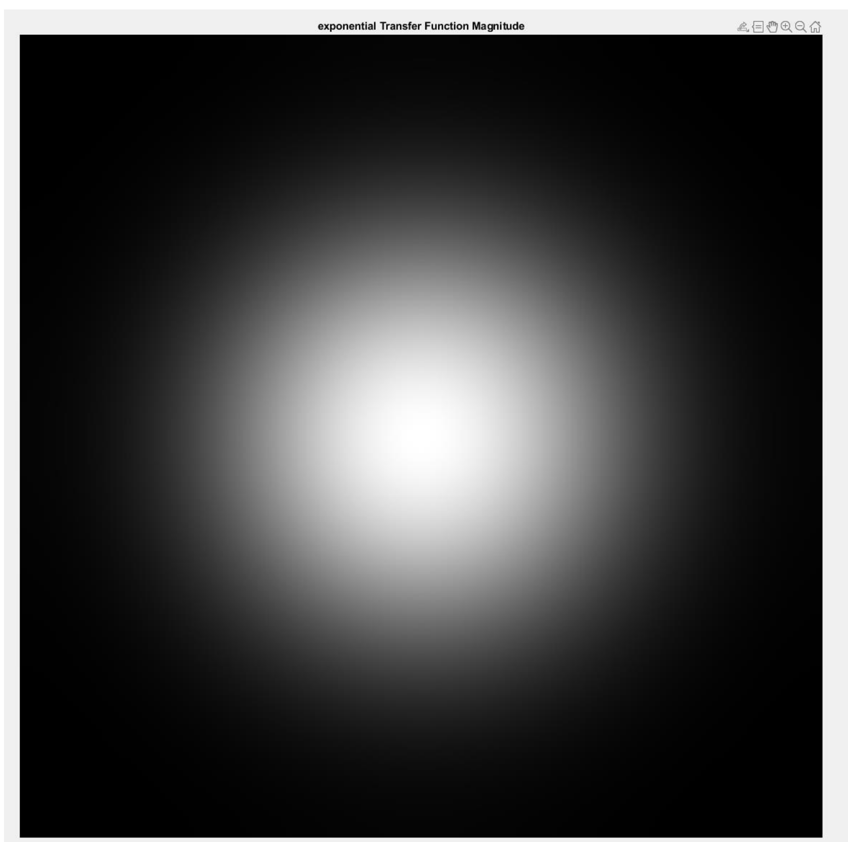

Figure 11: h2 exponential function Magintude Spectrum

c)

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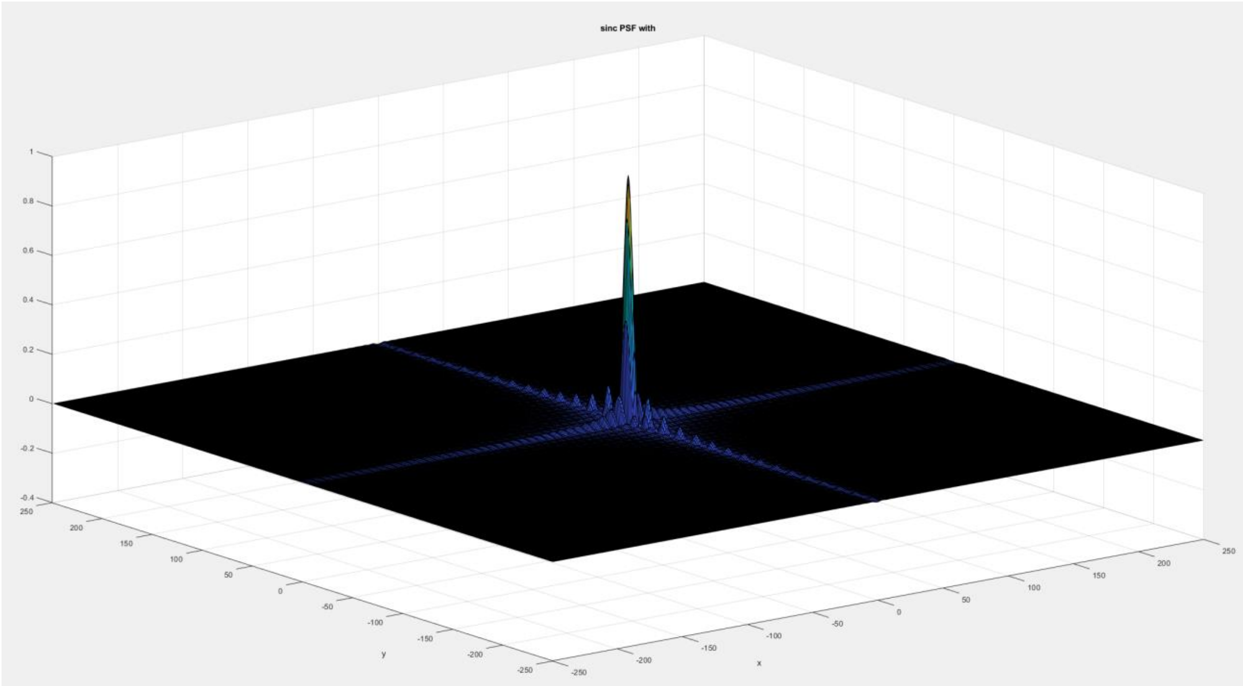

Figure 12: h3 sinc function PSF 3D graph

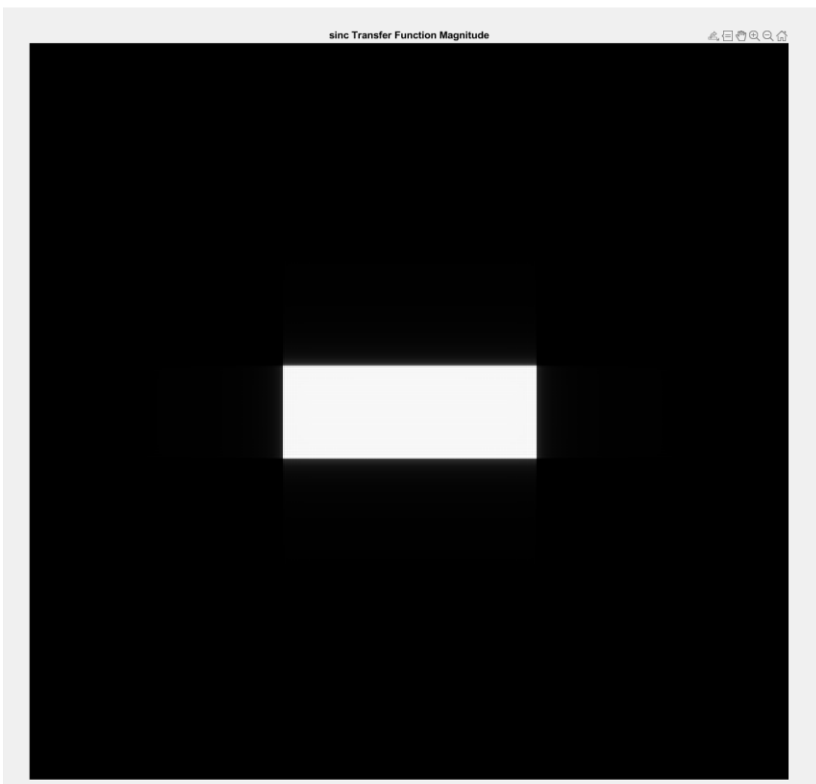

Figure 13: h3 sinc function Magintude Spectrum

c)

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Figure 14: Image resulted by using tri function system

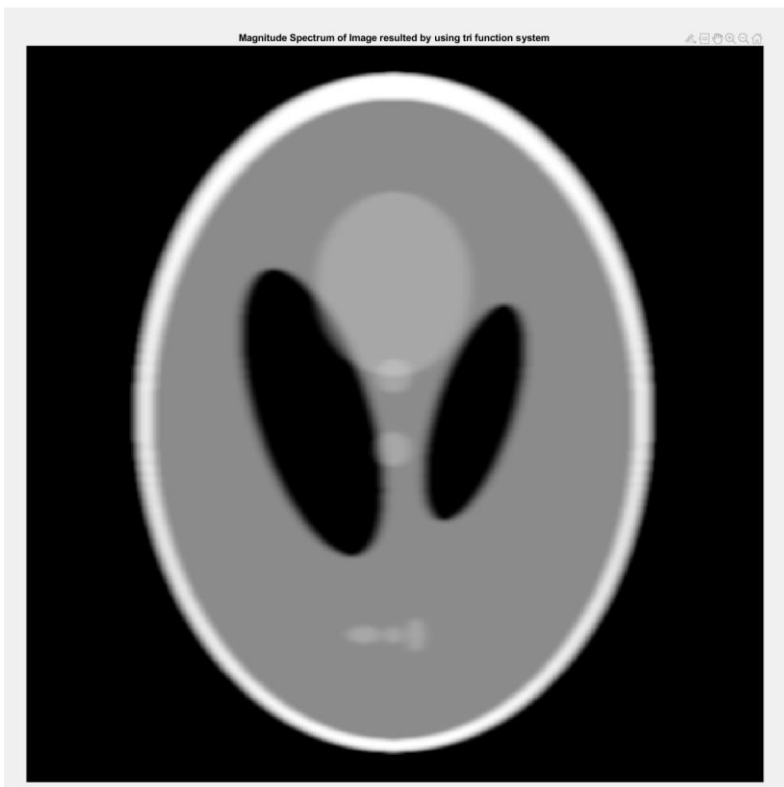

Figure 15: Magnitude Spectrum of the Image resulted by using tri function system

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Figure 16: Image resulted by using exp function system

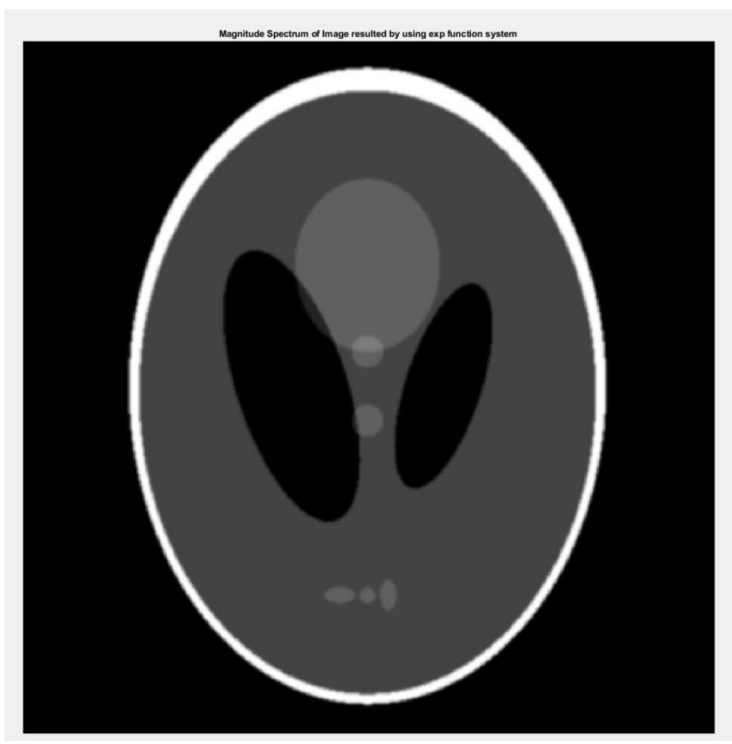

Figure 17: Magnitude Spectrum of the Image resulted by using exp function system

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Figure 18: Image resulted by using sinc function system

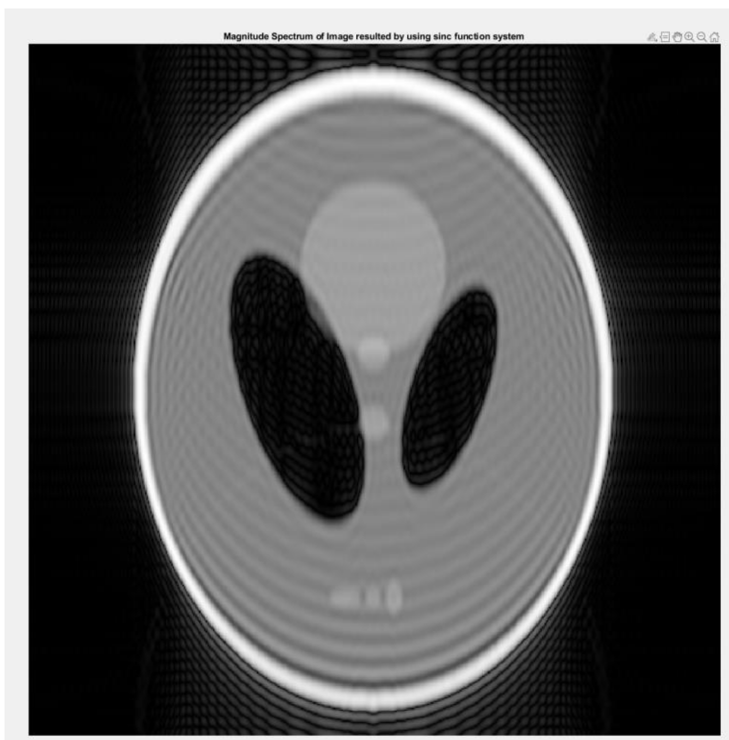

Figure 19: Magnitude Spectrum of the Image resulted by using sinc function system

d)

In figure 6 it can be seen the original image that we are working on. In figure 14 the image has resulted from the triangular point spread function. The smaller details such as bottom three circles are not visible in this image yet it is visible in its magnitude spectrum which is shown in figure 15. In figure 16 image resulted from the exponential point spread function can be seen. In this image, all small details and circles are more clear compared to the triangular function. In its magnitude spectrum in image 17 details can be also seen very clearly and it is very similar to the original picture. In the sinc function point spread function result which is shown in image 18 details are nearly gone. Its magnitude spectrum image is better but still has saturation and details are disturbed and not clear. Thus I believe h2 which is the exponential point spread function is the best system for this image.

References

[1] Unsplash, "Medical imaging X ray film pictures: Download free images on unsplash," *Medical Imaging X Ray Film Pictures | Download Free Images on Unsplash*. [Online]. Available: https://unsplash.com/s/photos/medical-imaging-x-ray-film. [Accessed: 15-Oct-2021].

[2] D. Cuete, "Normal CT brain: Radiology case," *Radiopaedia Blog RSS*. [Online]. Available: https://radiopaedia.org/cases/normal-ct-brain. [Accessed: 15-Oct-2021].

[3] S. N. S. C. I. E. N. C. E. P. H. O. T. O. LIBRARY, "Normal brain, PET scan - stock image - C039/3562," *Science Photo Library*. [Online]. Available: https://www.sciencephoto.com/media/934350/view/normal-brain-pet-scan. [Accessed: 15-Oct-2021].

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[5] "MRI Brain Scan Diagnostic Imaging: Melbourne Radiology," *Melbourne Radiology Clinic*, 18-Jun-2021. [Online]. Available: https://www.melbourneradiology.com.au/diagnostic-imaging/mri-scan-brain/. [Accessed: 15-Oct-2021].

Appendix

Matlab Code

```
clear all
% a
% Showing the ideal image
P = phantom('Modified Shepp-Logan',500);
imshow(P)
title("Ideal Image")
figure()
```

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```
% Showing the magnitude spectrum
F=fft2c(P);
imshow(log(abs(F)+1),[])
title("Magnitude Spectrum of Ideal Image")
figure()
%b
%i)-------------------------
xi=[-249:1:250];
yi=[-249:1:250];
psfi=zeros(500,500);
for k=1:500
    for l=1:500
psfi(k,l)=tri(xi(k)/3,yi(l)/8);
    end
end
surf(xi,yi,psfi)
title("tri(x/3,y/8) PSF")
xlabel("x")
ylabel("y")
figure()
%transfer function which is fourier of impulse response(psf)
Transferi=fft2c(psfi);
imshow(log(abs(Transferi)+1),[])
title("tri(x/3,y/8) Transfer Function Magnitude")
figure()
%ii)-----------------------------------------
xii=[-249:1:250];
yii=[-249:1:250];
psfii=zeros(500,500);
for k=1:500
    for l=1:500
psfii(k,l)=expo2(xii(k),yii(l));
    end
end
surf(xii,yii,psfii)
title("exponential PSF with theta=1")
xlabel("x")
ylabel("y")
figure()
Transferii=fft2c(psfii);
imshow(log(abs(Transferii)+1),[])
title("exponential Transfer Function Magnitude")
figure()
%iii)-------------------------
xiii=[-249:1:250];
yiii=[-249:1:250];
psfiii=zeros(500,500);
for k=1:500
    for l=1:500
        %using seperablity
psfiii(k,l)=sinc(xiii(k)/8)*sinc(yiii(l)/3);
    end
end
surf(xiii,yiii,psfiii)
```

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```
title("sinc PSF with")
xlabel("x")
ylabel("y")
figure()
Transferiii=fft2c(psfiii);
imshow(log(abs(Transferiii)+1),[])
title("sinc Transfer Function Magnitude")
figure()
%c---------------------------
%With h=tri
IdealImagei=F.*Transferi;
IdealImagei=ifft2c(IdealImagei);
imshow(IdealImagei);
title("Image resulted by using tri function system ")
figure()
%--magnitude
imshow(log(abs(IdealImagei)+1),[])
title("Magnitude Spectrum of Image resulted by using tri function system ")
figure()
%With h=exp
IdealImageii=F.*Transferii;
IdealImageii=ifft2c(IdealImageii);
imshow(IdealImageii);
title("Image resulted by using exp function system ")
figure()
%--magnitude
imshow(log(abs(IdealImageii)+1),[])
title("Magnitude Spectrum of Image resulted by using exp function system ")
figure()
%h=sinc
IdealImageiii=F.*Transferiii;
IdealImageiii=ifft2c(IdealImageiii);
imshow(IdealImageiii);
title("Image resulted by using sinc function system ")
figure()
%--magnitude
imshow(log(abs(IdealImageiii)+1),[])
title("Magnitude Spectrum of Image resulted by using sinc function system ")
figure()
function h=tri(a,b)
if and(abs(a)<=1,abs(b)<=1)
    h=((1-abs(a))*(1-abs(b)));
else
    h=0;
end
end
function h=expo2(a,b)
theta=1;
h=exp(-(a^2+b^2)/(2*theta^2))*1/(2*pi*theta*theta);
end
function d = fft2c(im)
 % d = fft2c(im)
 %
 % fft2c performs a centered fft2
```

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```
im = fftshift(fft2(ifftshift(im)));
d=im;
end
function im = ifft2c(d)
% im = fft2c(d)
%
% ifft2c performs a centered ifft2
im = fftshift(ifft2(ifftshift(d)));
end
```