EEE 473/573 Medical Imaging – Fall 2021-2022
Homework 1
Due 15 October 2021, Friday at 23:59

GUIDELINES FOR HOMEWORK SUBMISSION

1. NO submission via E-MAIL (all email submissions will be discarded).
2. Submit a single PDF file. Other file types will not be accepted. If there are any handwritten parts, you can scan them (make sure they are legible) and insert into the PDF file. Unclear presentation of results will be penalized heavily. No partial credits to unjustified answers.
3. For the part labeled as "MATLAB Question", you can choose to use MATLAB or other softwares (e.g., Python). Make sure to include the relevant codes at the end of the PDF file to be submitted. If your codes are missing, that question will NOT be graded.
4. This is a Turnitin submission. The Turnitin system requires the submitted file to contain at least 20 words in it. If you are submitting a Word file with scanned pages only, the file may be rejected by the system. You can type your name multiple times at the beginning of the file to overcome this problem.
5. Submission system will remain open for 1 day after the deadline. No points will be lost if you submit your assignment within 12 hours of the deadline. There will be a 50% penalty if you submit after 12 hours but within 24 hours past the deadline. No submissions beyond 24 hours past the deadline.

1) Find one image from each of the following medical imaging modalities: x-ray, CT, PET, ultrasound, and MRI. Clearly label the imaging system and indicate which organ/tissue is shown in the image (e.g., head, torso, heart, kidneys, liver etc.). For reference, include the URLs of the source webpages under each image or DOI number if images taken from an article.

2) For each system with the following input-output equation, determine whether the system is linear and determine whether it is shift-invariant:

   a) $g(x,y) = f(x,-1) + f(0,y)$

   b) $g(x,y) = \max(f(x,y),0) = \begin{cases} f(x,y), & f(x,y) \geq 0 \\ 0, & f(x,y) < 0 \end{cases}$

3) Given a continuous signal, $f(x,y) = e^{j2\pi(x+y)}$, evaluate the following, considering x_1 is the last digit and y_1 is the second-to-last digit of your student number. If any of the digits is 0, assume it is 2 instead.

   a) $f(x,y)\delta(x - x_1, \ y + y_1)$

   b) $f(x,y) * \delta(x - x_1, \ y + y_1)$

   c) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(x - x_1, \ 3y + y_1)f(x,y) \, dx \, dy$

   d) $f(x + 1, -y_1) * \delta(x - x_1, \ y + 1)$

4) Find the 2D Fourier transforms of the following continuous signals. You can make use of Fourier Transforms tables for any known relations. Assume, x_1 is the last digit and y_1 is the second-to-last digit of your student number. If any of the digits is 0, assume it is 2 instead.

   a) $g(x,y) = \delta\left(\frac{x}{x_1}, \ y_1 y - 1\right)$

   b) $g(x,y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{j2\pi(nx+my)}$

   c) $g(x,y) = sinc(3x + x_1, \ y_1 y - 1)$

d) $g(x,y) = rect\left(x_1 x, \dfrac{y}{y_1}\right) e^{j2\pi(u_0 x + 4v_0 y)}$

e) $g(x,y) = e^{-2\pi(4x^2 + y^2)} * \cos(2\pi x + \pi y)$

f) Show the following relations, assuming that $F(u,v) = \mathcal{F}_{2D}(f(x,y))$ and $f(x)$ is a real-valued signal.

   i.   If $f(x,y) = f(-x,-y)$, then $F^*(u,v) = F(u,v)$

   ii. If $f(x,y) = -f(-x,-y)$, then $F^*(u,v) = -F(u,v)$

PREPARATION 1) Centered FFT: In medical imaging, the preferred way to display the image and the Fourier domain data is such that the origin is at the center of the image or data array. The usual convention for the FFT in MATLAB, however, is that the origin is at the beginning of the array, or the upper left corner of a 2D array. To do a centered FFT, you want to do fftshift/ifftshift before/after the FFT. To do this, define the following functions in MATLAB:

```
function d = fft2c(im)
% d = fft2c(im)
%
% fft2c performs a centered fft2
im = fftshift(fft2(ifftshift(im)));
end

function im = ifft2c(d)
% im = fft2c(d)
%
% ifft2c performs a centered ifft2
im = fftshift(ifft2(ifftshift(d)));
end
```

When you type 'help fft2c' in Matlab, you will now see the commented text that gives the function usage. Pay attention to including "help" sections when you create your own functions/scripts.

You may also want the corresponding one dimensional versions fftc and ifftc. Note that fftshift and ifftshift give exactly the same result when the array size is even-valued, but are different otherwise.

PREPARATION 2) Displaying the magnitude spectrum: To display the magnitude spectrum as an image, we typically do the following:

>> imshow(log(abs(F)+1),[])

where F is the 2D Fourier transform of an image. In Fourier domain, the value at DC (i.e., at the origin of Fourier domain) is much larger than the values elsewhere. The "log" operation brings these values closer together, so that we can display the entire magnitude spectrum more easily. The addition of 1 is to avoid the log(0) problem.

5) MATLAB QUESTION: Generate a "phantom" image in MATLAB using the following command:

P = phantom('Modified Shepp-Logan',500);

This digital phantom presents an axial crosscut of a human body, showing the lungs, the heart, and a few blood vessels. Assume that this is our object of interest, with its physical x-axis and y-axis

ranging from $-20\ cm < x \leq 20\ cm$ and $-20\ cm < y \leq 20\ cm$ (i.e., 500 pixels corresponding a physical extent of 40 cm). "P" is our "ideal" image. Answer the following questions.

Hint: You may want to use the following functions in MATLAB: *imshow, meshgrid.*

a) Display the "ideal" image P and its magnitude spectrum.

b) We image this phantom using four different medical imaging systems with point spread functions (PSF) given below. Display the PSFs and the magnitudes of the transfer functions for these systems. Choose reasonable x and y values such that you capture a sufficiently large portion of the PSF.

    i. $h_1(x,y) = tri\left(\frac{x}{3}, \frac{y}{8}\right)$,   where $tri(x,y) = \begin{cases} (1 - |x|)(1 - |y|), & |x| \leq 1, |y| \leq 1 \\ 0, & \text{otherwise} \end{cases}$

    ii. $h_2(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$ with $\sigma = 1$.

    iii. $h_3(x,y) = sinc\left(\frac{x}{8}, \frac{y}{3}\right)$

c) Display the images resulting from each medical imaging system in part (b). Also, display the magnitude spectrum of each resulting image.

d) Explain what you see in the images and the resulting magnitude spectrums, and briefly comment on the quality of the images.