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EEE-473 Homework-2

1-2-3-4-5)

Hand written solutions for question 1 2 3 4 and 5 are scanned and given in this section. Matlab solution for question 6 is given in the other section. All the code can be seen in the appendix.

1)

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$$\mathcal{H}(\text{rect}(r)) = \text{jinc}(q) = \frac{J_1(\pi q)}{2q}, \quad q = r^2$$

$\Rightarrow$ Scaling property

$$H\left(\text{rect}\left(\frac{r}{a}\right)\right) = a^2\,\text{jinc}(aq) = \frac{J_1(a\pi q)}{2q}$$

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2 a) $h_1(x,y) = e^{-\pi\left(\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2\right)}$

$h_2(x,y) = \text{sinc}(4x,y)$

$\mathcal{F}_{2D}\left(e^{-\pi(x^2+y^2)}\right) = e^{-\pi(u^2+v^2)}$

From scaling property:

$\mathcal{F}_{2D}\left(e^{-\pi\left(\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2\right)}\right) = 3.2.\,e^{-\pi(4u^2+9v^2)}$

$= 6.\,e^{-\pi(4u^2+9v^2)} = H_1(u,v)$

$\mathcal{F}_{2D}(\text{sinc}(x,y)) = \text{rect}(u,v)$

From scaling property:

$\mathcal{F}_{2D}(\text{sinc}(4x,y)) = \frac{1}{4}\,\text{rect}\left(\frac{u}{4}, v\right)$

$MTF(H_1) = \frac{H_1(u,v)}{H_1(0,0)} = \frac{6.\,e^{-\pi(4u^2+9v^2)}}{6} = e^{-\pi(4u^2+9v^2)}$

$e^{-\pi(4u^2+9v^2)} = MTF_1$

$MTF(H_2) = \frac{H_2(u,v)}{H_2(0,0)} = \frac{\text{rect}\left(\frac{u}{4}, v\right).\frac{1}{4}}{\frac{1}{4}\,\underbrace{\text{rect}(0,0)}_{1}} = \text{rect}\left(\frac{u}{4}, v\right)$

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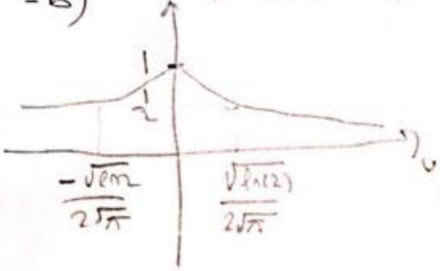
2b) $MTF_1(u,0) = e^{-\pi(4u^2)}$



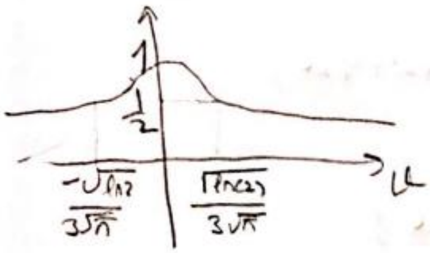
$$e^{-\pi(4u^2)} = \frac{1}{2}$$

$$-\pi 4u^2 \cdot \ln e = \ln\left(\frac{1}{2}\right)$$

$$+\pi 4u^2 = \ln(2)$$

$$u = \frac{\sqrt{\ln(2)}}{2\sqrt{\pi}}$$

$MTF_1(0,v) = e^{-\pi(9v^2)}$



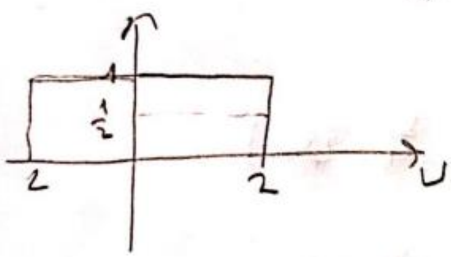
$$e^{-\pi(9v^2)} = \frac{1}{2}$$

$$-\pi 9v^2 \ln e = \ln\left(\frac{1}{2}\right)$$

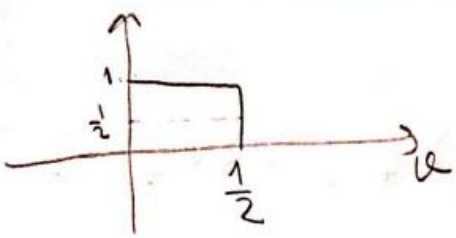
$$+\pi 9v^2 = \ln(2)$$

$$v = \frac{\sqrt{\ln(2)}}{3\sqrt{\pi}}$$

$$MTF_2(u,0) = rect\left(\frac{u}{4}, 0\right) = rect\left(\frac{u}{4}\right) \cdot \overset{1}{\overline{rect(0)}} = rect\left(\frac{u}{4}\right)$$



$$MTF_2(0,v) = rect(0,v) = rect(0) \cdot rect(v) = rect(v)$$



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2c)

$$\frac{B}{A} = \frac{2}{3}$$

$$H_1(u,v) = 6 \cdot e^{-\pi(4u^2+9v^2)}$$

from part a

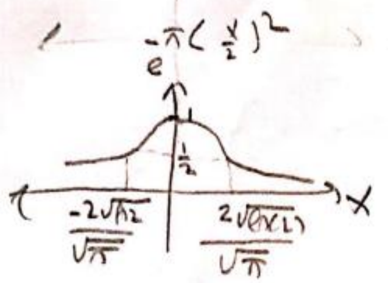
$$mg = \frac{2}{3} \cdot \frac{|H(u_0,v_0)|}{H(0,0)} = \frac{2}{3} e^{-13\pi}$$

$$H_2(u,v) = rect\left(\frac{u}{4}, v\right)$$

$$mg = \frac{2}{3} rect\left(\frac{1}{4}, 1\right)$$

3a) $h_1 = e^{-\pi\left(\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2\right)}$

for x axis : $h_1(x,0) = e^{-\pi\left(\frac{x}{2}\right)^2}$



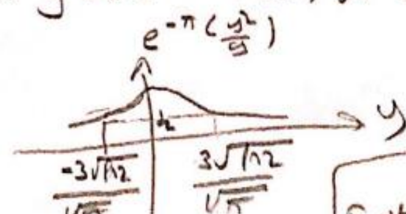
$$e^{-\pi\frac{x^2}{4}} = \frac{1}{2}$$

$$\pi\frac{x^2}{4} = \ln(2)$$

$$x = \mp\frac{2\sqrt{\ln(2)}}{\sqrt{\pi}}$$

$$\boxed{FWHM = \frac{4\sqrt{\ln 2}}{\sqrt{\pi}}}$$

for y axis $h_1(0,y) = e^{-\pi\frac{y^2}{9}}$



$$e^{-\pi\frac{y^2}{9}} = \frac{1}{2}$$

$$\pi\frac{y^2}{9} = \ln(2)$$

$$y = \mp\frac{3\sqrt{\ln(2)}}{\sqrt{\pi}}$$

$$\boxed{FWHM = \frac{6\sqrt{\ln 2}}{\sqrt{\pi}}}$$

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$h_2 = \mathrm{sinc}(4x, y)$, $h_2(x, 0) = \mathrm{sinc}(4x)$

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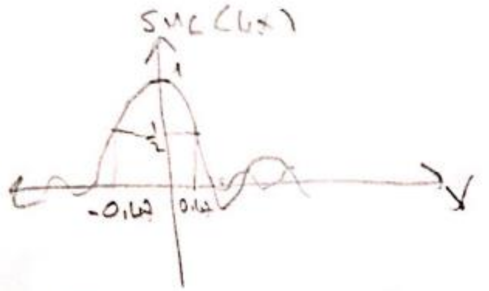
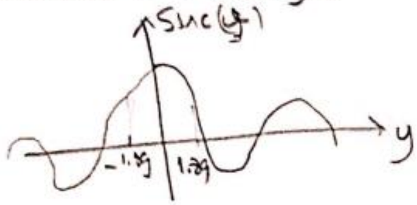


$\mathrm{sinc}(4x) = \frac{1}{2}$

from arcsinc $\rightarrow$ $y = \frac{1.89}{4} = 0.47$

$\mathrm{FWHM} = 0.47 \cdot 2 = 0.94$

$h_2(0, y) = \mathrm{sinc}(y)$



$y = \mp 1.89$

$\hookrightarrow$ from arcsinc $\left(\frac{1}{2}\right)$

$\mathrm{FWHM} = 2 \times 1.89 = 3.78$

b) along the x:

$\mathrm{FWHM}_x = \sqrt{\mathrm{FWHM}_{1x}^2 + \mathrm{FWHM}_{2x}^2} = \sqrt{\frac{16 \ln 2}{\pi} + 0.94^2}$

$\cong 2.1$

along y:

$\mathrm{FWHM}_y = \sqrt{\mathrm{FWHM}_{1y}^2 + \mathrm{FWHM}_{2y}^2} = \sqrt{\frac{36 \ln(2)}{\pi} + 3.78^2}$

$\cong 4.72$

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4a) prevelance = $\frac{183+320}{183+72+320+7425} = \frac{503}{8000}$

$\frac{503}{8000} = \%6.28$

Sensitivity = $\frac{183}{183+320} = \frac{183}{503} = \%36$

Specificity: $\frac{7425}{7425+72} = \%99$

PPV: $\frac{183}{183+72} = \frac{183}{255} = \%71.7$

NPV: $\frac{7425}{7425+320} = \frac{7425}{7745} = \%95.8$

b) Prevelance = $\frac{143+360}{143+39+360+7458} = \%6.28$

Sensitivity: $\frac{143}{503} = \%28.4$

Specifity: $\frac{7458}{7497} = \%99.4$

PPV: $\frac{143}{182} = \%78.5$

NPV: $\frac{7458}{7818} = \%95.3$

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Prevelance didnt change becuse test threshhold doesnt affect how many percentage of subjects have the disease

Sensitivity decreased and Specificity increased from a to b. This is because threshold is increased for test result to conclude subject is positive thus positive test results decreased and negative test results increased

PPV increased and NPV decreased from a to b

c) For this ANN b is better

This is because when threshold is increased there is a significant increase in PPV and slight decrease in NPV. Becuse negative test results were alredy reliable from %95.8 NPV, %95.3 NPV from b still a good confidence interval. But positive test results truthness increased from %71.7 to %78.5

Thus b is better

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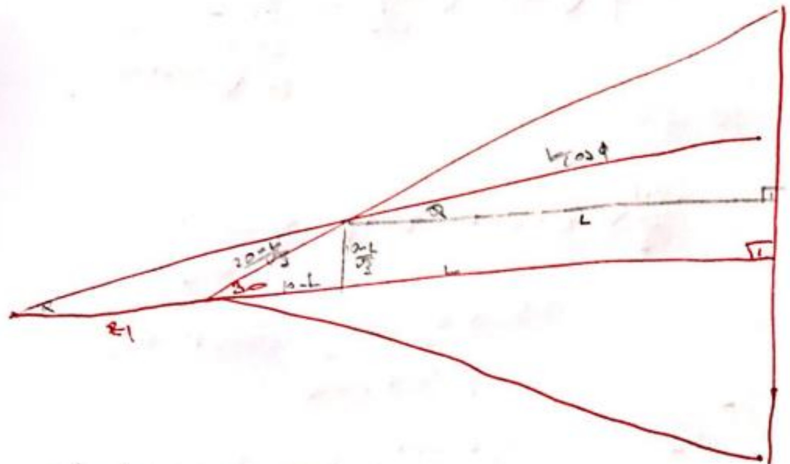
Question 5)



$$\frac{\frac{0.1-L}{\sqrt{3}\cdot\sin\phi}}{\frac{0.1+z_1}{\cos\phi}} = \frac{0.1-L}{\sqrt{3}(0.1+z_1)} \cot\phi = \frac{0.1-L+z_1}{0.1+z_1}$$

$$(0.1-L)\cot\phi = (0.1-L+z_1)\sqrt{3}$$

$$0.1\cot\phi - L\cot\phi = 0.1\sqrt{3} - \sqrt{3}L + z_1\sqrt{3}$$

$$0.1\cot\phi - 0.1\sqrt{3} - \sqrt{3}z_1 = L(\cot\phi - \sqrt{3})$$

$$L = \frac{0.1(\cot\phi - \sqrt{3}) - \sqrt{3}z_1}{\cot\phi - \sqrt{3}} = 0.1 - \frac{z_1\sqrt{3}}{\cot\phi - \sqrt{3}}$$

$$\frac{L}{\cos\phi} = \text{path} = \boxed{\frac{0.1}{\cos} - \frac{z_1\sqrt{3}}{\frac{\cos^2}{\sin} - \sqrt{3}\cos}}$$

$$I_0 = \frac{I_s}{4\pi d^2} = \frac{I_s}{4\pi}$$

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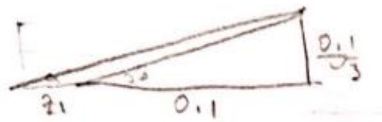
$$I_d = \frac{I_s}{4\pi} \cos^3\phi \cdot \exp\left(-\mu_o \left(\frac{0.1}{\cos} - \frac{z_1\sqrt{3}}{\frac{\cos^2}{|\sin|} - \sqrt{3}\cos}\right)\right)$$

$\mu_o = 0.1\ cm^{-1} = 10\ m^{-1}$



$$\frac{-0.1}{\sqrt{3}} < (z+0.1)\tan\phi < \frac{0.1}{\sqrt{3}}$$

Meaning ray doesnt miss prism core

$(z+0.1)\cdot\tan\phi \geq -\frac{0.1}{\sqrt{3}}$ or $(z+0.1)\cdot\tan\phi \leq \frac{0.1}{\sqrt{3}}$

Meaning ray misses the prism

For $\frac{-0.1}{\sqrt{3}} < (z+0.1)\tan\phi < \frac{0.1}{\sqrt{3}}$) Ray doest miss the prism

$$I_x = \frac{I_s}{4\pi} \cos^3\phi \cdot \exp\left(-\mu_o \cdot \left(\frac{0.1}{\cos} - \frac{z_1\sqrt{3}}{\frac{\cos^2}{|\sin|} - \frac{\sqrt{3}}{\cos}}\right)\right)$$

For $(z+0.1)\tan\phi \geq -\frac{0.1}{\sqrt{3}}$ or $(z+0.1)\tan\phi \leq \frac{0.1}{\sqrt{3}}$

$$I_x = \frac{I_s}{4\pi} \cdot \cos^3\phi$$

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For $z_1 = 0.5$

Ray doesn't miss condition: $\frac{-0.1}{\sqrt{3}} < 0.6 \tan\phi < \frac{0.1}{\sqrt{3}}$

$$I_\alpha = \frac{I_s}{4\pi} \cos^3\phi \, \exp\left(-10 \cdot \left(\frac{0.1}{\cos\phi} - \frac{\sqrt{3} \cdot 0.5}{2 \cdot \left(\frac{\cos^2\phi}{|\sin\phi|} - \frac{\sqrt{3}}{\cos}\right)}\right)\right)$$

Ray missing condition: $0.6 \tan\phi \geq -\frac{0.1}{\sqrt{3}}$ or $0.6 \tan\phi \leq \frac{0.1}{\sqrt{3}}$:

$$I_\alpha = \frac{I_s}{4\pi} \cos^3\phi$$

For $z_1 = 0.3$

Ray doesn't miss condition: $-\frac{0.1}{\sqrt{3}} < 0.9 \tan\phi < \frac{0.1}{\sqrt{3}}$:

$$I_\alpha = \frac{I_s}{4\pi} \cos^3\phi \, \exp\left(-10 \left(\frac{0.1}{\cos\phi} - \frac{0.8\sqrt{3}}{\frac{\cos^2\phi}{|\sin\phi|} - \frac{\sqrt{3}}{\cos}}\right)\right)$$

Ray miss condition: $0.9 \tan\phi \leq -\frac{0.1}{\sqrt{3}}$ or $0.9 \tan\phi \geq \frac{0.1}{\sqrt{3}}$

$$I_\alpha = \frac{I_s}{4\pi} \cos^3\phi$$

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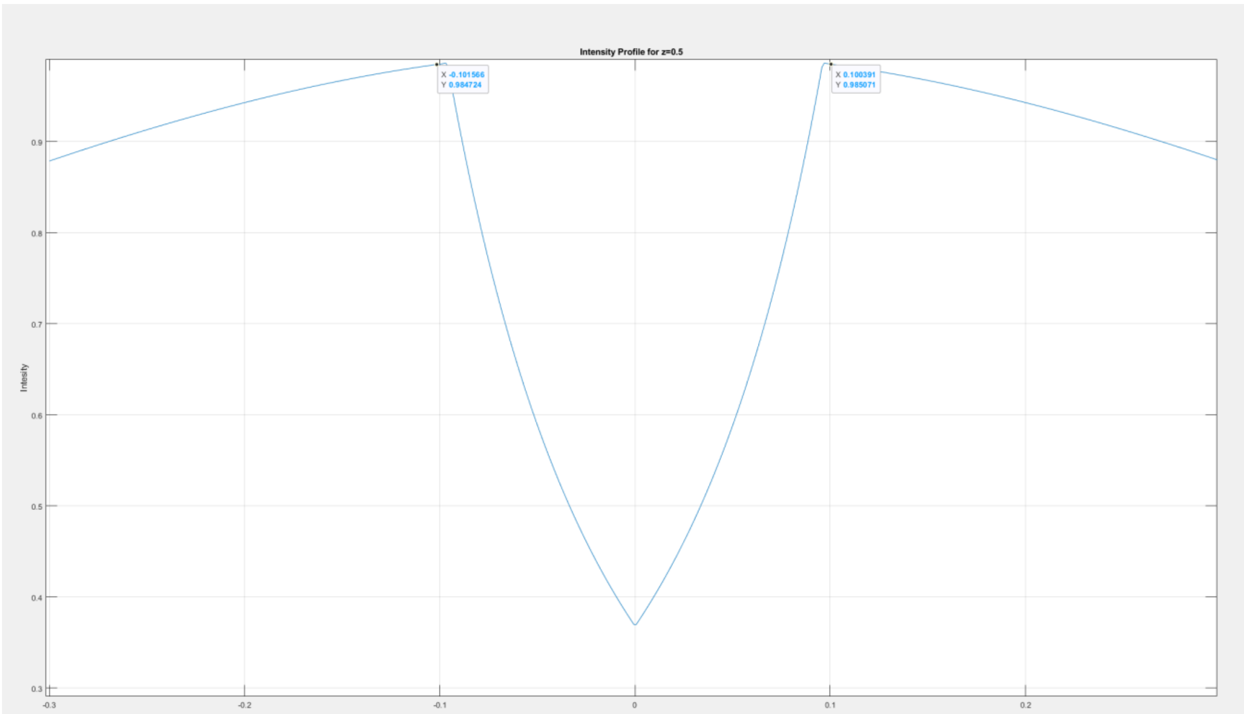
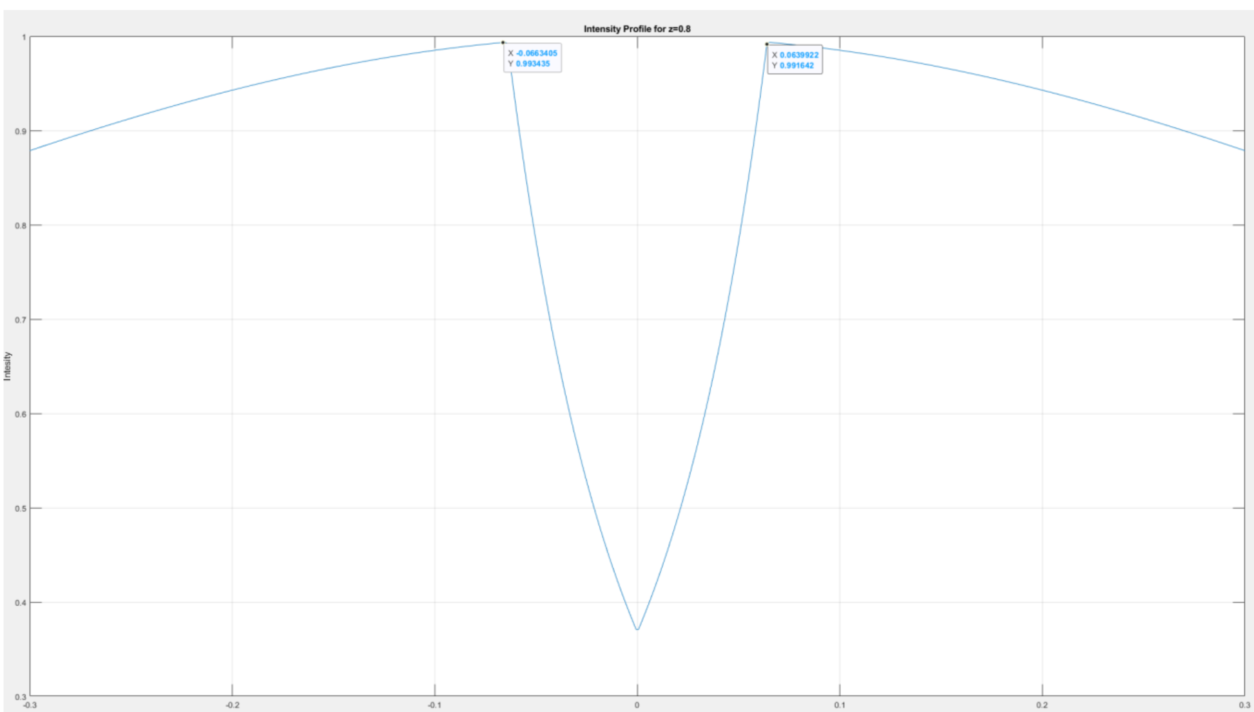
Question 6(Matlab)

a)



Plot 1: Intensity Profile when z=0.5



Plot 2: Intensity Profile when z=0.8

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b)

From plots it can be seen that when parameter z(distance between the source and the cone decreases more of the source rays goes through the cone. This results in distance between origin and x plane of the detector gets affected by the attenuation coefficent from the cone. When z increases detector interval that is affected by the attenuation decreases as it can be from the plots. In plot 1 when z=0.5 it can be seen that attenuation affected interval between -0.1 and 0.1 meter and the rest of the detector range from -0.3 to 0.3 meter is not affected by the attenuation. On the other hand when z=0.8 interval range of the detector that is affected by the attenuation range decreased as it can be seen from the plot 2. The range interval of the detector that is affected by the attenuation is between -0.06 to 0.06 meter on the detector. The rest between -0.3 to 0.3 meter is not affected by the atteunated ray but it is affected by the not attenuated ray.

Appendix

Matlab Code

```
clear all
% For z=0.5
I0=1;
x=linspace(-0.3,0.3,512);
q=atand(x/1);
Ix=zeros(512,1)
for i=1:512
if(and(0.6*tand(q(i))>=-0.1/sqrt(3),0.6*tand(q(i))<=0.1/sqrt(3)) )
    co=I0*cosd(q(i))^3;
    a=0.1/cosd(q(i));
    bnum=sqrt(3)*0.5;
    bdem=((cosd(q(i))^2)/abs(sind(q(i))))-sqrt(3)/cosd(q(i));
    Ix(i)=co*exp(-10*(a-bnum/bdem));
else
    Ix(i)=I0*cosd(q(i))^3;
    x(i)
end
end

plot(x,Ix)
title('Intensity Profile for z=0.5')
xlabel("x")
ylabel("Intesity")
grid on
figure()
%%----------------------------
% For z=0.8

I0=1;
x=linspace(-0.3,0.3,512);
q=atand(x/1);
Ix=zeros(512,1)
for i=1:512
if(and(0.9*tand(q(i))>=-0.1/sqrt(3),0.9*tand(q(i))<=0.1/sqrt(3)) )
    co=I0*cosd(q(i))^3;
    a=0.1/cosd(q(i));
```

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```
    bnum=sqrt(3)*0.8;
    bdem=((cosd(q(i))^2)/abs(sind(q(i))))-sqrt(3)/cosd(q(i));
    Ix(i)=co*exp(-10*(a-bnum/bdem));
else
    Ix(i)=I0*cosd(q(i))^3;
    x(i)
end
end

plot(x,Ix)
title('Intensity Profile for z=0.8');
xlabel("x");
ylabel("Intesity")
grid on
```