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EEE-473 Homework-3

1-2-3-4)

Hand written solutions for question 1 2 3 and 4 are scanned and given in this section. Matlab solution for question 5 is given in the other section. All the code can be seen in the appendix.

Homework 3

Question 1

$$f(x,y) \xrightarrow[\text{Transform}]{\text{2D Radon}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x\cos\theta + y\sin\theta - l)\,dx\,dy$$

$l = x\cos\theta + y\sin\theta$

$$= g(l,\theta)$$

$$x = l\cos\theta - s\sin\theta$$
$$y = l\sin\theta + s\cos\theta$$

$f(x-x_0, y-y_0)$

$$\hookrightarrow l' = \underbrace{x\cos\theta + y\sin\theta}_{l} - x_0\cos\theta - y_0\sin\theta$$

$$l - x_0\cos\theta - y_0\sin\theta$$

Thus:

$$\text{Radon}_{2D}(f(x-x_0, y-y_0)) = g(l',\theta) = g(l - x_0\cos\theta - y_0\sin\theta, \theta)$$

Question 2

$f(x,y)$ is rotationally symetric: $g(l,\theta) = g(l,0)$

$$g(l,\underset{0}{\theta}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{-x^2-y^2}{2}}\,\delta(x\cos(\theta)(\theta) + y\sin\theta - l)\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{-x^2-y^2}{2}}\,\delta(x-l)\,dx\,dy$$

$$= \int_{-\infty}^{\infty} e^{\frac{-l^2-y^2}{2}}\,dy$$

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$$= e^{-\frac{\ell^2}{2}} \int_{-\infty}^{\infty} e^{-\frac{y^2}{2}} dy$$

$$\int_{-\infty}^{\infty} e^{-\frac{y^2}{2}} = \sqrt{2\pi}$$

$$g(\ell,\theta) = \sqrt{2\pi}\, e^{-\ell^2}$$

b) $g(\ell,\theta) = \sqrt{2\pi}\, e^{-\ell^2}$

$$\hat{g}(\ell,\theta) = F_{1D}^{-1}\left(F_{1D}\left(\sqrt{2\pi}\, e^{-\ell^2}\right) |\rho| \cdot e^{-\frac{\rho^2}{4}}\right.$$

$$G(\rho,\theta) = \int_{-\infty}^{\infty} e^{-\ell^2} \cdot \sqrt{2\pi} \cdot e^{-j2\pi\rho\ell} = \sqrt{2}\,\pi\, e^{-\pi^2\rho^2}$$

$$G(\rho,\theta) = \sqrt{2}\,\pi\, e^{-\pi^2\rho^2}$$

$$\hat{g}(\ell,\theta) = F_{1D}^{-1}\left(e^{-(\pi^2+\frac{1}{4})\cdot\rho^2} \cdot |\rho| \cdot \sqrt{2} \right)$$

CamScanner ile tarandı

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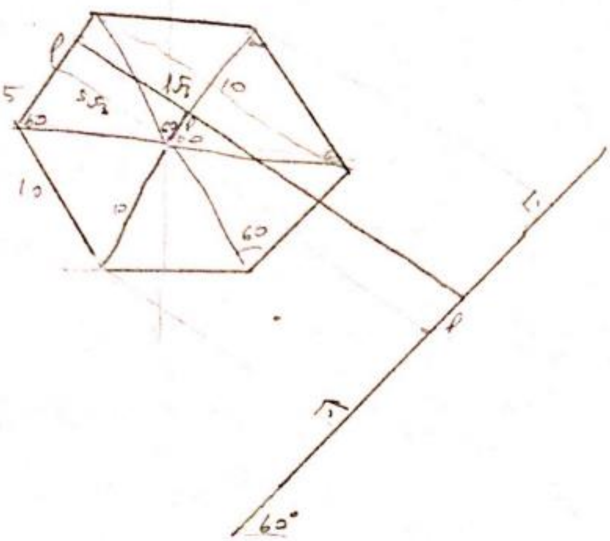
Question 3



$g(l,60)$ is 0 if $l > 10$ or $l < -10$ as it is shown in the figure.

for $0 < l \leq 5$:

μ_6 length: $\dfrac{5-l}{5} = \dfrac{x}{5\sqrt{3}} \Rightarrow 5x = 25\sqrt{3} - 5\sqrt{3}\,l$

$\underline{x = 5\sqrt{3} - \sqrt{3}\,l}$

μ_1 length $= \mu_2$ length: $l\sqrt{3}$

μ_3 length $= \mu_6$ length $= 5\sqrt{3} - \sqrt{3}\,l$

for $5 < l < 10$

μ_1 length $=$ μ_2 length $= \dfrac{\overbrace{5-(l-5)}^{10-l}}{5} = \dfrac{x}{5\sqrt{3}} \Rightarrow 50\sqrt{3} - 5\sqrt{3}\,l = 5x$

$\underline{x = 10\sqrt{3} - \sqrt{3}\,l}$

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for $-5 < l < 0$ (from symetry)

v_6 length $= v_3$ length : $(5\sqrt{3} + \sqrt{3}\, l)$

$v_4 = v_5$ length : $-l\sqrt{3}$

for $-10 < l < -5$

$v_5 = v_4$ length $= 10\sqrt{3} + \sqrt{3}\, l$

So $g(l, 60°)$:

$(5\sqrt{3} - \sqrt{3}\, l)\underbrace{0.55}_{v_6+v_3} + l\sqrt{3} \cdot \underbrace{(0.3)}_{v_1+v_2}$: for $0 < l < 5$

$(10\sqrt{3} - \sqrt{3}\, l)\underbrace{(0.3)}_{v_1+v_2}$: for $5 < l < 10$

$(5\sqrt{3} + \sqrt{3}\, l)\underbrace{0.55}_{v_6+v_3} - l\sqrt{3}\underbrace{(0.5)}_{v_4+v_5}$: for $-5 < l < 0$

$(10\sqrt{3} + \sqrt{3}\, l) \cdot \underbrace{(0.5)}_{v_4+v_5}$ for $-10 < l < -5$

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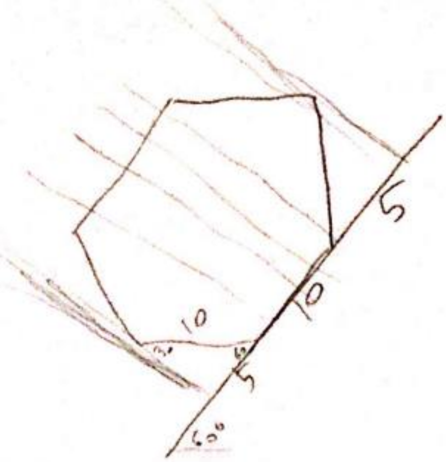
4)

a)



Detector crossing from edge has the smallest length that covering Fov

$$10+5+5=20cm$$

$$x=20 \Rightarrow \text{Shortest Fov}$$

b)

256 elements on the detector

$$N_{projections} \geq \frac{\pi}{2} \cdot 256 \cong 403$$

$N_{projections} \geq 403$ for avoid aliasing

Thus:

$$g(l,\theta) = 256 \times 403$$

But reconstructed image should have resolution of

$$256 \times 256$$

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Question 5(Matlab)

a)

Image 1: Phantom is displayed

b)

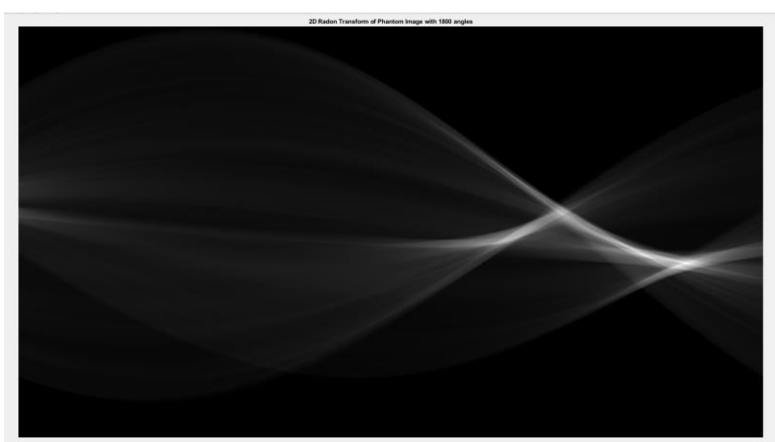

Image 2: 2D Radon Transform of the Phantom with 1800 angles

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Image 3: Reconstructed Image with 1800 angles

c)

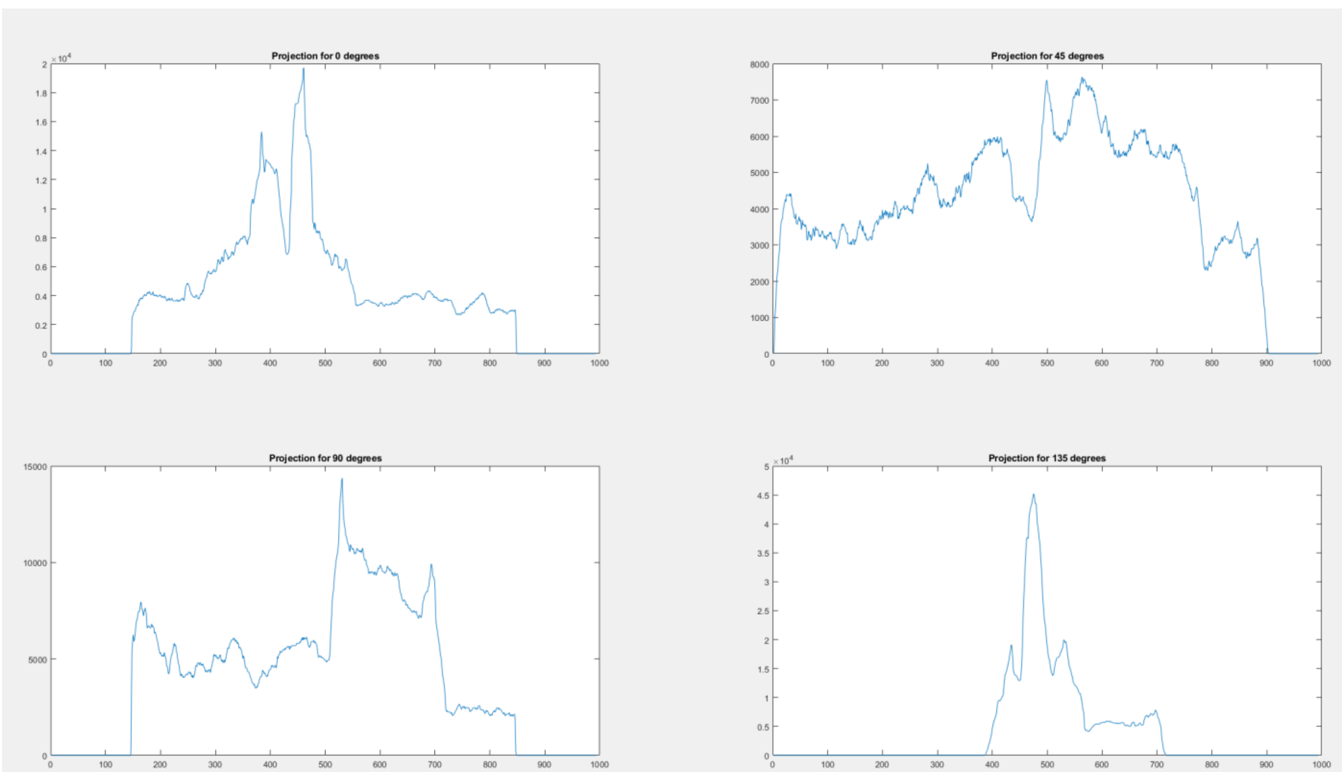

Image 4: Projections of P with 0, 45,90 and 135 degrees.

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d)

Image 5: 2D Radon Transform of the Phantom with 60 angles

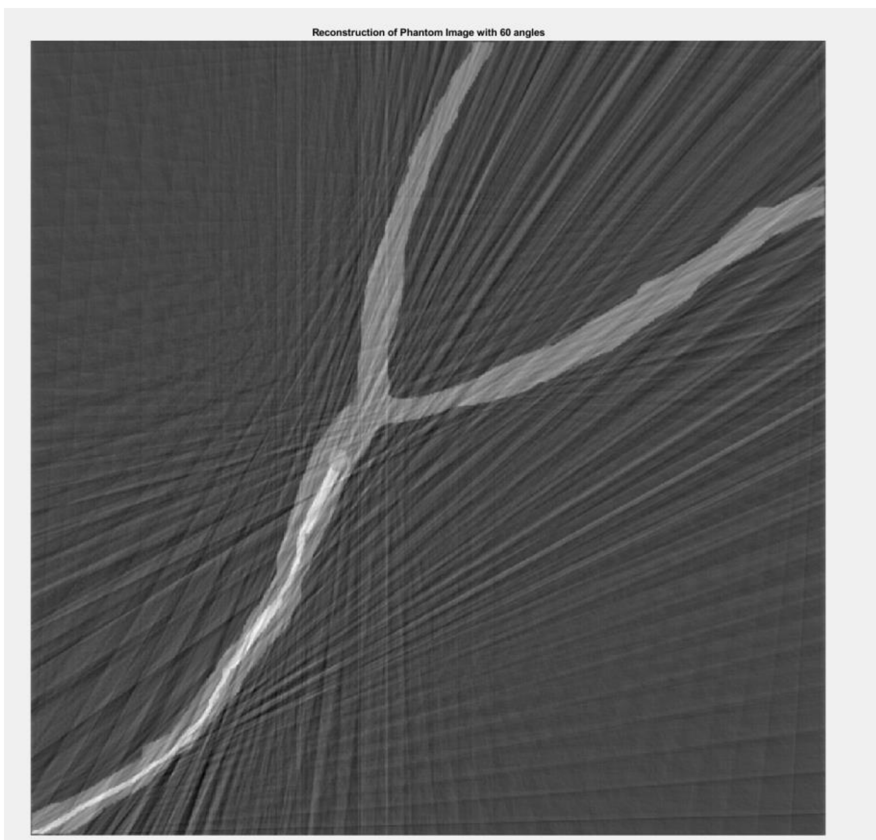

Image 6: Reconstruction of Phantom Image with 60 angles(clear aliasing)

Because there are not sufficent number of projections taken, clear aliasing can be seen in the reconstructed image.

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e)

i,ii,iii)

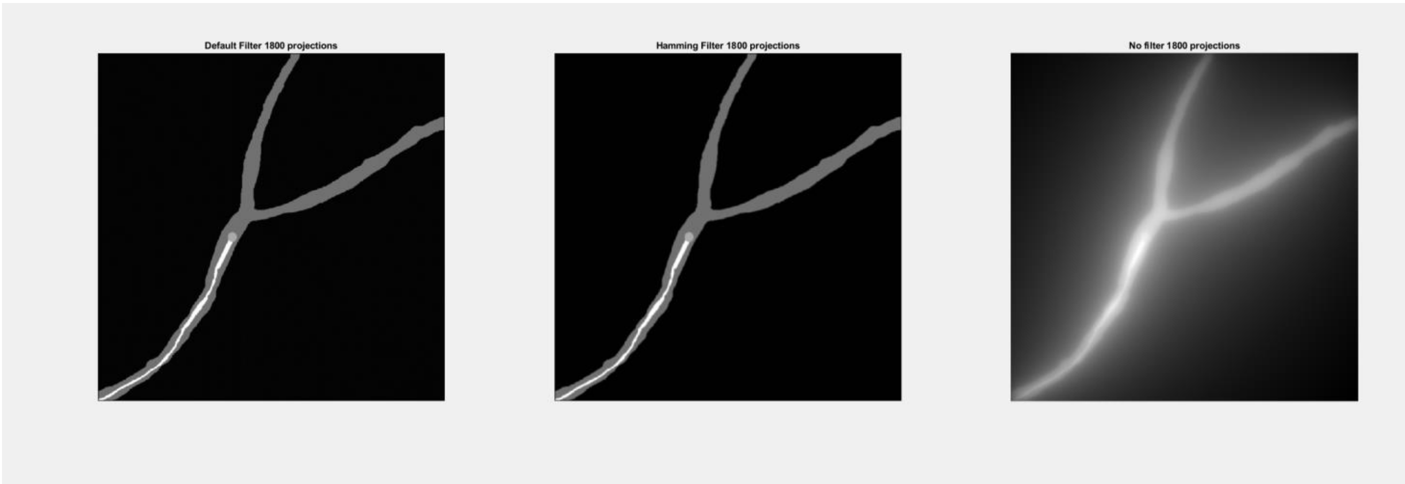

Image 7: Reconstruction of Phantom Image with 1800 angles with Ram-Lak filter, Hamming Filter and No Filter

There are no artifacts and RamLak Filter(Default Filter) is better for reconstruction result with sufficent number of projections because there is no need for smoothing.

f)

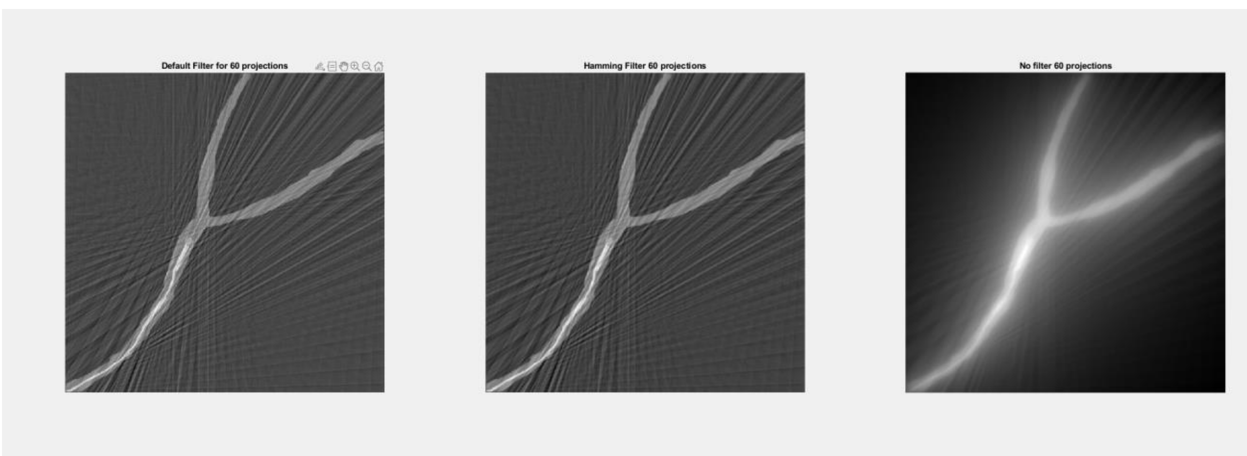

Image 8: Reconstruction of Phantom Image with 60 angles with Ram-Lak filter, Hamming Filter and No Filter

There is clear aliasing in the reconstructed images for all filters due to small amount of projections that is used for reconstructing the images. Disturbtion from these aliasing is reduced with Hamming Filter and thus best result is obtained from Hamming Filter as it can be seen from the images.

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Appendix

Matlab Code

```
% a-------
% Loading the image
P = double(rgb2gray(imread('vessel_and_catheter.png')));
imshow(P,[])
title("Phantom Image")
figure()
% b------------
theta=linspace(0,180,1800);
Rb=radon(P,theta);
imshow(Rb,[])
title("2D Radon Transform of Phantom Image with 1800 angles")
figure()
Rbinverse=iradon(Rb,theta);
imshow(Rbinverse,[])
title("Reconstructed Image")
figure()
%c------------
t=linspace(1,993,993);
Rb0=Rb(:,1);
Rb45=Rb(:,451);
Rb90=Rb(:,901);
Rb135=Rb(:,1351);
subplot(2,2,1);
plot(t,Rb0)
title("Projection for 0 degrees")
subplot(2,2,2);
plot(t,Rb45)
title("Projection for 45 degrees")
subplot(2,2,3);
plot(t,Rb90)
title("Projection for 90 degrees")
subplot(2,2,4);
plot(t,Rb135)
title("Projection for 135 degrees")
figure()
% d--------
thetad=linspace(0,180,60);
Rbd=radon(P,thetad);
imshow(Rbd,[])
title("2D Radon Transform of Phantom Image with 60 angles")
figure()
Rbinversed=iradon(Rbd,thetad);


imshow(Rbinversed,[])
title("Reconstruction of Phantom Image with 60 angles")
figure()
% e-----------------
RbInverseHamming=iradon(Rb,theta,'Hamming');
```

```
RbInverseNofilter=iradon(Rb,theta,'linear','none');
subplot(1,3,1)
imshow(Rbinverse,[]);
title("Default Filter 1800 projections");
subplot(1,3,2)
imshow(RbInverseHamming,[]);
title("Hamming Filter 1800 projections");
subplot(1,3,3);
imshow(RbInverseNofilter,[])
title("No filter 1800 projections");
figure()
% f---------------------------
RbdInverseHamming=iradon(Rbd,thetad,'Hamming');
RbdInverseNofilter=iradon(Rbd,thetad,'linear','none');
subplot(1,3,1)
imshow(Rbinversed,[]);
title("Default Filter for 60 projections");
subplot(1,3,2)
imshow(RbdInverseHamming,[]);
title("Hamming Filter 60 projections");
subplot(1,3,3);
imshow(RbdInverseNofilter,[])
title("No filter 60 projections");
```