Name: Cemal Güven Adal

ID: 21703986

EEE-473 Homework-4

1-2-3)

1) a) $a = \gamma \int_0^{t} B_1^e(t)\, dt$

$$B_1^e(t) = \begin{cases} t.2 & , 0 \le t \le \frac{T}{2} \\ \frac{1}{10} & , \frac{T}{2} \le t \le \frac{3T}{2} \\ \frac{4}{10} - \frac{2}{10T} t & , \frac{3T}{2} \le t \le 2T \end{cases}$$

In rage $0 \le t \le \frac{T}{2}$

$$a = \gamma \int_0^{t} \frac{2t}{10T}\, dt = \frac{\gamma t^2}{10T}$$

In rage $\frac{T}{2} \le t \le \frac{3T}{2}$

$$a = \gamma \left(\int_0^{\frac{T}{2}} \frac{2t}{10T} \right) + \gamma \int_{\frac{T}{2}}^{t} \frac{1}{10}$$

$$a = \gamma \frac{T}{40} + \gamma \left(\frac{t}{10} - \frac{T}{20} \right) = \gamma \left(-\frac{T}{40} + \frac{4t}{40} \right)$$

In the rage $\frac{3T}{2} < t < 2T$

$$a = \gamma \left(-\frac{T}{40} + \frac{6T}{40} \right) + \gamma \int_{\frac{3T}{2}}^{t} \frac{4}{10} - \frac{2}{10T} t$$

$$= \gamma . \left(\frac{5T}{40} \right) + \gamma . \left(\frac{4t}{10} - \frac{t^2}{10T} - \frac{6T}{10} + \frac{9T}{20} \right)$$

$$= \gamma . \frac{5T}{40} + \gamma . \left(\frac{8t - 3T}{20} - \frac{t^2}{10T} \right)$$

CamScanner ile tarandı

Name: Cemal Güven Adal

ID: 21703986

b) at the end $t = 2T$. Thus

$$a = \frac{\gamma T}{8} + \gamma \cdot \left(\frac{16T - 3T}{20} - \frac{4T}{10} \right)$$

$$= \frac{\gamma T}{8} + \gamma \left(\frac{5T}{20} \right)$$

$$= \gamma \cdot \left(\frac{T}{8} + \frac{T}{4} \right) = \boxed{\gamma \cdot \left(\frac{3T}{8} \right)}$$

Name: Cemal Güven Adal

ID: 21703986

2) $M_z(t) = M_0 (1 - e^{-\frac{t}{T_1}}) + M_0 \cos a \cdot e^{-\frac{t}{T_1}}$

$M_z^n(t) = M_0 (1 - e^{-\frac{t}{T_1}}) + M_z^{ss} \cos a \; e^{-\frac{t}{T_1}}$

$M_z^{ss} = M_0 \dfrac{1 - e^{-\frac{TR}{T_1}}}{1 - \cos a \; e^{-\frac{TR}{T_1}}}$

$M_z^n(TR) = M_z^{ss}(TR) = M_0 \dfrac{(1 - e^{-\frac{TR}{T_1}})}{1 - \cos a \; e^{-\frac{TR}{T_1}}}$

$M_z^{n+1} = M_0 \cos a \; e^{-\frac{nTR}{T_1}} \cdot e^{-\frac{TR}{T_1}} + M_0 (1 - e^{-\frac{TR}{T_1}}) + e^{-\frac{TR}{T_1}} M_0 \cdot (1 - e^{-\frac{nTR}{T_1}})$

$\Rightarrow M_z^{ss} = \dfrac{M_0 (1 - e^{-\frac{TR}{T_1}})}{(1 - e^{-\frac{TR}{T_1}} \cdot \cos a)}$

$M_{(x,y)}^{(t)} = M_z^{ss} \, e^{j\phi} \cdot e^{-\frac{t}{T_2}} \cdot \sin a$

$M_{(x,y)}(t) = M_0 \dfrac{1 - e^{-\frac{TR}{T_1}}}{1 - e^{-\frac{TR}{T_1}}} \cdot e^{j\phi} \cdot e^{-\frac{t}{T_2}} \cdot \sin a$

Name: Cemal Güven Adal

ID: 21703986

3)

a) $0.1 \cdot 42.58 \cdot 0.5 = 2,12$ kHz BW

b) $z = 10$

$\frac{10}{\Delta z} = \frac{y}{2} = \frac{x}{1}$ $a = \frac{x}{2} = 1$cm

5 $b = \frac{y}{2} = 1,25$ cm

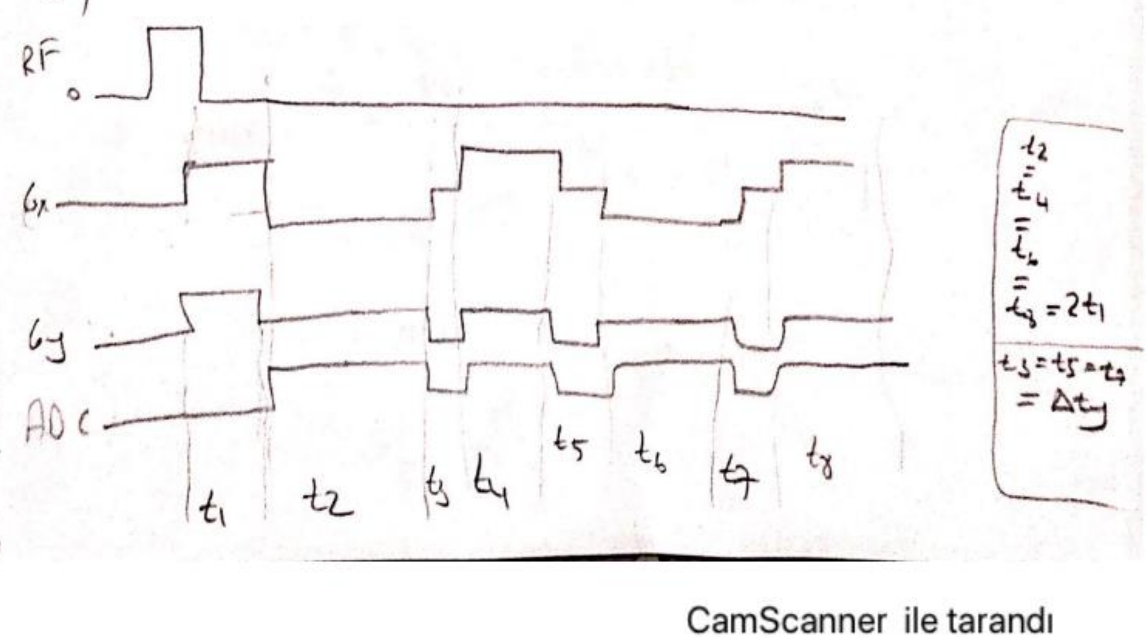
c) $FOV_y = \frac{1}{\frac{1}{10}} = 10$

$\# G_y \Delta t_y = 42.58 \frac{MHz}{T} \cdot 0.1 \Delta t_y$

$\Delta t_y = \frac{1}{42.58}$ $\Delta t_y = \frac{1}{42.58} = 0,02$ ms

d) $FOV_x = \frac{f_s}{\# G_x}$ $f_s = 10 \cdot 42.58 \cdot 0.1 = 42.58$ kHz

e)



Name: Cemal Güven Adal

ID: 21703986

Question 4(Matlab)

a)

a) $$\frac{I_{m_1}(x,y)}{I_{m_2}(x,y)} = \frac{A.M_0(x,y)\sin a\; e^{-\frac{TE_1}{T_2(x,y)}}}{A.M_0(x,y)\sin a\; e^{-\frac{TE_2}{T_2(x,y)}}}$$

$$= e^{\frac{-TE_1+Te_2}{T_2(x,y)}} \Rightarrow \frac{Te_2 - Te_1}{\ln\left(\frac{I_{m_1}(x,y)}{I_{m_2}(x,y)}\right)} = T_2(x,y)$$

$$\frac{(Te_2 - Te_1)}{T_2(x,y)} = \ln\left(\frac{I_{m1}}{I_{m2}}\right)$$

b)

Image1: T2 Map for noisefree dataset

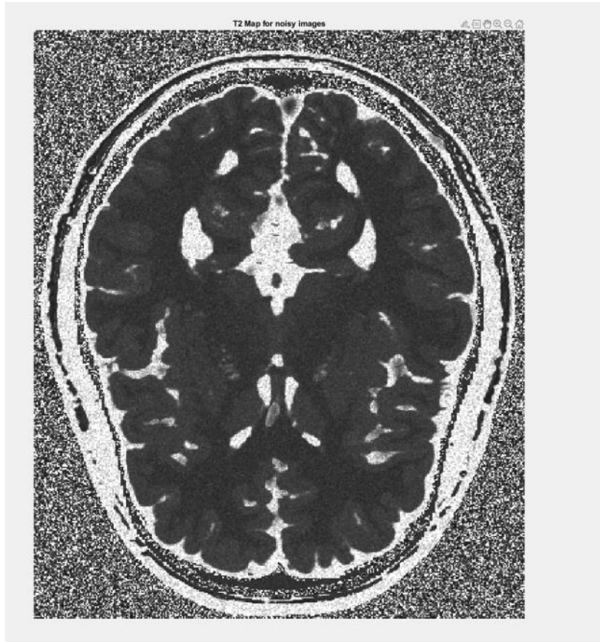


Image2: T2 Map for noisy dataset

c)

Image3: White Matter selected Image for noisefree dataset

```
T2_est =

   70.0748
```

T2 estimated value is 70.0748

d)

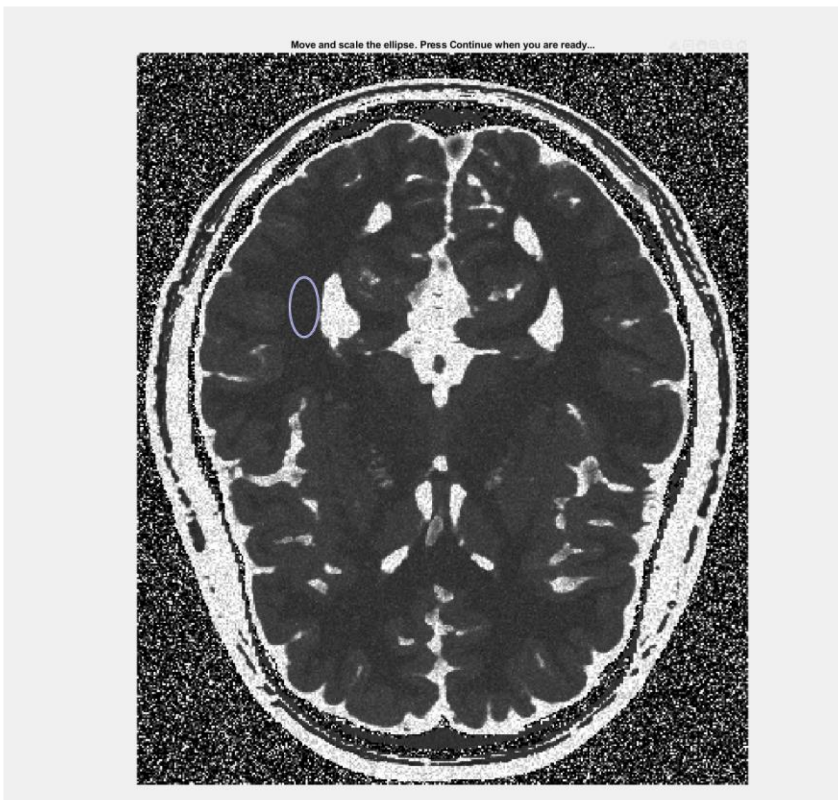


Image4: White Matter selected Image for noised dataset

```
T2_est =

   70.6878
```

T2 Estimated value is 70.6878

Percantage deviation is %0.87

e)

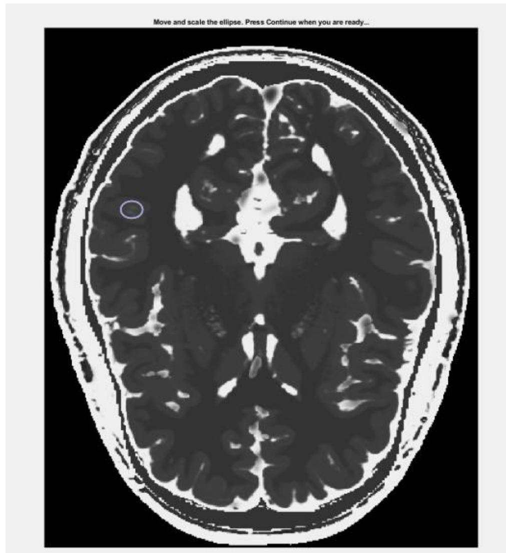


Image5: Gray Matter selected Image for noise free dataset

```
T2_est =

   89.4257
```

T2 estimation is 89.4257

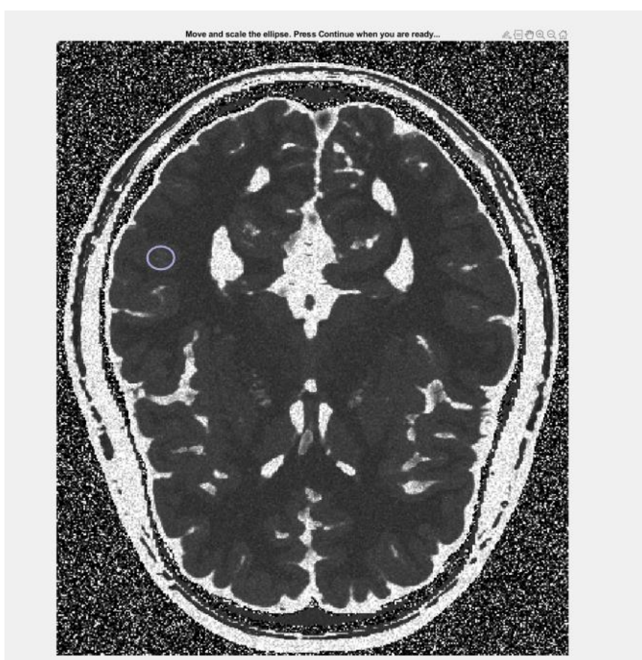


Image6: Gray Matter selected Image for noised dataset

Name: Cemal Güven Adal

ID: 21703986

```
T2_est =

   88.6461
```

T2 estimation is 88.6461

Percantage deviation is %0.87

f)

Image7: CSF selected Image for noise free dataset

```
T2_est =

   316.8685
```

T2 estimation is 316.8685

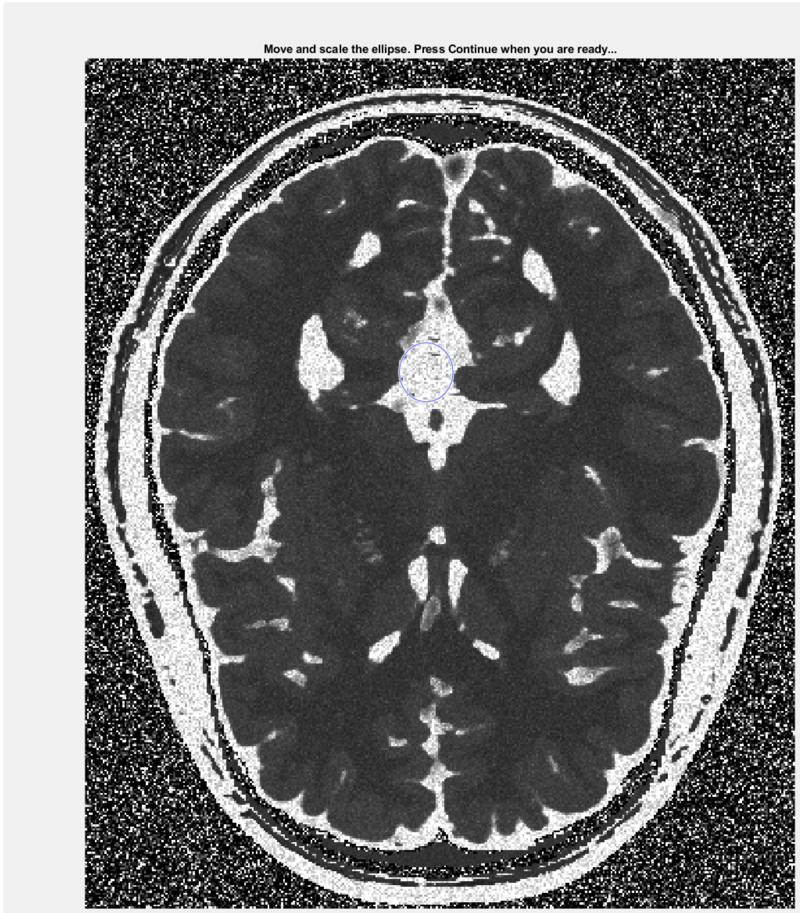


Image8: CSF selected Image for noised dataset

```
T2_est =

  328.7995
```

T2 estimated is 328.7995

Percentage deviation is %3.62

g)

CSF noisy T2 estimation showed the largest deviation from the noise free version. In the brain gray matter and white matter is surronded by csf. Gray matter amd white matter has a larger contrast compared to CSF thus in T2 map they have less deviation but because csf has less contrast deviations are greeater in T2 mapping.

Name: Cemal Güven Adal

ID: 21703986

Appendix

Matlab Code

```
% b--------------
P=load("brainT2_mri.mat");
im1=P.image1;
im2=P.image2;
im1noisy=P.image1_noisy;
im2noisy=P.image2_noisy;
TE1=P.TE(1);
TE2=P.TE(2);
res1=T2(TE2,TE1,im1,im2);
imshow(abs(res1),[0 350])
title('T2 Map for not noisy Images')
figure;

res2=T2(TE2,TE1,im1noisy,im2noisy);
imshow(abs(res2),[0 350])
title('T2 Map for noisy images')
figure;

% c--------------
imshow(res1,[0 350])
mask = roiellipse; % type "help roiellipse" to see how to use it
T2_est = mean(res1(mask))
figure;
%D-----------Noisy
imshow(res2,[0 350])
mask = roiellipse; % type "help roiellipse" to see how to use it
T2_est = mean(res2(mask))
figure;
%e--------------------
%not noisy
imshow(res1,[0 350])
mask = roiellipse; % type "help roiellipse" to see how to use it
T2_est = mean(res1(mask))
figure;
%noisy
imshow(res2,[0 350])
mask = roiellipse; % type "help roiellipse" to see how to use it
T2_est = mean(res2(mask))
figure;
%f------------------------------
%not noisy
imshow(res1,[0 350])
mask = roiellipse; % type "help roiellipse" to see how to use it
T2_est = mean(res1(mask))
figure;
%noisy
imshow(res2,[0 350])
mask = roiellipse; % type "help roiellipse" to see how to use it
T2_est = mean(res2(mask))
```

Name: Cemal Güven Adal

ID: 21703986

```
figure;

function C=T2(TE2,TE1,ima1,ima2)
C=(TE2-TE1)./(log(ima1./ima2));

end
```