## EGR 103L - Fall 2016

## Laboratory 8 - Intermediate Curve Fitting

# $\begin{array}{c} {\rm CEMAL~YAGCIOGLU} \\ {\rm Lab~Section~5D,~Wednesday~11.45AM~-~2.35PM} \\ {\rm ~30~OCTOBER,~2016} \end{array}$

I have adhered to the Duke Community Standard in completing this assignment. I understand that a violation of the Standard can result in failure of this assignment, failure of this course, and/or suspension from Duke University.

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## 1 Palm 6.10

St value is equal to 57550.

Order	Equation $(d = f(v))$
1	(5.6857e + 00) + (-8.5857e + 01)x
2	$(5.0893e - 02) + (1.1054e + 00)x(2.3571e + 00)x^2$
3	$(1.3889e - 04) + (3.2143e - 02)x + (1.8790e + 00)x^2 + (-7.1429e + 00)x^3$

Order	$S_r$	$r^2$	d(65) (ft)	d(100) (ft)
1	9.7714e+02	9.8302e-01	2.8371e+02	4.8271e+02
2	1.0179e+01	9.9982e-01	2.8923e+02	6.2182e+02
3	8.9286e+00	9.9984e-01	2.8894e+02	6.4107e+02

Although r squared value increases with higher number of terms, 2nd degree polynomial and 3rd degree polynomial are both good representations of the given data. The increasing r squared value when passing from second degree to third degree is due to side effect of mathematics and does not add a scientifically significant term.

## 2 Palm 6.16

Model is: y = 5.75ln(x) + 9.91. Thus a1=5.75 and a2=9.91. St, Sr and r2 values are respectively 160, 0.4145, 0.997. Estimates for x=2.5 and x=11 are respectively 15.2, 23.7.

## 3 Chapra 15.12

Model is :  $y = 3.7450e - 01 + (9.8644e - 01)x + (8.4564e - 01)x^{-1}$ St, Sr and r2 values are respectively 6.9480e+00, 2.7651e-03, 9.9960e-01. Estimates for x=1.5 and x=4.5 are respectively 2.4179e+00, 5.0014e+00.

## 4 Chapra 14.12

Model is :  $W = (4.1489e - 01) * A^{3.7991e - 01}$ St, Sr and r2 values are respectively 7.0102e-03, 2.0240e-04, 9.7113e-01. Estimate for W=95 is 2.3404e+00(meters squared).

## 5 Chapra 14.14

Model is :  $k = ((1.0345e + 01) * c^2)/(2.0897e + 00 + c^2)$ St, St and r2 values are respectively 4.4527e-01, 1.9178e-04, 9.9957e-01.

## 6 Chapra 15.7

Model is :  $OC = (-1.0493e - 01)c + (-4.3704e - 05)T^3 + (5.7444e - 03)T^2 + (-3.3642e - 01)T + 1.4027e + 01$ St, Sr and r2 values are respectively 1.0400e + 02, 1.2784e + 00, 9.8771e - 01.

Estimate for T=12 and c=15 is 9.1678e+00.

For this model, r2 value is high, indicating a good mathematical fit. Also relative error for given point is small, which tells that exptrapolation can be accurately used meaning its a good mathematical fit. Relative error is equal to 8.5605e-03.

## 7 Chapra 15.10

Model is:  $p(t) = (4.1375e + 00)e^{-1.5t} + (2.8959e + 00)e^{-0.3t} + (1.5349e + 00)e^{-0.05t}$ 

St, Sr and r2 values are respectively 2.0409e+01, 8.0348e-02, 9.9606e-01.

In this model, r2 value is high indicating a mathetically fit model. Also, as expected for decay, all graphs show decrease. Thus its a mathematically good model.

## 8 Chapra 15.10 Alternate

Model is:  $p(t) = (5.6328e + 00)e^{-1.5t} + (-4.4272e + 00)e^{-0.3t} + (7.8906e + 00)e^{-0.2t}$ 

St, Sr and r2 values are respectively 2.0409e+01, 7.9611e-02, 9.9610e-01.

This model has a higher r2 value, however Decay B starts from a negative value which is not scientificially accurate as a population cannot be negative. Thus, it is not a scientifically accurate model.

## 9 Chapra 14.11 (Linearized)

St is equal to 5.4240e+04.

Fit	Equation	$S_r$	$r^2$	Tiger M. (watts)
Exp.	$M = (5.3509e + 00) e^{((1.1138e - 02) m)}$	4.2972e+04	2.0773e-01	4.9640e+01
Power	$M = (3.3893e + 00) m^{(7.2656e - 01)}$	1.1874e + 02	9.9781e-01	1.5920e+02
Sat. G.	$M = (2.1011e + 01) \frac{m}{(3.4857e + 00) + m}$	6.6866e+04	-2.3279e-01	2.0651e+01

## 10 Chapra 14.11 (Nonlinear Regression)

St is equal to 5.4240e+04.

Fit	Equation	$S_r$	$r^2$	Tiger M. (watts)
Exp.	$M = (2.6088e + 01) e^{((5.8543e - 03) m)}$	3.7901e+03	9.3012e-01	8.4128e+01
Power	$M = (3.5436e + 00) m^{(7.2345e - 01)}$	6.2197e + 01	9.9885e-01	1.6373e + 02
Sat. G.	$M = (5.5198e + 02) \frac{m}{(4.1748e + 02) + m}$	2.7645e+01	9.9949e-01	1.7878e + 02

## A Codes and Output

#### A.1 ONE.m

```
% [ONE.m]
    % [Cemal Yagcioglu]
 2
   % [November 20,2016]
   % I have adhered to all the tenets of the
    % Duke Community Standard in creating this code.
    % Signed: [cy111]
7
   clear
8
9
    % V data
10 \quad v = 20:10:70;
11 % d data
   d = [45 80 130 185 250 330];
   plot(v,d,'ko')
   hold on
14
15
   % Linear a
    x=linspace(20,70,100)
16
    p1 = polyfit(v,d,1);
17
18
    y1=polyval(p1,x)
19
    y11=polyval(p1,v)
    plot(x,y1,'b-')
20
21
    est1=polyval(p1,[65 100])
22
    hold on
23
24
    % Quadratic a
25
    p2 = polyfit(v,d,2);
   y2=polyval(p2,x)
27
    y22=polyval(p2,v)
   plot(x,y2,'r--')
   est2=polyval(p2,[65 100])
30
    hold on
31
    % Cubic a
32
    p3 = polyfit(v,d,3);
33
    y3=polyval(p3,x)
    y33=polyval(p3,v)
34
    plot(x,y3,'c-.')
    legend('data points','linear','quadratic','cubic','location','northwest')
36
37
    xlabel('v (mi/hr)')
38
    ylabel('d (ft)')
    title('Distance vs Velocity Plot(cy111)')
40
    print -depsc DistanceVelocityPlot
    est3=polyval(p3,[65 100])
42
    St = sum((d - mean(d)).^2)
43
44
    % Compute sum of the squares of the estimate residuals
45
    Sr1 = sum((d - y11).^2)
    Sr2 = sum((d - y22).^2)
46
47
    Sr3 = sum((d - y33).^2)
48
    % Compute the coefficient of determination
49
    r21 = (St - Sr1) / St
50
    r22 = (St - Sr2) / St
51
    r23 = (St - Sr3) / St
52
```

#### A.2 TWO.m

```
% [TWO.m]
 2
    % [Cemal Yagcioglu]
 3
    % [November 20,2016]
    % Based on: General Linear Regression Example Code
 4
    % Written by: Michael R. Gustafson II
    % I have adhered to all the tenets of the
    % Duke Community Standard in creating this code.
    % Signed: [cy111]
9
10
    clear
    %% Rename and create model data
11
    xx=1:1:10
   yy=[10 14 16 18 19 20 21 22 23 23]
    lnx=log(xx)
14
15
    x=lnx(:)
16
    y=yy(:)
17
18
19
    %% Define model equation and A matrix
20
             yeqn = @(coefs, x) coefs(1)*x.^1 + coefs(2)*x.^0;
21
22
                = [x.^1 x.^0];
23
24
25
    \mbox{\%} Determine the function coefficients
26
    MyCoefs = A y
27
28
    %% Generate estimates and model
29
    yhat = yeqn(MyCoefs, x);
30
    x1 = log(2.5)
31
    x2=log(11)
32
    est = yeqn(MyCoefs, [x1 x2])
33
    %% Calculate statistics
34
35
    \% Compute sum of the squares of the data residuals
36
    St = sum((y - mean(y)).^2)
37
    \% Compute sum of the squares of the estimate residuals
38
39
    Sr = sum((y - yhat).^2)
40
    \% Compute the coefficient of determination
41
42
    r2 = (St - Sr) / St
```

#### A.3 THREE.m

```
% [THREE.m]
    % [Cemal Yagcioglu]
 2
 3
    % [November 20,2016]
    % Based on: General Linear Regression Example Code
 4
    % Written by: Michael R. Gustafson II
    % I have adhered to all the tenets of the
    % Duke Community Standard in creating this code.
    % Signed: [cy111]
8
9
10
    %% Initialize the workspace
11
    clear; format short e
12
13
14
    %% Load and manipulate the data
15
16
    x=1:1:5
17
    y=[2.2 2.8 3.6 4.5 5.5]
    x=x,
18
19
    y=y,
20
21
22
23
    %% Define model equation and A matrix
24
25
            yeqn = @(coefs, x) coefs(1)*x.^0 + coefs(2)*x.^1 + coefs(3)*x.^-1;
                                        [x.^0 x.^1 x.^{-1}];
26
            Α
27
28
29
    %% Determine the function coefficients
30
    MyCoefs = A y
31
    %% Generate estimates and model
    yhat = yeqn(MyCoefs, x);
33
34
    x1=1.5
35
    x2=4.5
36
    est = yeqn(MyCoefs, [x1 x2])
37
38
39
    %% Calculate statistics
40
    % Compute sum of the squares of the data residuals
    St = sum((y - mean(y)).^2)
41
42
    \% Compute sum of the squares of the estimate residuals
43
    Sr = sum((y - yhat).^2)
44
45
46
    % Compute the coefficient of determination
47
    r2 = (St - Sr) / St
```

#### A.4 FOUR.m

```
% [FOUR.m]
    % [Cemal Yagcioglu]
 2
    % [November 20,2016]
 3
    % Based on: Linearized Regression Example Code
 4
    % Written by: Michael R. Gustafson II
    % I have adhered to all the tenets of the
    % Duke Community Standard in creating this code.
    % Signed: [cy111]
8
9
10
    %% Initialize the workspace
11
    clear; format short e
12
13
   %% Rename and create model data
    W=[70 75 77 80 82 84 87 90]
14
    A=[2.10 2.12 2.15 2.20 2.22 2.23 2.26 2.30]
15
16
    lnW=log(W)
17
    lnA=log(A)
    x=lnW,
18
19
    y=lnA'
20
21
    %% Define model equation and A matrix
22
            yeqn = @(coefs, x) coefs(1)*x.^1 + coefs(2)*x.^0;
23
24
                                        [x.^1]
25
26
    %% Determine the function coefficients
27
    MyCoefs = A y
28
29
    %% Generate estimates and model
30
    yhat = yeqn(MyCoefs, x);
31
    %% Calculate statistics
    \% Compute sum of the squares of the data residuals
33
34
    St = sum((y - mean(y)).^2)
35
36
37
    % Compute sum of the squares of the estimate residuals
    Sr = sum((y - yhat).^2)
38
39
40
    % Compute the coefficient of determination
    r2 = (St - Sr) / St
41
42
43
    finalb = MyCoefs(1)
44
    finala = exp(MyCoefs(2))
45
46
    est=exp(yeqn(MyCoefs, log(95)))
```

#### A.5 FIVE.m

```
% [FIVE.m]
    % [Cemal Yagcioglu]
 2
    % [November 20,2016]
 3
    % Based on: Linearized Regression Example Code
 4
    % Written by: Michael R. Gustafson II
    % I have adhered to all the tenets of the
     \mbox{\ensuremath{\mbox{\%}}} Duke Community Standard in creating this code.
8
    % Signed: [cy111]
10
    %% Initialize the workspace
11
     clear; format short e
12
13
    %% Rename and create model data
    c = [0.5 \ 0.8 \ 1.5 \ 2.5 \ 4]
14
    k = [1.1 \ 2.5 \ 5.3 \ 7.6 \ 8.9]
15
    Divk=1./k
16
17
    Divc=1./(c.^2)
    x=Divc'
18
19
    y=Divk'
20
     xmodel=linspace(min(x), max(x), 100)
21
22
     %% Define model equation and A matrix
23
24
             yeqn = @(coefs, x) coefs(1)*x.^1 + coefs(2)*x.^0;
25
                                          [x.^1
                                                            x.^0];
26
             Α
27
28
29
     %% Determine the function coefficients
30
    MyCoefs = A y
31
     %% Generate estimates and model
     yhat = yeqn(MyCoefs, x);
33
34
35
36
37
    %% Calculate statistics
38
     % Compute sum of the squares of the data residuals
40
     St = sum((y - mean(y)).^2)
41
42
     \% Compute sum of the squares of the estimate residuals
     Sr = sum((y - yhat).^2)
43
44
45
     % Compute the coefficient of determination
46
     r2 = (St - Sr) / St
47
48
    kmax = 1/MyCoefs(2)
49
     cs = MyCoefs(1)*kmax
50
51
     ymodel=(kmax.*xmodel.^2)./(cs+xmodel.^2)
52
     yvalues=(kmax.*c.^2)./(cs+c.^2)
                          'ko',...
53
     plot(c,
                  k,
54
                  yvalues, 'ks',...
          С,
```

#### A.6 SIX.m

```
% [SIX.m]
          % [Cemal Yagcioglu]
  2
           % [November 20,2016]
  3
          % Based on: Linearized Regression Example Code
  4
          % Written by: Michael R. Gustafson II
           % I have adhered to all the tenets of the
  7
           % Duke Community Standard in creating this code.
  8
           % Signed: [cy111]
  9
          clear
          load('DOCtable.mat')
10
11
12
          %% Initialize the workspace
13
          format short e
          figure(1); clf
14
15
           %%% Create data set - this takes the place of loading the set
16
17
           %% for this demonstration; otherwise, the end of this step
           \%\% should have x1m, x2m, and ym defined as matrices
18
19
           %[x1m, x2m] = meshgrid(0:.1:1, 0:.15:3);
           %T , c
20
           %ym - OC
21
22
23
           %% Rename and create vectors and model data
           x1v = T(:);
25
           x2v = c(:);
           yv = OC(:);
26
27
           [x1model, x2model] = meshgrid(...
28
                     linspace(min(x1v), max(x1v), 19),...
29
                     linspace(min(x2v), max(x2v), 17));
30
31
           %% Define model equation and A matrix
32
                              yeqn = @(coefs, x1e, x2e) coefs(1)*x2e.^1 + coefs(2)*x1e.^3 + coefs(3)*x1e.^2 + coefs(4)*x1e.^3 + coefs(3)*x1e.^2 + coefs(4)*x1e.^3 + coefs(3)*x1e.^3 + co
33
34
                              Α
                                                                                                                  [x2v.^1
                                                                                                                                                               x1v.^3
                                                                                                                                                                                                           x1v.^2 x1v.^1
                                                                                                                                                                                                                                                      x1v
35
36
37
           %% Determine the function coefficients
38
           MyCoefs = A \yv
39
40
           %% Generate estimates and model
           yhat = yeqn(MyCoefs, x1v, x2v);
41
42
           ymodel = yeqn(MyCoefs, x1model, x2model);
43
44
           %% Calculate statistics
           St = sum((yv - mean(yv)).^2)
45
           Sr = sum((yv - yhat).^2)
46
47
           r2 = (St - Sr) / St
48
49
           %% Generate plots
50
           %figure(1); clf
51
52
           %surfc(T, c, OC);
           %xlabel('x1'); ylabel('x2'); zlabel('y Data')
53
54
           %view(145, 15)
```

```
% Model data
56 figure(1)
57 surfc(x1model, x2model, ymodel)
58 title('Oxygen Concentration in Water(cy111)')
    xlabel('T values(C)'); ylabel('c values(g/L)'); zlabel('OC values(mg/L)')
60
   view(145, 15)
61
    colormap autumn
62
    print -depsc OxygenConcentration
63
    Test=12
64
   cest=15
   est = yeqn(MyCoefs, Test, cest)
65
66 relerr=(est-9.09)/9.09
```

#### A.7 SEVEN.m

```
% [SEVEN.m]
    % [Cemal Yagcioglu]
 2
    % [November 20,2016]
 3
 4
    % Based on: General Multidimensional Linear Regression Example Code
    % Written by: Michael R. Gustafson II
    % I have adhered to all the tenets of the
     \mbox{\ensuremath{\mbox{\%}}} Duke Community Standard in creating this code.
8
    % Signed: [cy111]
    %% Initialize the workspace
    clear; format short e
10
11
12
    t = [0.5 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 9]
    p = [6 \ 4.4 \ 3.2 \ 2.7 \ 2 \ 1.9 \ 1.7 \ 1.4 \ 1.1]
14
15
16
    y=p'
17
18
19
     %% Rename and create model data
20
     xmodel = linspace(min(x), max(x), 100);
21
22
23
     %% Define model equation and A matrix
24
25
             yeqn = @(coefs, x) coefs(1).*exp(-1.5.*x) + coefs(2).*exp(-0.3.*x) + coefs(3).*exp(-0.05.*x)
                                                            \exp(-0.3.*x) = \exp(-0.05.*x);
26
             Α
                                          [exp(-1.5.*x)]
27
28
     %% Determine the function coefficients
29
     MyCoefs = A y
30
31
     %% Generate estimates and model
32
     yhat = yeqn(MyCoefs, x);
33
     ymodel = yeqn(MyCoefs, xmodel);
34
35
     %% Calculate statistics
     \% Compute sum of the squares of the data residuals
36
     St = sum((y - mean(y)).^2)
37
38
39
     % Compute sum of the squares of the estimate residuals
40
     Sr = sum((y - yhat).^2)
41
     % Compute the coefficient of determination
42
    r2 = (St - Sr) / St
43
44
45
     %% Generate plots
46
     figure(1)
47
    plot(x,
                           'ko',...
                  у,
                  yhat, 'ks',...
48
          х,
          xmodel, ymodel, 'k-');
49
50
     xlabel('Time(s)')
      ylabel('Population')
51
52
      title('Population vs. Time First Model(cy111)')
53
      legend('Data', 'Estimates', 'Model')
54
      print -depsc Decay11
```

```
55
56
      figure(2)
      tmodel=linspace(0,9,100)
57
58
59
      plot(tmodel,MyCoefs(1).*exp(-1.5.*tmodel),'r--')
60
      hold on
     plot(tmodel,MyCoefs(2).*exp(-0.3.*tmodel),'b-')
61
62
      \quad \text{hold on} \quad
      plot(tmodel,MyCoefs(3).*exp(-0.05.*tmodel),'g-.')
63
64
      title('Decay of A,B,C First Model(cy111)')
      legend('Decay A', 'Decay B', 'Decay C')
65
66
      print -depsc Decay12
```

#### A.8 EIGHT.m

```
% [EIGHT.m]
    % [Cemal Yagcioglu]
 2
    % [November 20,2016]
 3
    % Based on: General Linear Regression Example Code
 4
    % Written by: Michael R. Gustafson II
    % I have adhered to all the tenets of the
    % Duke Community Standard in creating this code.
8
    % Signed: [cy111]
    %% Initialize the workspace
    clear; format short e
10
11
12
    t = [0.5 1 2 3 4 5 6 7 9]
    p = [6 \ 4.4 \ 3.2 \ 2.7 \ 2 \ 1.9 \ 1.7 \ 1.4 \ 1.1]
14
15
16
    y=p'
17
18
19
    %% Rename and create model data
20
    xmodel = linspace(min(x), max(x), 100);
21
22
23
    %% Define model equation and A matrix
24
25
             yeqn = @(coefs, x) coefs(1).*exp(-1.5.*x) + coefs(2).*exp(-0.3.*x) + coefs(3).*exp(-0.2.*x);
                                                          \exp(-0.3.*x) = \exp(-0.2.*x);
26
             Α
                                         [exp(-1.5.*x)]
27
28
    %% Determine the function coefficients
29
    MyCoefs = A y
30
31
    %% Generate estimates and model
    yhat = yeqn(MyCoefs, x);
    ymodel = yeqn(MyCoefs, xmodel);
33
34
35
    %% Calculate statistics
    \% Compute sum of the squares of the data residuals
36
    St = sum((y - mean(y)).^2)
37
38
39
    % Compute sum of the squares of the estimate residuals
40
    Sr = sum((y - yhat).^2)
41
    % Compute the coefficient of determination
42
    r2 = (St - Sr) / St
43
44
45
    %% Generate plots
46
    figure(1)
47
    plot(x,
                          'ko',...
                  у,
                  yhat, 'ks',...
48
          х,
         xmodel, ymodel, 'k-');
49
50
    xlabel('Time(s)')
     ylabel('Population')
51
52
     title('Population vs. Time Second Model(cy111)')
53
     legend('Data', 'Estimates', 'Model')
54
     print -depsc Decay21
```

```
55
     figure(2)
56
     tmodel=linspace(0,9,100)
57
58
     plot(tmodel,MyCoefs(1).*exp(-1.5.*tmodel),'r--')
     hold on
59
     plot(tmodel,MyCoefs(2).*exp(-0.3.*tmodel),'b-')
60
     hold on
61
     plot(tmodel,MyCoefs(3).*exp(-0.2.*tmodel),'g-.')
62
     title('Decay of A,B,C Second Model(cy111)')
63
64
     legend('Decay A', 'Decay B', 'Decay C')
     print -depsc Decay22
65
```

#### A.9 NINE.m

```
% [NINE.m]
    % [Cemal Yagcioglu]
 2
    % [November 20,2016]
 3
    % Based on: Linearized Regression Example Code
 4
    % Written by: Michael R. Gustafson II
    % I have adhered to all the tenets of the
    % Duke Community Standard in creating this code.
8
    % Signed: [cy111]
9
    %% Initialize the workspace
    clear; format short e
10
11
12
13
    %% Load and manipulate the data
14
    Mass=[400 70 45 2 0.3 0.16]
15
    Metabolism=[270 82 50 4.8 1.45 0.97]
16
17
    %% Rename and create model data
18
    x = Mass'
19
20
    y = Metabolism'
21
    xmodel = linspace(min(x), max(x), 100);
22
23
    %% Define the model equation; transform the variables; find the linearized fit; transform back
24
25
    %model = 'exponential'
26
    %switch model
27
         case 'exponential'
28
             yeqnE = @(coefs, x) coefs(1).*exp(coefs(2).*x);
29
             xiE = x;
30
             etaE = log(y);
             PE = polyfit(xiE, etaE, 1);
31
             MyCoefsE(1) = exp(PE(2));
32
             MyCoefsE(2) = PE(1)
33
34
    %
         case 'power law'
35
             yeqnP = @(coefs, x) coefs(1).*x.^coefs(2);
             xiP = log10(x);
36
             etaP = log10(y);
37
38
             PP = polyfit(xiP, etaP, 1);
39
             MyCoefsP(1) = 10^(PP(2));
40
             MyCoefsP(2) = PP(1)
41
         case 'sat growth'
42
             yeqnS = @(coefs, x) coefs(1).*x./(coefs(2)+x);
43
             xiS = 1./x;
             etaS = 1./y;
44
             PS = polyfit(xiS, etaS, 1);
45
             MyCoefsS(1) =
                             1/PS(2);
46
             MyCoefsS(2) = PS(1)/PS(2)
47
48
49
    %% Generate estimates and model
50
51
52
    yhatE
             = yeqnE(MyCoefsE, x);
     ymodelE = yeqnE(MyCoefsE, xmodel);
53
54
```

```
55
     yhatP = yeqnP(MyCoefsP, x);
56
     ymodelP = yeqnP(MyCoefsP, xmodel);
57
58
     yhatS = yeqnS(MyCoefsS, x);
     ymodelS = yeqnS(MyCoefsS, xmodel);
 59
 60
 61
     %% Calculate statistics
 62
     \% Compute sum of the squares of the data residuals
 63
     St = sum((y - mean(y)).^2)
 64
 65
     % Compute sum of the squares of the estimate residuals
     SrE = sum((y - yhatE).^2)
 66
 67
     SrP = sum((y - yhatP).^2)
     SrS = sum((y - yhatS).^2)
 68
 69
 70
     % Compute the coefficient of determination
     r2E = (St - SrE) / St
 71
 72
     r2P = (St - SrP) / St
     r2S = (St - SrS) / St
73
 74
 75
     %% Generate plots
 76
     figure(1)
 77
     plot(x,
                           'ko', xmodel, ymodelE, 'k-')
                  у,
 78
     title('Metabolism as a Function of Mass(Exponential) (cy111)')'
 79
     xlabel('Mass(kg)')
 80
      ylabel('Metabolism(watt)')
 81
      print -depsc M11
 82
 83
 84
     figure(2)
                                                   'ko')
 85
     plot(xmodel,log(ymodelE), x,
                                        log(y),
      xlabel('Mass(kg)')
 86
      ylabel('log(Metabolism)')
 87
 88
      title('Linearized Exponential Model (cy111)')
 89
      print -depsc M12
90
91
     figure(3)
92
     plot(x,
                           'k-',xmodel, ymodelP, 'k:')
                  у,
93
     xlabel('Mass(kg)')
      ylabel('Metabolism(watt)')
94
      title('Metabolism as a Function of Mass(Power) (cy111)')
95
96
      print -depsc M13
97
98
     figure(4)
99
     plot(log10(xmodel),log10(ymodelP), log10(x),
                                                        log10(y),
                                                                       'ko')
100
      xlabel('log10(Mass)')
      ylabel('log10(Metabolism)')
101
102
      title('Linearized Power Model (cy111)')
103
      print -depsc M14
104
     figure(5)
105
106
     plot(x,
                           'ko', xmodel, ymodelS, 'k--')
                  у,
107
     xlabel('Mass(kg)')
108
      ylabel('Metabolism(watt)')
109
      title('Metabolism as a Function of Mass(Saturation) (cy111)')
```

```
110
      print -depsc M15
111
112
    figure(6)
113
     plot(1./xmodel,1./ymodelS, 1./x, 1./y, 'ko')
     xlabel('1/Mass(1/kg)')
114
    ylabel('1/Metabolism(1/watt)')
115
    title('Linearized Saturation Model (cy111)')
116
117
     print -depsc M16
118
119
     EstE = yeqnE(MyCoefsE, 200)
     EstP = yeqnP(MyCoefsP, 200)
120
121
     EstS = yeqnS(MyCoefsS, 200)
```

#### A.10 TEN.m

```
% [TEN.m]
    % [Cemal Yagcioglu]
 2
    % [November 20,2016]
 3
    % Based on: Nonlinear Regression Example Code
 4
    % Written by: Michael R. Gustafson II
    % I have adhered to all the tenets of the
    % Duke Community Standard in creating this code.
8
    % Signed: [cy111]
    %% Initialize the workspace
    clear; format short e
10
11
    figure(1); clf
12
    %% Load and manipulate the data
    Mass=[400 70 45 2 0.3 0.16]
14
15
    Metabolism=[270 82 50 4.8 1.45 0.97]
16
17
    %% Rename and create model data
18
    x = Mass'
19
    y = Metabolism'
20
    xmodel = linspace(min(x), max(x), 100);
21
22
             yeqnE = @(coefs, x) coefs(1).*exp(coefs(2).*x);
23
             InitGuessE = [5.3509 \ 0.01138]
24
25
             yeqnP = @(coefs, x) coefs(1).*x.^coefs(2);
             InitGuessP = [3.3893 \ 0.72656]
26
27
28
             yeqnS = @(coefs, x) coefs(1).*x./(coefs(2)+x);
29
             InitGuessS = [21.011 \ 3.4857]
30
31
    %% Determine the function coefficients
    fSSRE = @(coefs, x, y) sum(( y - yeqnE(coefs, x) ).^2)
32
     [MyCoefsE, SrE] = fminsearch(@(MyCoefsDummy) fSSRE(MyCoefsDummy, x, y), InitGuessE)
33
34
35
    fSSRP = @(coefs, x, y) sum(( y - yeqnP(coefs, x) ).^2)
     [MyCoefsP, SrP] = fminsearch(@(MyCoefsDummy) fSSRP(MyCoefsDummy, x, y), InitGuessP)
36
37
    fSSRS = @(coefs, x, y) sum(( y - yeqnS(coefs, x) ).^2)
38
39
    [MyCoefsS, SrS] = fminsearch(@(MyCoefsDummy) fSSRS(MyCoefsDummy, x, y), InitGuessS)
40
41
    %% Generate estimates and model
42
    yhatE = yeqnE(MyCoefsE, x);
43
    ymodelE = yeqnE(MyCoefsE, xmodel);
44
45
    yhatP
             = yeqnP(MyCoefsP, x);
46
    ymodelP = yeqnP(MyCoefsP, xmodel);
47
48
             = yeqnS(MyCoefsS, x);
    yhatS
49
    ymodelS = yeqnS(MyCoefsS, xmodel);
50
51
52
    %% Calculate statistics
53
    \% Compute sum of the squares of the data residuals
    St = sum((y - mean(y)).^2)
54
```

```
55
 56
     \% Compute sum of the squares of the estimate residuals
     SrE = sum((y - yhatE).^2)
57
     SrP = sum((y - yhatP).^2)
 58
     SrS = sum((y - yhatS).^2)
 59
 60
61
     % Compute the coefficient of determination
 62
     r2E = (St - SrE) / St
     r2P = (St - SrP) / St
 63
 64
     r2S = (St - SrS) / St
 65
 66
 67
     %% Generate plots
     figure(1)
 68
     plot(x,
                           'ko', xmodel, ymodelE, 'k-')
 69
                  у,
     title('Nonlinear Metabolism as a Function of Mass(Exponential) (cy111)')
 70
      xlabel('Mass(kg)')
 71
 72
      ylabel('Metabolism(watt)')
 73
      print -depsc M21
 74
 75
     figure(2)
 76
 77
     plot(xmodel,log(ymodelE), x,
                                       log(y),
                                                   'ko')
 78
      xlabel('Mass(kg)')
 79
      ylabel('log(Metabolism)')
      title('Nonlinear Transformation Linearized Exponential Model (cy111)')
 80
 81
      print -depsc M22
 82
 83
     figure(3)
     plot(x,
                           'k-',xmodel, ymodelP, 'k:')
 84
                   у,
      xlabel('Mass(kg)')
 85
 86
      ylabel('Metabolism(watt)')
      title('Nonlinear Metabolism as a Function of Mass(Power) (cy111)')
 87
 88
      print -depsc M23
 89
     figure(4)
90
91
     plot(log10(xmodel),log10(ymodelP), log10(x),
                                                        log10(y),
                                                                      'ko')
92
     xlabel('log10(Mass)')
93
      ylabel('log10(Metabolism)')
94
      title('Nonlinear Transformation Power Model (cy111)')
 95
     print -depsc M24
96
     figure(5)
                           'ko', xmodel, ymodelS, 'k--')
97
     plot(x,
                  у,
     xlabel('Mass(kg)')
98
99
      ylabel('Metabolism(watt)')
      title('Nonlinear Metabolism as a Function of Mass(Saturation) (cy111)')
100
     print -depsc M25
101
     figure(6)
102
     plot(1./xmodel,1./ymodelS, 1./x,
                                            1./y,
                                                       'ko')
103
104
      xlabel('1/Mass(1/kg)')
      ylabel('1/Metabolism(1/watt)')
105
      title('Nonlinear Transformation Saturation Model (cy111)')
106
107
      print -depsc M26
108
109
```

- 110 EstE = yeqnE(MyCoefsE, 200)
  111 EstP = yeqnP(MyCoefsP, 200)
  112 EstS = yeqnS(MyCoefsS, 200)

## B Figures

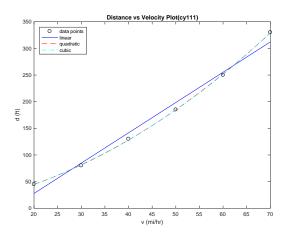


Figure 1: Distance Velocity Plot

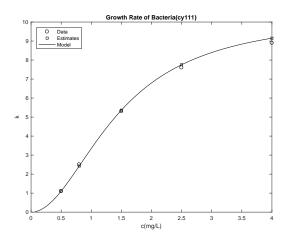


Figure 2: Bacteria Plot

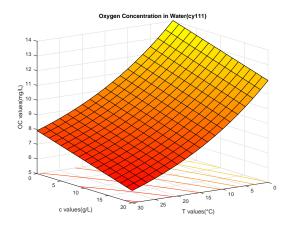


Figure 3: Oxygen Concentration Surface

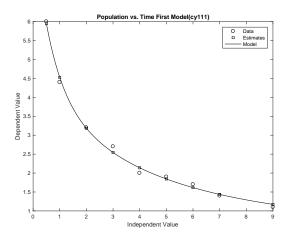


Figure 4: Population vs Time First Model

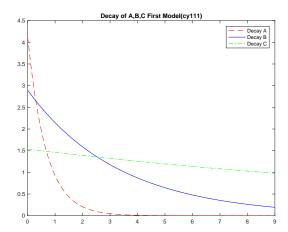


Figure 5: Decay of A,B,C First Model

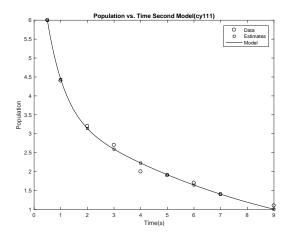


Figure 6: Population vs Time Second Model

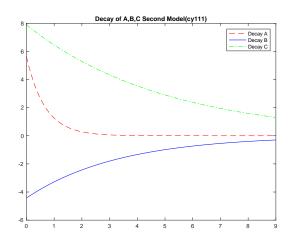


Figure 7: Decay of A,B,C Second Model

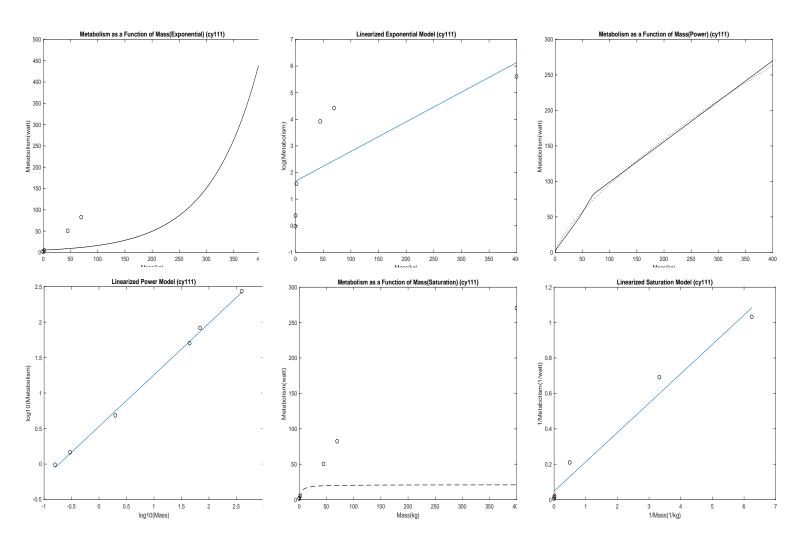


Figure 8: Untransformed and Transformed Plots for Linearized Metabolism Models

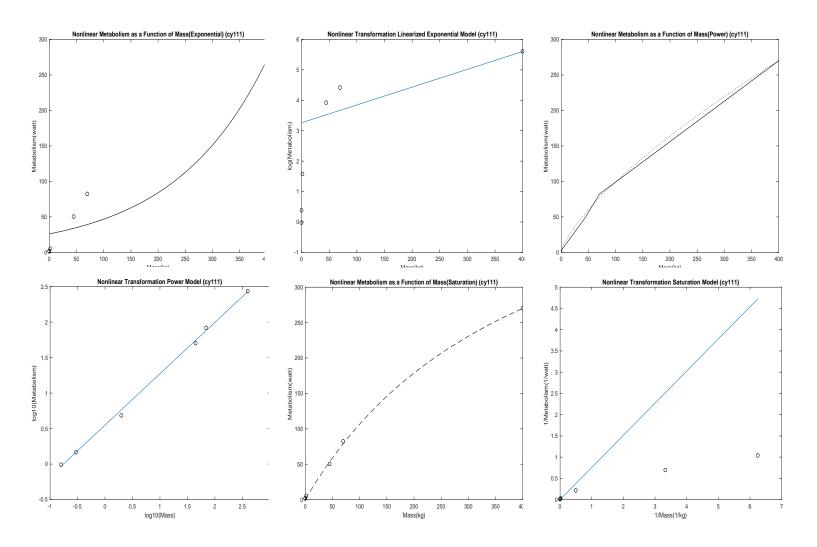


Figure 9: Untransformed and Transformed Plots for Unlinearized Metabolism Models