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Grasp Planning: How to Choose a Suitable Task Wrench Space

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***Abstract*— For the evaluation of grasp quality, different measures have been proposed that are based on wrench spaces. Almost all of them have drawbacks that derive from the non-uniformity of the wrench space, composed of force and torque dimensions. Moreover, many of these approaches are computationally expensive.**

In this paper, we address the problem of choosing a proper task wrench space to overcome the problems of the non-uniform wrench space and show how to integrate it in a well-known, high precision and extremely fast computable grasp quality measure.

I. Introduction

With the development of flexible and highly integrated dexterous gripping devices (e.g. the DLR Hands I and II), the research on grasp and manipulation analysis and planning can be applied to the real world. So the need for efficient and implementable methods to perform these analysis and planning tasks increases.

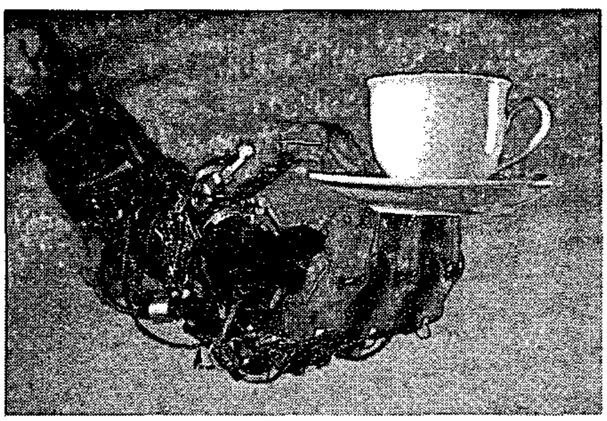

Fig. 1. The DLR Hand II executing a special grasping task

One main topic in this field is the static analysis of forces and torques that can be applied to an object through grasp contacts. The evaluation of the efficiency of a grasp to counteract disturbances in order to keep the object firmly fixed in the gripper is the next important research topic.

A basic quality criterion for a grasp is the force closure property, first proposed for grasping applications by Lakshminarayana [5]. Efficient tests for this property have been developed using different models, like the grasp matrix or the grasp wrench spaces [1], [3], [8]. Also, many approaches deal with the construction of a force closure grasp.

The force closure property, however, is only a minimal quality requirement for a grasp. It is more relevant, how efficiently a grasp can compensate for arbitrary disturbances or balance a special set of disturbances that is expected when executing a desired task. To quantify this efficiency of a grasp, the concept of wrench spaces can be used. The set of all wrenches that can be applied to the object through the grasp contacts is called the *Grasp Wrench Space (GWS)*.

A commonly used and efficient way to approximate the GWS is to calculate the convex hull over the discretized friction cones [9], [10], [11]. The problem with all these approaches is the discretization of the friction cones, where significant errors may be introduced when approximating the cone with only a few vectors to achieve fast computation (e.g. 4 vectors lead to an error of $\sim$ 30 %, 8 vectors still $\sim$ 8 % [2]). Moreover, Teichmann and Mishra [13] showed that there are problems to be expected with this method for large friction coefficients. This GWS approximation corresponds to the idea that the sum of all applied forces is constrained to one, which has only a weak physical interpretation for multifingered grippers.

Ferrari and Canny [3] proposed a method for calculating a physically well interpretable GWS approximation, where the forces applied in the contact sum up to the number of contacts, by calculating the convex hull over the Minkowski sum of the friction cones. The drawback with directly calculating this GWS approximation is that it is computationally expensive.

To rate the quality of a grasp, task directed and task independent measures were introduced. Kirkpatrick et al. [4] use the largest wrench sphere that just fits within the GWS as a task independent quality measure of the grasp. The measure is not scale invariant and depends on the selection of the torque origin (r in fig. 5). To achieve invariance to the selection of the reference point, Li and Sastry [6] propose to use the volume of an ellipsoid generated by the grasp matrix as a measure of grasp quality. They also suggest to model the task wrench space as a six dimensional ellipsoid and fit it in the GWS. The problem with this approach is how to model the task ellipsoid for a given task, which they state to be quite complicated.

Pollard [11] introduces the Object Wrench Space (OWS) which incorporates the object geometry into the grasp evaluation. With her approach, however, this OWS represents the best grasp that can be achieved for an object and gives no direct measure for an arbitrary grasp. To rate the quality of a grasp, the largest inscribed sphere is used. To achieve scale invariance, the torque component of the wrenches is scaled with the length of the longest object axis.

Strandberg [12] proposes to evaluate grasps using disturbance forces in order to overcome the problem of torque origin selection and to take the geometry of the object into account. The quality evaluation method is very reliable, however, the complexity of this approach is very high as the geometric information has to be evaluated for each grasp candidate.

In the next section, we want to illustrate shortly some drawbacks of the so far proposed grasp measures. Then we give a description of exact Grasp and Task Wrench Spaces that are physically motivated and propose a very intuitive grasp measure using these wrench space definitions. The final part of the paper deals with a very efficient method for calculating this new grasp measure.

II. A COMPARISON OF PROPOSED QUALITY MEASURES AND THEIR INVARIANCES

The different methods mentioned above for rating the quality of a grasp in a task independent manner have different properties as to scaling the grasped object, changing the reference point, and scaling torque axes. Here we shortly present the invariances of the different methods with some simple examples to illustrate in which case the quality measures are intuitive and physically correct and where they may fail.

We compare the largest inscribed sphere method and the same method with uniform scaled torques (as suggested by Pollard) with the volume of the GWS. The first example compares the same geometric grasp configuration applied at different locations on the object and the change of reference point (fig. 2). In the second example, the grasp and object are scaled by a factor two (fig. 3). In the last example, we apply the same grasp configuration to different objects (fig. 3).

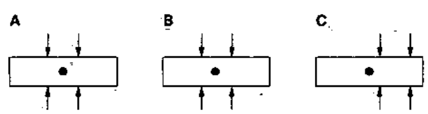

Fig. 2. The same grasp configuration applied at different locations on an object (case A and C) and the change of reference point with the same grasp on the same object (case A and B)

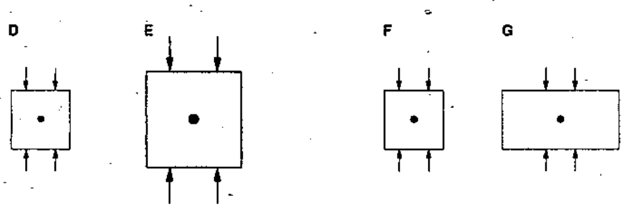

Fig. 3. The same grasp and object topology scaled by a factor of two (D and E) and the same grasp configuation applied to different objects (F and G).

How we expect the different quality measures to compare in the different grasp situations is listed in the table of fig. 4.

For the results on human intuition in fig. 4, some remarks to explain the background of our decisions may be useful. For the first example in fig. 2, we would qualify situation A and B as equal grasps but C as inferior since we move away from the center of mass. The scaled grasp and object (fig. 3) situations lead to an equal qualification as there is no obvious

method	*fig. 2*	*fig. 3*	*fig. 3*
InscrBall	$A \geq B$ $A \geq C$	$D \leq E$	$F = G$
InscrBall (torque scaled)	$A \geq B$ $A \geq C$	$D = E$	$F \geq G$
volume of GWS	$A = B$ $A = C$	$D < E$	$F = G$
Human *Intuition*	$A = B$ $A > C$	$D = E$	$F > G$

Fig. 4. Comparing the different grasp measures with human intuition in some simple cases

reason why grasps on larger object should be better than those on smaller. For the last case, F is superior to G as the torque disturbances that are likely to occur are much larger in G than in F, while the grasp abilities are the same.

III. A PHYSICALLY MOTIVATED MODEL TO SPECIFY GRASPS AND GRASPING TASKS

There are three main questions regarding the static part of grasping: What are the forces/torques that can be applied to the object by the grasp? Which disturbances are expected to act on the object? The third question is about the quality of the chosen grasp. A good quality measure for a grasp is a scalar that describes how well the grasp can resist the expected disturbances.

For all three questions above various models and measures have been developed. From a physical or mechanical point of view, however, all can be modeled similarly and in a simple manner.

There are only forces and torques acting on the object, either as a disturbance anywhere on the object or in the grasp contacts to counteract the disturbances. Both, the set of disturbance forces/torques and the set of possible grasp forces/torques, are usually represented in a vector space [3], [10], [11].

A. The Grasp Wrench Space

Let us assume that the grasp consists of k point contacts with friction. So in each contact a force within the friction cone can be applied to the object (Fig. 5).

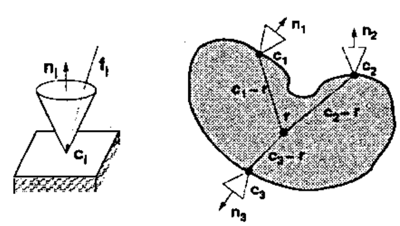

Fig. 5. A single contact point in 3D illustrating the friction cone and a sample k-contact grasp (k=3) on a planar object.

The length of the applied force vector is normalized to a unit force as we assume that each finger can apply the same magnitude of force and only one contact arises for each finger (precision grasp).

The direction of the force $\mathbf{f}_i$ that can be applied at contact point $\mathbf{c}_i$ is constrained by the friction cone specified by the friction coefficient μ, the contact point $\mathbf{c_i}$, and the contact normal $\mathbf{n_i}$. The constraint can be written as:

$$\|\mathbf{f_i} - (\mathbf{f_i} \cdot \mathbf{n_i})\mathbf{n_i}\| \quad \leq \quad -\mu(\mathbf{f_i} \cdot \mathbf{n_i}). \qquad (1)$$

Any force acting at a contact point on the object also creates a torque relative to a reference point $\mathbf{r}$ that can be arbitrarily chosen. Often the center of mass is used as that reference point to give it a physical meaning. The torque τ_i corresponding to $\mathbf{f_i}$ is then:

$$\tau_i = (\mathbf{c_i} - \mathbf{r}) \times \mathbf{f_i}$$

For convenience, these force and torque vectors can be concatenated to a *wrench* :

$$\mathbf{w_i} = \begin{pmatrix} \mathbf{f_i} \\ (\mathbf{c_i} - \mathbf{r}) \times \mathbf{f_i} \end{pmatrix}$$

Next we specify the set of wrenches that can be created by friction cone unit forces acting in one contact. We call this set the *Cone Wrench Space (CWS)*. It is used to clarify the construction of the Grasp Wrench Space.

$$\mathrm{CWS}_{\mathbf{c_i}} = \left\{ \mathbf{w_i} \middle| \mathbf{w_i} = \begin{pmatrix} \mathbf{f_i} \\ (\mathbf{c_i} - \mathbf{r}) \times \mathbf{f_i} \end{pmatrix} \wedge \right.$$
$$\left. \|\mathbf{f_i} - (\mathbf{f_i} \cdot \mathbf{n_i})\mathbf{n_i}\| \leq -\mu(\mathbf{f_i} \cdot \mathbf{n_i}) \wedge \|\mathbf{f_i}\| \leq 1 \right\} \qquad (2)$$

The *Grasp Wrench Space (GWS)* should contain all wrenches that a given grasp can counterbalance by applying forces in its k contacts. This space can be composed from all k cone wrench spaces in the following manner:

$$\mathrm{GWS} = \left\{ \mathbf{w} \middle| \mathbf{w} = \sum_{\mathbf{i}=1}^{\mathbf{k}} \mathbf{w_i} \wedge \mathbf{w_i} \in \mathrm{CWS}_{\mathbf{c_i}} \right\} \qquad (3)$$

Note that equation 3 is an exact description of the GWS. It corresponds to the idea that each finger of the manipulator is capable to exert a unit magnitude of force to the object. The only drawback of this kind of definition is that it is only descriptive but not constructive. To find the linear combination of finger forces to counterbalance a disturbance wrench is difficult. This problem is addressed in section IV.

B. The Task Wrench Space

The wrenches that are expected to occur for a given task can be specified as a so-called *Task Wrench Space (TWS)*. For the TWS two cases can be distinguished, either the task to be executed is known and a specification in the wrench space is given or the task is unknown and no specification exists.

1) Given Task Specification: If there is a detailed description of the task given by a set of wrenches that are applied to the object during the manipulation one can use the convex hull over these task disturbance wrenches as a Task Wrench Space. Li and Sastry [6] propose to approximate the task wrench space by a task ellipsoid but they state that the data acquisition is difficult.

2) Unknown Task Specification: If one knows nothing about a grasping task, one at least can assume that a grasp should hold an object (1) against gravity, (2) against forces and torques arising from accelerating the object (which has the same effect for translational accelerations) and (3) against forces that result from contacts of the object with the environment.

A commonly used approach to model an unknown task wrench space is to use a unit sphere in the wrench space. With this approach it is assumed that the probability for every wrench direction to occur as a disturbance is equal. However, this has no physical or mechanical interpretation. Torques are typically caused by forces acting on the boundary of the object and therefore a general task wrench space is not uniform for most objects.

A more natural way to describe an unknown TWS that takes the object geometry into acount is the *Object Wrench Space (OWS)* as introduced by Pollard [11]. In this work, we want to combine the idea of the task ellipsoid [6] with the concept of the object wrench space. This enables us to automatically obtain a task independent, yet physically motivated description of the wrench space that takes all possible disturbances into account and thus is a kind of generalization over all task dependent wrench spaces.

C. The Object Wrench Space

The OWS should contain any wrench that can be created by a distribution of n disturbance forces acting anywhere on the surface of the object. As we are interested in the effect of a normalized disturbance on the object, the sum of the length of all n forces should be 1. By contrast, the number of forces that act on the object is unlimited (so $n \in \{1..\infty\}$; see fig. 6 for illustration). The OWS can again be composed of the union of cone wrench spaces, in the following way:

$$\mathrm{OWS} \quad = \quad \left\{ \mathbf{w} \middle| \mathbf{w} = \sum_{\mathbf{i}=1}^{\mathbf{n}} \alpha_\mathbf{i} \mathbf{w_i} \quad \wedge \right.$$
$$\left. \sum_{i=1}^{n} \alpha_i = 1 \wedge \mathbf{w_i} \in \mathrm{CWS}_i \wedge n \in \{1..\infty\} \right\} \ (4)$$

This description represents the resulting wrenches of any possible disturbance of a certain magnitude that act on the surface. To add gravity, which acts not on the surface but in the center of mass, one can merge this OWS with the wrench space that is produced by forces of any direction acting in the center of mass. If the reference point $\mathbf{r}$ is equal to the center of mass, then one merges a sphere in the force domain to the OWS, scaled with the mass of the object. For the general case, the "mass wrench space" (MWS) generated by gravity g acting in any direction in the center of mass $(\mathbf{m})$ (dependent on the object rotation) can be written as:

$$\mathrm{MWS} = \left\{ \mathbf{w} \middle| \mathbf{w} = \begin{pmatrix} \mathbf{f} \\ (\mathbf{m} - \mathbf{r}) \times \mathbf{f} \end{pmatrix} \wedge \|\mathbf{f}\| \leq \mathbf{m} \cdot \mathbf{g} \right\} \quad (5)$$

Such an OWS $\cup$ MWS describes the general case where nothing about the task is known very naturally, as any possible disturbance and also gravity is represented. The drawback is again that there is no constructive description to really calculate the set.

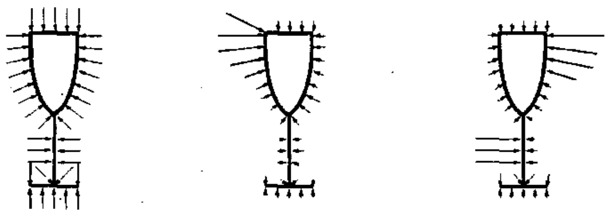

Fig. 6. Illustration of different force distributions that produce the wrench set of the OWS. Each distribution contributes one single wrench to the OWS set. The length of all force vectors sum to the unit length.

D. *The Physically Motivated Grasp Quality Measure QM_{BF}*

As stated in the introduction, many different quality metrics for grasps have been introduced but almost all of them have drawbacks that arise from the different units or scaling in the force/torque dimensions in the grasp wrench space or are even dependent on the selection of the reference point. With the OWS defined above, we can propose a quality metric that overcomes all these drawbacks and rates grasps in a physically interpretable and intuitive way. Of course, the non-uniformity of the wrench space remains. However, our concept generates a physically interpretable scaling between forces and torques automatically.

We take the ability of a grasp to counteract the possible disturbances on an object as a measure of the grasp quality. The Grasp Wrench Space (GWS) of a given grasp $\mathbf{C_k}$ represents the capabilities of the grasp, while the Object Wrench Space (OWS) of a given object $\mathbf{o}$ defines which disturbances may occur. So the largest factor by which we can scale the OWS to just fit in the GWS gives us a measure of the grasp quality. Formally expressed, we get

$$\mathrm{QMS_{BF}}(\mathbf{C_k}, \mathbf{o}) \quad = \quad \Big\{k; k \to \mathrm{Max} \mid \forall \mathbf{x} \in \mathrm{OWS}_{\mathbf{o}} : k \cdot \mathbf{x} \in \mathrm{GWS}_{\mathbf{C_k}} \Big\} \tag{6}$$

With this measure, we are independent of the selection of the reference point as we use it for the creation of both wrench spaces. Moving the reference point can be expressed by a linear transformation that is norm conserving and so it takes no effect on the scaling factor.

It should be noted that the above descriptions are an exact view on the static grasp situation which is the base for the grasp planning problem that we want to address with our grasp quality measure. The question of how to control the forces exerted by the manipulator to resist certain disturbances in a real dynamic grasp situation is not addressed. See [7] for a review. Also, no other phenomena that may occur when the grasped object dynamically touches the enviroment are considered.

The problem with our described measure is to find a way to efficiently calculate the scaling factor of the task wrench space. This problem is considered in the following sections.

IV. A Modified Grasp Qualification Procedure

The next problem to be solved is to integrate the OWS concept in a computationally efficient manner into a grasp qualification procedure: Our grasp measure calculation uses the more complex but physically more relevant Minkowski-sum based grasp quality measure [2]. For cases where no task wrench space is given, we use the radius of the largest inscribed sphere. Our grasp quality measure calculation is of extremely high performance. This stems from the fact that we calculate the GWS not completely but iteratively only at its relevant, "weakest" regions, i.e. where the inscribed sphere touches it. For obtaining the weakest wrench direction during the iterative GWS computation, we project it into the grasp contact friction cones. This way we easily get the linear combination of CWS wrenches that sum up to the largest possible wrench in this direction. (BTW this also avoids the need for a friction cone discretisation, which is a major error cause, as mentioned above).

As motivated in section III-B.2 we now want to compare the OWS (and no longer the largest inscribed sphere) with the GWS of the grasp that is actually evaluated. That means we search the largest scaling factor for a given OWS to fit it into a GWS. In order to keep this algorithm of the same complexity, we cannot use the sampled OWS directly. Instead, we circumscribe the OWS with an ellipsoid and use the corresponding inverse linear mapping for the GWS. Thus we reduce the problem to the above mentioned "sphere fitting" problem with an additional linear mappping per GWS vector (see fig. 7).

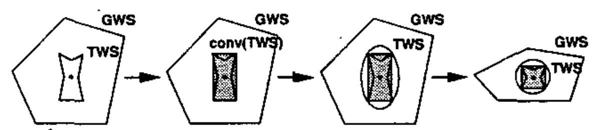

Fig. 7. Approximating the OWS with an ellipsoid:
1. The sampled OWS (exact space, exact quality measure)
2. Convex Hull over the sampled OWS (approx. space, exact quality measure)
3. Enclosing ellipsoid (approx. space, approx. quality measure)
4. Linear transformation of ellipsoid and GWS (sphere algorithm applicable)

A. *Calculating an OWS approximation ellipsoid*

We are looking now for the smallest ellipsoid (spanned by the quadratic form $x^T\mathbf{Q}x \le 1$, $\mathbf{Q}$ symmetric and positive definite) that encloses the QWS, more formally, we look for $\mathbf{Q}$ that fulfills

$$\forall x \in \mathbf{OWS} : x^T\mathbf{Q}x \le 1 \quad \wedge \quad \forall \mathbf{Q}' \neq \mathbf{Q} : V(\mathbf{Q}') < V(\mathbf{Q})$$

$V(\mathbf{Q}) \;=\; 1/\sqrt{det(\mathbf{Q})}$ being the volume of the ellipsoid spanned by $\mathbf{Q}$.

In the following, we outline a four step procedure to efficiently calculate a small (not necessarily the smallest) OWS approximation ellipsoid: (1) Sample a discrete set of OWS wrenches, (2) find a special, analytically describable hull $HULL$(OWS) for these, (3) circumscribe an ellipsoid to $HULL$(OWS) and (4) integrate this OWS representation in our grasp qualification algorithm.

1) OWS sampling: Given a polyhedral object model, we get a set of wrenches ($\overline{OWS}$) that sample and approximate the corresponding OWS by calculating the wrenches generated from friction cone approximation forces at the corners of all polygons of the object. Of course, one might use this set directly to compute the smallest enclosing ellipsoid. This, however, is computationally expensive and doesn't allow for keeping the spherical structure of the OWS's force dimensions.

2) An OWS hull: Let's start with a short discussion of the OWS structure: First, by definition, the forces generating the OWS have all unit length and can have any direction (on most objects), thus the OWS projection to the force dimensions can be tightly enclosed by a unit sphere. To add the effects of gravity for the (most relevant) case that we use the center of mass as our reference point, we add a scaling factor $c = Max(f_{contact}, f_{gravity})$ and get

$$Hull(\text{OWS}|_{\text{forces}}) = \left\{ f \,\middle|\, \frac{1}{c}\|f\|_2 = 1 \right\}$$

Next, the form and size of the OWS projected to its torque dimensions $OWS|_{torques}$ is determined by the object geometry. From examples with different test objects we can see that this projection can be approximated by a 3 dimensional ellipsoid without introducing a large error (see fig. 8).

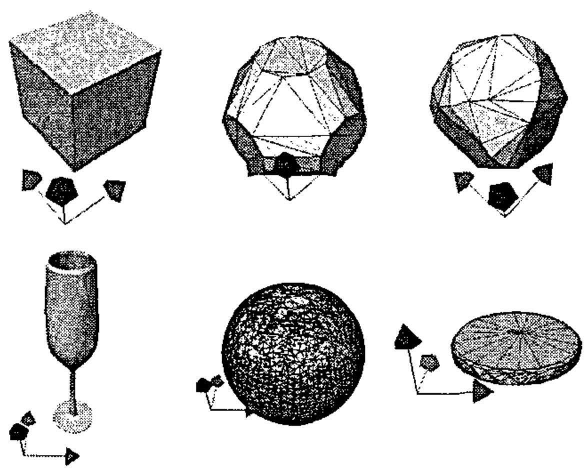

Fig. 8. The structure of the OWS projections in force and torque space for two sample objects a cube and a champagne glass. In force space (middle) one can see that not every direction can be generated by a single disturbance force, due to the limited surface normal directions on a cube. For the glass the force space is almost a perfect sphere.
The torque space for the cube is symmetric in all coordinate axes and the enclosing ellipsoid would be a sphere in this special case. For the glass the torque space is flat for torques round the symmetry axis of the glass.

Assuming that we already have given the ellipsoid which encloses $OWS|_{torques}$ as a quadratic form with the symmetric matrix $\mathbf{W}$ (see sect. IV-A.5), we get

$$Hull(\text{OWS}|_{\text{torques}}) = \{ t' \mid t'^T \mathbf{W} t' = 1 \}$$

with $t' = t - t_{origin}$ where t_{origin} is the center of the torque enclosing ellipsoid (again see sect. IV-A.5). Finally, we can combine all this information (independence of the force and torque dimensions can be expressed as maximum norm) with $w = (f^T, t^T)^T$

$$Hull(OWS) = \left\{ w \,\middle|\, \left\| \frac{1}{c}\|f\|_2, t^T \mathbf{W} t \right\|_\infty = 1 \right\}$$

For people like us who like some rough visual or lower dimensional interpretation of the things they deal with: In 3D this corresponds to a cylinder, but instead of being bounded by its height and a circle, this 6D object is bounded by a sphere and a ellipsoid in its force and torque dimensions, respectively.

3) The OWS approximation ellipsoid: The next thing to be done now is to find a "small enclosing 6D ellipsoid" for this "cylinder": We choose a coordinate system for the following in which the torque ellipsoid is centered at the origin and its main axes are aligned with the coordinate system axes of the torque dimensions. Let's call this transformation $\mathbf{M}$. In this coordinate system $\mathbf{M}$ the shape of $\overline{\mathbf{W}}$ is

$$\overline{\mathbf{W}} = \mathbf{M}\mathbf{W} = \begin{pmatrix} t_1 & 0 & 0 \\ 0 & t_2 & 0 \\ 0 & 0 & t_3 \end{pmatrix}$$

As there is no need for a transformation in the force dimensions (a sphere), we need to look for a quadratic form

$$x^T \mathbf{Q} x \leq 1, \quad \mathbf{Q} = \begin{pmatrix} k_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_6 \end{pmatrix}$$

To determine the $k_1..k_6$ we again come from the 3D intuition: If we want to calculate the smallest enclosing ellipsoid for a cylinder, we know that this ellipsoid needs to touch the cylinder in the cylinder's bounding circle, which means, in particular, that the projection to the cylinders "circle" dimensions also looks like a circle.

In 6D this requires that the ellipsoid should touch the "bounding force sphere", and projected to the force dimension it looks like a sphere:

$$k_1 = k_3 \tag{7}$$
$$k_2 = k_3 \tag{8}$$

The property of touching the unit force sphere means in 2D that the edge points of a rectangle touch its smallest enclosing ellipse. Its 6D analog is

$$x^T \mathbf{Q} x = 1 \quad \forall x \in \left\{ \begin{pmatrix} 0 \\ 0 \\ c \\ t_4 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ c \\ 0 \\ t_5 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ c \\ 0 \\ 0 \\ t_6 \end{pmatrix} \right\}$$

expressed by the following equations

$$c^2 k_3 + t_4^2 k_4 = 1 \tag{9}$$
$$c^2 k_3 + t_5^2 k_5 = 1 \tag{10}$$
$$c^2 k_3 + t_5^2 k_5 = 1 \tag{11}$$

The equations 7 through 11 determine a family of ellipsoid generating quadratic forms

$$\mathbf{Q_{k_3}} = \begin{pmatrix} k_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-c^2k_3}{t_4^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-c^2k_3}{t_5^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-c^2k_3}{t_6^2} \end{pmatrix}$$

To compute k_3 corresponding to the smallest ellipsoid, we minimize the trace (a measure for the volume of an axis aligned ellipsoid) of the matrix $\mathbf{S}$ that transforms a unit sphere to the ellipsoid corresponding to $\mathbf{Q} = (\mathbf{SS})^{-1}$. As $\mathbf{tr(S)} = \frac{1}{\sqrt{\mathbf{tr(G)}}}$, we need to minimize

$$\mathbf{tr(S)} = \frac{1}{\sqrt{k_3^3 \frac{1-c^2k_3}{t_4^2} \frac{1-c^2k_3}{t_5^2} \frac{1-c^2k_3}{t_6^2}}}$$

and get the (not surprising) solution $k_3 = c^2/2$, which results in the ellipsoid generating matrix

$$\mathbf{Q} = \begin{pmatrix} \frac{c^2}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{c^2}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{c^2}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{c^2}{2t_4^2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{c^2}{2t_5^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{c^2}{2t_6^2} \end{pmatrix}$$

In fig. 10 through 13 we plotted several 2D projections of $\overline{OWS}$ for a champagne glass (fig. 9), $Hull(\overline{OWS})$ (the inner circle and the rectangle, resp.) and the final ellipsoid, resulting from $\mathbf{Q}$ (Note that the plot axes are scaled so that the outer ellipsoid is mapped to a circle). Although the projections look rather conservative, it should be noted that $\overline{OWS}$ *touches* the ellipsoid and that the shape of $\overline{OWS}$ is approximated very well (especially in the force dimensions, $\mathbf{Q}$ adds no distortion at all!).

4) Integration in Our Grasp Measure Algorithm: The incorporation of $\mathbf{Q}$ in the previously mentioned grasp quality measure is straightforward:

For the incremental GWS calculation we map each GWS vector with the inverse mapping which maps a 6D unit sphere to our ellipsoid: Note that we determined $\mathbf{Q}$ in a rotated and translated coordinate system $\mathbf{M}$ (see above). $\mathbf{M}$ is derived from the mean $\mu_{\overline{\mathbf{T}}}$ and the covariance matrix $\Sigma_{\overline{\mathbf{T}}}$ of the torques of $\overline{OWS}$ (see sec. IV-A.5). The translational part of $\mathbf{M}$ is $\mu_{\overline{\mathbf{T}}}$, the rotational part is the smallest rotation $\mathbf{R}$ of $\Sigma_{\overline{\mathbf{T}}}$'s main axes to the coordinate axes (note that there is no rotation and translation in the force dimensions, only in the torque dimensions). Now we just need to map each GWS vector $\mathbf{w}$ to

$$\mathbf{w}' = \mathbf{R}^{-1}\left(\mathbf{w} - \begin{pmatrix} 0 \\ \mu_{\overline{\mathbf{T}}} \end{pmatrix}\right)$$

and can use our high performance, high precision, physically well motivated grasp quality measure not only for a spherical task wrench space but also for a physically motivated task wrench space with a neglectible loss of performance!

5) Enclosing Torque Ellipsoid: The only thing that remains to be done now is to determine the smallest enclosing ellipsoid for the OWS projected into torque space. Although there may be better algorithms, we use first and second order statistics here because of high computational performance and sufficiently good results:

Given an OWS as a set of sampled wrenches ($\overline{OWS}$, see sec. IV-A.1) , we project the wrenches to a set of torques in torque space $\overline{\mathbf{T}} = \overline{OWS}|_{torque}$ and calculate the mean $\mu_{\overline{\mathbf{T}}}$ and the covariance matrix $\Sigma_{\overline{\mathbf{T}}}$.

$\Sigma_{\overline{\mathbf{T}}}^{-1}$ is already very similar to a quadratic form of an *enclosing* ellipsoid, we just need to add a scaling factor

$$\sigma = max\left(t'^T \Sigma_{\overline{\mathbf{T}}}^{-1} t' \mid t' \in \overline{\mathbf{T}} - \mu_{\overline{\mathbf{T}}}\right)$$

and get

$$\mathbf{Q_T} = (\sigma^2 \Sigma_{\overline{\mathbf{T}}})^{-1}$$

V. Conclusion

In this paper we give a well defined and pysically motivated description of a general task wrench space based on an OWS for cases where no exact task specification is known. From this TWS we derive a quality measure QM_{BF} and show how it can be implemented very efficiently.

The main improvement this measure adds to our grasp planner is for grasping long, thin objects, e.g. a screwdriver: While using a sphere as a general task wrench space specification as we did before results in good grasps without accounting for different torques in different object dimensions, using our TWS approximation takes care of those now.

It has also to be mentioned that the computation time of the new measure is only slightly above the time for the old one. That is due to the preprocessing of the grasp object to calculate the ellipsoid approximation of the OWS (mainly the calculation of the mean and the covariance matrix), which takes some 10 ms on a 2 GHz Pentium IV and is done only once per object. The online mapping of the GWS increases the computational complexity for calculating the grasp quality by ~ 100 multiplications per iteration step (number of contacts $\cdot 6^2$) , which still leads to a small total computation time of $\sim 10 - 20$ ms for the grasp quality.

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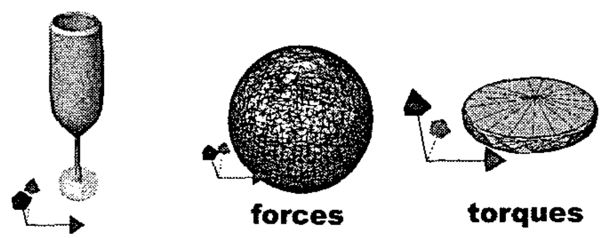

Fig. 9. The object and its sampled OWS in force and torque space for which the OWS projections were calculated.

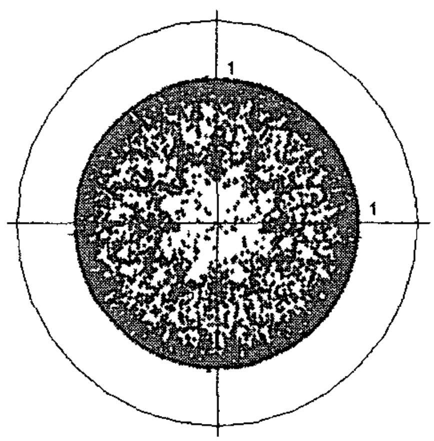

Fig. 10. Projection to force dimensions 1 and 2.
The inner circle is the projection of the $Hull(\overline{OWS})$, the outer circle is the projection of the enclosing ellipsoid, resulting from **Q**.

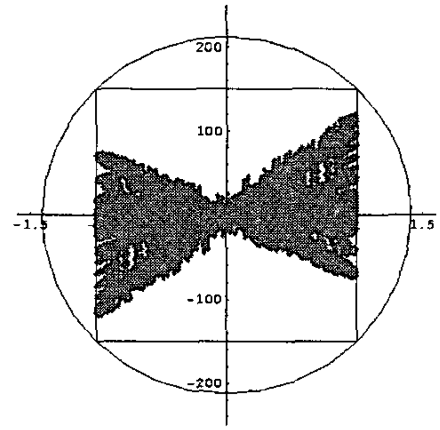

Fig. 11. Projection to force dimensions 2 and torque dimension 4.
The box is the projection of the $Hull(\overline{OWS})$, the outer circle is the projection of the enclosing ellipsoid, resulting from **Q**. (Note that the plot axes are scaled so that the outer ellipsoid is mapped to a circle)

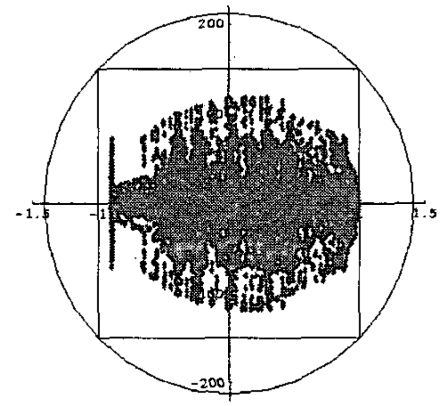

Fig. 12. Projection to force dimensions 3 and torque dimension 4 (for explanation see fig. 11)

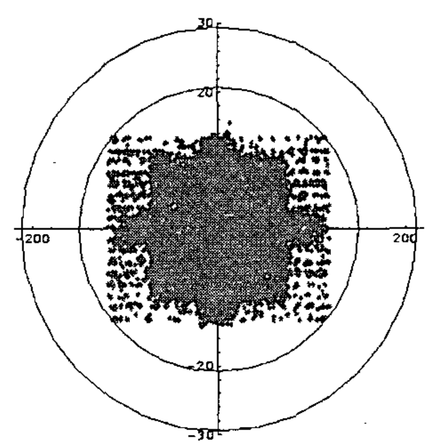

Fig. 13. Projection to torque dimensions 4 and 6 (for explanation see fig. 10)