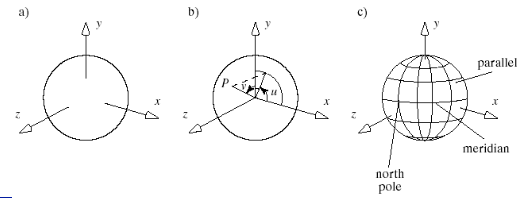
Computer Graphics

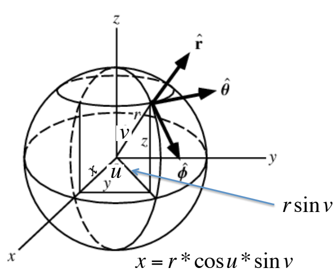
Modeling Shapes with Polygonal Meshes
Ch 6.5-6

Generic Sphere

- Center (0, 0, 0), radius 1;
- $F(x, y, z) = x^2 + y^2 + z^2 - 1 = 0$, or $F(P)=|P|^2-1$.
- $P(u, v) = (\cos u \cos v, \sin u \cos v, \sin v)$,
  with $0 \le u \le 2\pi$, $-\pi/2 \le v \le \pi/2$



Spherical Coordinates



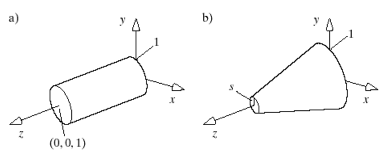
$$x = r * \cos u * \sin v$$
$$y = r * \sin u * \sin v$$
$$z = r * \cos v$$

Sphere (2)

- u-contours are longitude lines (meridians), v-contours are latitude lines (parallels).
- The normal vector to (*x*, *y*, *z*) is radially outward.
- What is the parametric form of **n**(*u*, *v*) ?

Generic Tapered Cylinder

- Axis coincides with *z*-axis; circular cross section of radius 1 at base, s when z = 1; extends in *z* from 0 to 1.
- The tapered cylinder with an arbitrary value of *s* provides formulas for the generic cylinder and cone by setting *s* to 1 or 0, respectively.



Generic Tapered Cylinder (2)

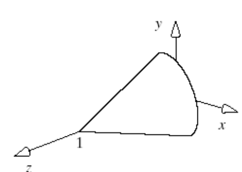
- The wall of the tapered cylinder is given by the implicit form $F(x,y,z) = x^2 + y^2 - (1+(s-1)z)^2 \quad 0 < z < 1$
  for 0 < z < 1, and by the parametric form
  $$P(u,v) = ((1+(s-1)v)\cos(u), (1+(s-1)v)\sin(u), v)$$
  $$0 \le u \le 2\pi \qquad 0 \le v \le 1$$
- When the tapered cylinder is a solid object, we add two circular discs at its ends: a **base** and a **cap**. The cap is a circular portion of the plane z = 1, characterized by the inequality $x^2 + y^2 < s^2$, or given parametrically by $P(u, v)$ = (v *cos*(u), v *sin*(u), 1) for v in [0, s].

Generic Tapered Cylinder (3)

- The normal vector to the wall of the tapered cylinder is
  n(x, y, z) = (x, y, -(s - 1)(1+ (s - 1)z)),
  or in parametric form
  n(u, v) = (*cos*(u), *sin*(u), 1 – s).
- For the generic cylinder the normal is simply
  (*cos*(u), *sin*(u), 0).
- The normal is directed radially away from the axis of the cylinder. For the tapered cylinder it is also directed radially, but shifted by a constant z-component.

Generic Cone

- A cone whose axis coincides with the *z*-axis, has a circular cross section of maximum radius 1, and extends in *z* from 0 to 1. It is a tapered cylinder with small radius of s = 0.



Generic Cone (2)

- Wall: $F(x, y, z) = x^2 + y^2 - (1 - z)^2 = 0$ for $0 < z < 1$; parametric form
  $P(u, v) = ((1-v) \cos(u), (1- v) \sin(u), v)$ for azimuth u in $[0, 2\pi]$ and v in $[0, 1]$.
- Using the results for the tapered cylinder again, the normal vector to the wall of the cone is
  $(x, y, 1-z)$.

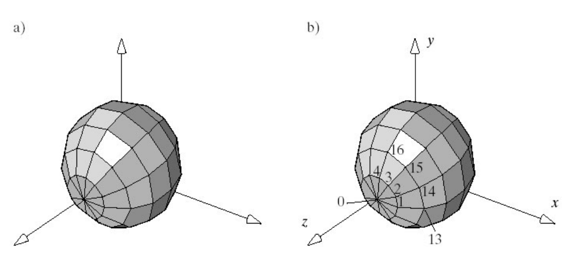
Normal vectors to general surfaces

Surface	n(u,v) at p(u, v) Parametric form	F(x, y, z) Implicit form
Sphere	P(u, v)	(x, y, z)
Tapered cylinder	(cos(u), sin(u), 1-s)	(x, y, -(s-1)(1+(s-1)z))
Cylinder	(cos(u), sin(u), 0)	(x, y, 0)
Cone	(cos(u), sin(u), 1)	(x, y, 1-z)

Mesh for the Generic Sphere

- We slice the sphere along azimuth lines and latitude lines.
- We slice the sphere into nSlices slices around the equator and nStacks stacks from the South Pole to the North Pole.
- The figure (next slide) shows the example of 12 slices and 8 stacks.
- The larger nSlices and nStacks are, the better the mesh approximates a true sphere.

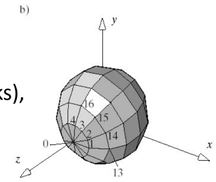
Mesh for the Generic Sphere (2)



Mesh for the Generic Sphere (3)

- To make slices we need nSlices values of *u* around the equator between 0 and 2π. Usually these are chosen to be equal-spaced: $u_i = i*(2\pi/\text{nSlices})$, $i = 0, 1, \ldots, \text{nSlices} - 1$.
- We put half of the stacks above the equator and half below. The top and bottom stacks will consist of triangles; all other faces will be quadrilaterals.

This requires we define (nStacks + 1) values of latitude: $v_j = \pi - j*(\pi/\text{nStacks})$, $j = 0, 1, \ldots, \text{nStacks}$.



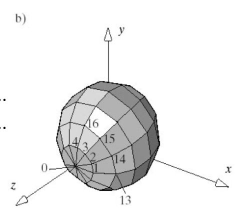
Mesh for the Generic Sphere (4)

- The vertex list: put the north pole in pt[0], the bottom points of the top stack into the next 12 vertices, etc. There will be 86 points.
- The normal vector list: norm[k] is the normal for the sphere at vertex pt[k] in parametric form; **n**(*u*,*v*) is evaluated at (*u*,*v*) used for the points.
  - For the sphere this is particularly easy since norm[k] is the same as pt[k].

Mesh for the Generic Sphere (5)

- The face list: Put the top triangles in the first 12 faces, the 12 quadrilaterals of the next stack down in the next 12 faces, etc.
- The first few entries in the face list will contain the data

number of vertices:	3	3	3	...
vertex indices:	0 1 2	0 2 3	0 3 4	...
normal indices:	0 1 2	0 2 3	0 3 4	...

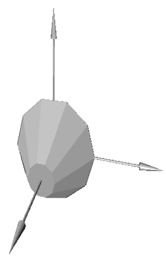


General Meshes

- Ultimately we need a method, such as makeSurfaceMesh(), that generates appropriate meshes for a given surface *P*(*u*, *v*).
- Some graphics packages have routines that are highly optimized for triangles, making triangular meshes preferable to quadrilateral ones.
- We can use the same vertices, but alter the face list by replacing each quadrilateral with two triangles.
  - For instance, a face that uses vertices 2, 3, 15, 14 might be subdivided into two triangles, one using 2, 3, 15 and the other using 2, 15, 14.

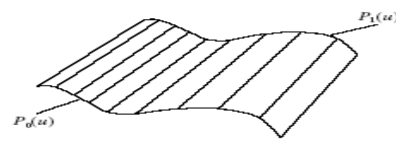
Mesh for the Tapered Cylinder

- We use nSlices = 10 and nStacks = 1.
- A decagon is used for the cap and base.
- If you prefer to use only triangles, the walls, the cap, and the base could be dissected into triangles.



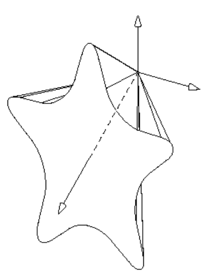
Ruled Surfaces

- Ruled Surface: through every point, there passes at least one straight line lying entirely on the surface.
- Made by moving the ends of a straight line along curves.



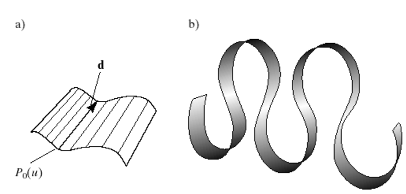
Ruled Surfaces (2)

- A cone is a ruled surface for which one of the curves, say, $P_0(u)$, is a *single* point $P_0(u) = P_0$, the apex of the cone,.



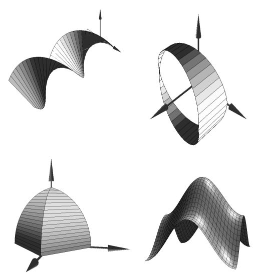
Ruled Surfaces (3)

- A cylinder is a ruled surface for which $P_1(u)$ is a translated version of $P_0(u)$: $P_1(u) = P_0(u) + \mathbf{d}$, for some vector **d**.



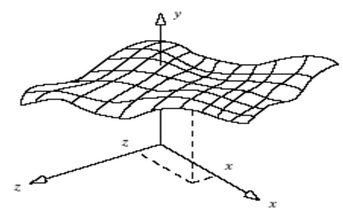
Ruled Surfaces (5)

- Double helix: $P_0(u)$ and $P_1(u)$ are both helixes that wind around each other.
- Möbius strip (has only one edge).
- Vaulted roof made up of four ruled surfaces.
- Coons patch named after the legendary graphicist Steven Coons.



Surfaces which are Functions of Two Variables

- Define a single-valued height function y = f(x, z) of any sort you wish.



Surfaces which are Functions of Two Variables (2)

- Parametric form: P(u, v) = (u, f(u, v), v)
- Normal vector **n**(u, v) = $-\left(\frac{\partial f}{\partial u}, -1, \frac{\partial f}{\partial v}\right)$
- Thus *u*-contours lie in planes of constant *x*, and *v*-contours lie in planes of constant *z*.