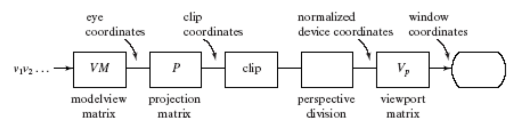
Computer Graphics

Projection: Orthographic vs. Perspective Ch7.4

Projections of 3-D Objects

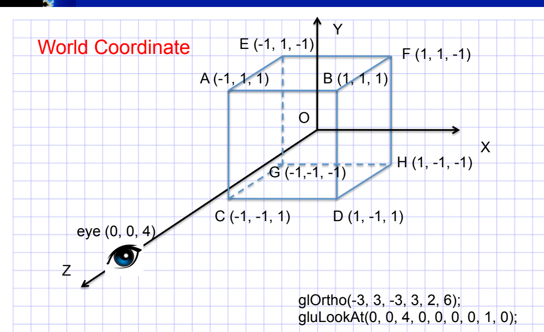
- The graphics pipeline: vertices start in world coordinates; after MV, in eye coordinates, after P, in clip coordinates; after perspective division, in normalized device coordinates; after V, in screen coordinates.



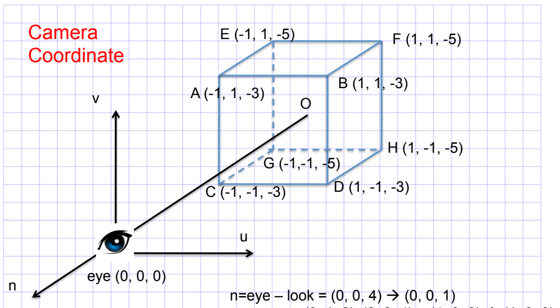
Projections of 3-D Objects (2)

- Each vertex *v* is multiplied by the modelview matrix (*VM*), containing all of the modeling transformations for the object; the viewing part (*V*) accounts for the transformation set by the camera's position and orientation. When a vertex emerges from this matrix it is in **eye coordinates**, that is, in the coordinate system of the eye.
- The figure shows this system: the eye is at the origin, and the near plane is perpendicular to the *z*-axis, located at *z* = -*N*.

World vs. Camera Coordinates

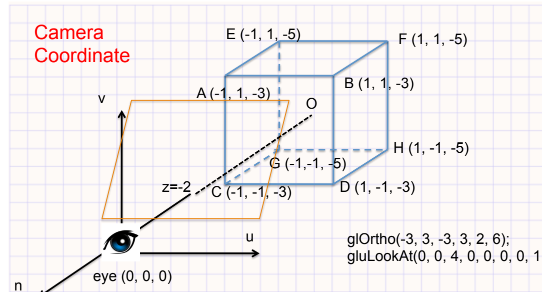


World vs. Camera Coordinates(2)



n=eye – look = (0, 0, 4) → (0, 0, 1)
u=up x n = (0, 1, 0)x(0, 0, 4) = (4, 0, 0) → (1, 0, 0)
v=n x u = (0, 0, 4)x(4, 0, 0) → (0, -16, 0)→ (0, -1, 0)

World vs. Camera Coordinates(3)

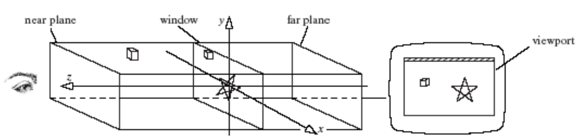


glOrtho(-3, 3, -3, 3, 2, 6);
gluLookAt(0, 0, 4, 0, 0, 0, 0, 1, 0);

Near plane : z=2; Far plane: z=6 (world coordinate)
Near plane: z=-2, Far plane: z=-6 (camera coordinate)

Orthographic Projection Matrix P

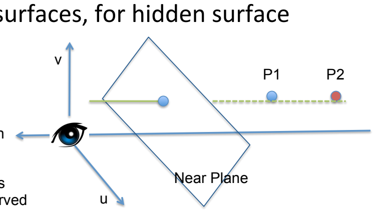
- The **view volume** of the camera is a rectangular parallelepiped (Orthographic Projection)



The x and y coordinates of each point are preserved using the orthographic projection

Orthographic Project Matrix P (2)

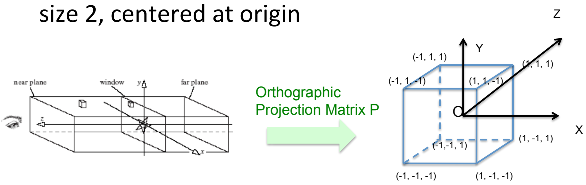
- We need to add depth information
- Depth information tells which surfaces are in front of other surfaces, for hidden surface removal.



The x and y coordinates of each point are preserved using the orthographic projection

Orthographic Project Matrix P (3)

- Instead of Euclidean distance, we use a pseudo-depth, $-1 \le P_z' \le 1$ for $-N > z > -F$. This quantity is faster to compute than the Euclidean distance.
- The view volume is transformed into a cube of size 2, centered at origin



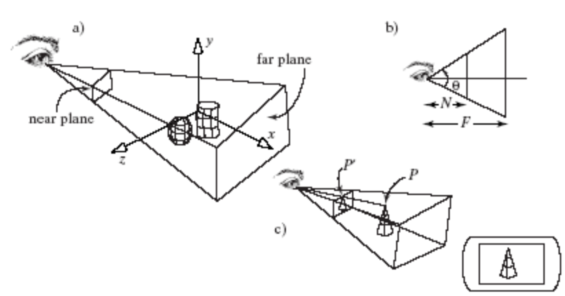
Orthographic Project Matrix P (4)

- Translate the view volume to center at the origin
- Scale the view volume to x:[-1, 1], y:[-1, 1], z:[-1:1]

$$P = ST = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & 0 \\ 0 & \frac{2}{top-bottom} & 0 & 0 \\ 0 & 0 & \frac{2}{far-near} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -\frac{left+right}{2} \\ 0 & 1 & 0 & -\frac{top+bottom}{2} \\ 0 & 0 & 1 & -\frac{far+near}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

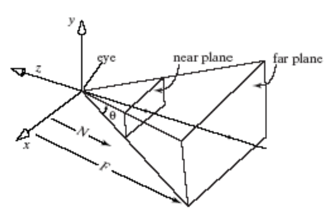
$$P = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Perspective Projection



Perspective projection in OpenGL

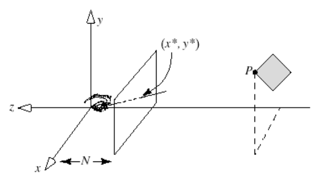
glFrustum(left, right,bott, top, N, F); or
gluPerspective(viewAngle, aspect, N, F);



top = N*tan(PI/180*viewAngle/2)
bott = -top
right = top*aspect
left = -right

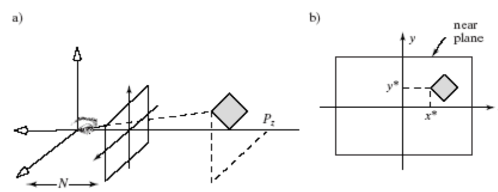
Perspective Projections of 3-D Objects

- A vertex located at *P* in eye coordinates is passed through the next stages of the pipeline where it is projected to a certain point (*x**, *y**) on the near plane, clipping is carried out, and finally the surviving vertices are mapped to the viewport on the display.



Perspective Projections of 3-D Objects (4)

- We erect a local coordinate system on the near plane, with its origin on the camera's *z*-axis. Then it makes sense to talk about the point x* units right of the origin, and y* units above the origin.

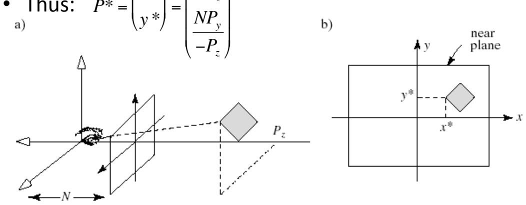


Perspective Projection of 3-D Objects (5)

- (P_x, P_y, P_z) projects to (x*, y*).
- By similar triangles, we get: $\frac{x^*}{P_x} = \frac{N}{-P_z} \Rightarrow\Rightarrow x^* = \frac{NP_x}{-P_z}$

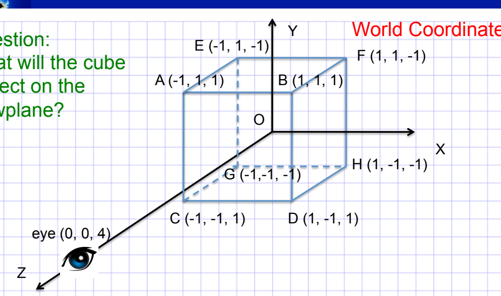
$$\frac{y^*}{P_y} = \frac{N}{-P_z} \Rightarrow\Rightarrow y^* = \frac{NP_y}{-P_z}$$

- Thus: $P^* = \begin{pmatrix} x^* \\ y^* \end{pmatrix} = \begin{pmatrix} \frac{NP_x}{-P_z} \\ \frac{NP_y}{-P_z} \end{pmatrix}$



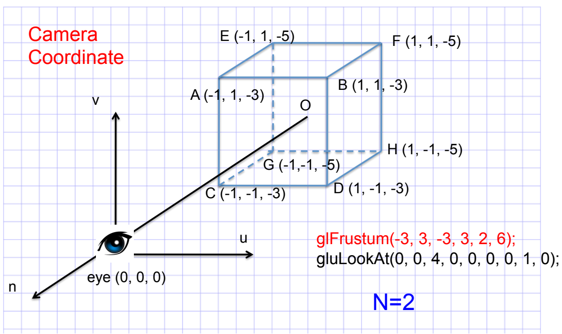
Practice Question

Question:
What will the cube project on the viewplane?




```
glFrustum(-3, 3, -3, 3, 2, 6);
gluLookAt(0, 0, 4, 0, 0, 0, 0, 1, 0);
```

Practice Question

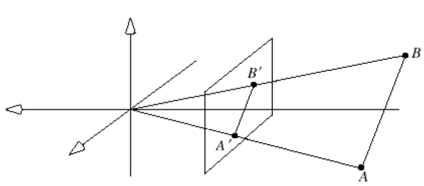


Perspective Projection Properties

- $|P_z|$ is larger for points further away from the eye, and, because we divide by it, causes objects further away to appear smaller (perspective foreshortening).
- We do not want $P_z \geq 0$; generally these points (at or behind eye) are clipped.
- Projection to a plane other than N simply scales P*; since the viewport matrix will scale anyway, we might as well project to N.

Perspective Projection Properties (2)

- Straight lines project to straight lines. Consider the line between *A* and *B*. *A* projects to *A'* and *B* projects to *B'*.
- In between: consider the plane formed by *A, B,* and the origin. Since any two planes intersect in a straight line, this plane intersects the near plane in a straight line. Thus line segment *AB* projects to *line segment A'B'*.

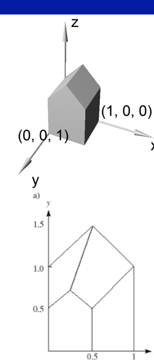


Barn Example

- **View #1**: gluPerspective(30, 1.5, 1, 2);
  gluLookAt(0, 0, 2, 0, 0, 0, 0, 1, 0);

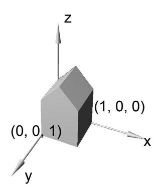
The near plane coincides w/ the front of the barn.

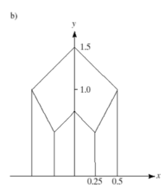
- In camera coordinates all points on the front wall of the barn have P_z = -1 and those on the back wall have P_z = -2. So any point (P_x, P_y, P_z) on the front wall projects to $P' = (P_x, P_y)$ and any point on the back wall projects to $P' = (P_x/2, P_y/2)$.
- The foreshortening factor is two for points on the back wall. Note that edges on the rear wall project at half their true length. Also note that edges of the barn that are actually parallel in 3D *need not* project as parallel.



Barn Example (2)

- In view #2, the camera has been moved right, but everything else is the same:
  gluPerspective(30, 1.5, 1, 2);
  gluLookAt(0.5, 0, 2, 0, 0, 0, 0, 1, 0);





Perspective Projection of Lines

- Straight lines are transformed to straight lines.
- Lines that are parallel in 3D project to lines, but not necessarily parallel lines. If not parallel, they meet at some vanishing point.
- If $P_z \geq 0$, lines that pass through the camera undergo a catastrophic "passage through infinity"; such lines must be clipped.
- Perspective projections usually produce geometrically realistic pictures. But realism is strained for very long lines parallel to the viewplane.

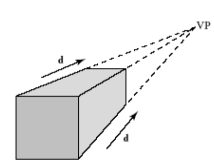
Projection of Straight Lines (2)

- For any line in 3D: P = A + **c**t, its projected line on viewplane:
  $p(t) = -N\,([A_x + c_xt]/[A_z + c_zt],\ [A_y + c_yt]/[A_z + c_zt])$
  $= -N/[A_z + c_zt]\ (A_x + c_xt,\ A_y + c_yt)$
- Point A → $p(0) = -\,N/A_z\ (A_x, A_y)$.

- Effect of projection → on parallel lines:
- If the line is parallel to plane N, $c_z = 0$, and
  $p(t) = -\,N/A_z\ (A_x + c_xt,\ A_y + c_yt)$.
- This is a line with slope c_y/c_x and all lines with direction **c**→ a line with this slope.
- **Thus if two lines in 3D are parallel to each other *and* to the viewplane, they project to two parallel lines.**

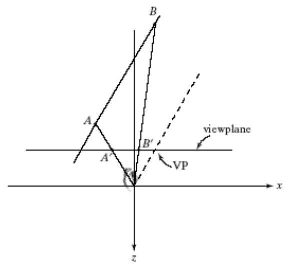
Projection of Straight Lines (3)

- If the line is not parallel to plane N (near plane), look at limit as t becomes ∞ for
  $p(t) = -N/[A_z + c_zt]\ (A_x + c_xt,\ A_y + c_yt)$,
  ➔ $p(t) = -N/c_z\ (c_x, c_y)$, a constant.
  – All lines with direction **c** reach this point as t becomes ∞; it is called the vanishing point.
- Thus all parallel lines share the same vanishing point.
- In particular, these lines project to lines that are *not* parallel.

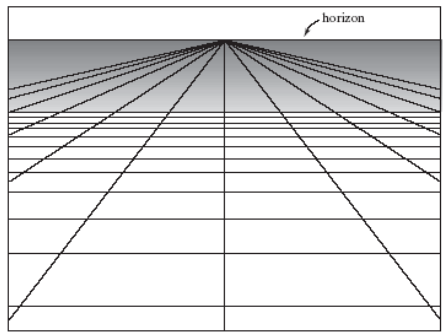


Projection of Straight Lines (≤)

- Geometry of vanishing point: *A* projects to *A*', *B* projects to *B*', etc. Very remote points on the line project to *VP* as shown.
- Line from eye to VP becomes parallel to line AB.

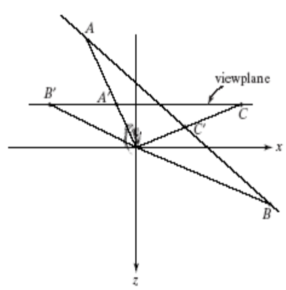


Example: horizontal grid in perspective



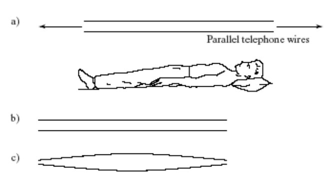
Projection of Straight Lines (5)

- Lines that pass behind the eye have a different geometry for the vanishing point; as C approaches the eye plane, its projection moves infinitely far to the right.
- When it reaches the eye plane, it jumps infinitely far to the left.



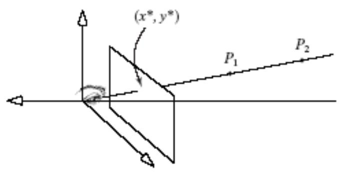
Eye Effects

- Note that technically the eye is not a planar surface, but a curved one. This fact results in anomalies such as a slight curve appearing in the view of very long parallel lines.



Incorporating Perspective in the Graphics Pipeline

- We need to add depth information (destroyed by projection).
- Depth information tells which surfaces are in front of other surfaces, for hidden surface removal.



Incorporating Perspective in the Graphics Pipeline (2)

- We use a projection point
  $(x^*, y^*, z^*) = [N/(-P_z)](P_x, P_y, (a + bP_z))$,
  Pseudo-depth

  and choose a and b so that
  $P_z^* = -1$ when $P_z = -N$ and 1 when $P_z = -F$.
- Result:
  $a = -(F + N)/(F - N)$,
  $b = -2FN/(F - N)$.
- P_z^* increases (becomes more positive) as P_z decreases (becomes more negative, moves further away).

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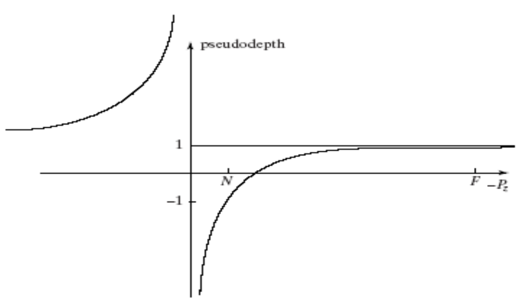
Illustration of Pseudo-depth Values



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Incorporating Perspective in the Graphics Pipeline (3)

- Pseudodepth values bunch together as $-P_z$ gets closer to *F*, causing difficulties for hidden surface removal.
- When *N* is much smaller than *F,* as it normally will be, pseudodepth can be approximated by

$$pseudodepth \approx 1 + \frac{2N}{P_z}$$

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