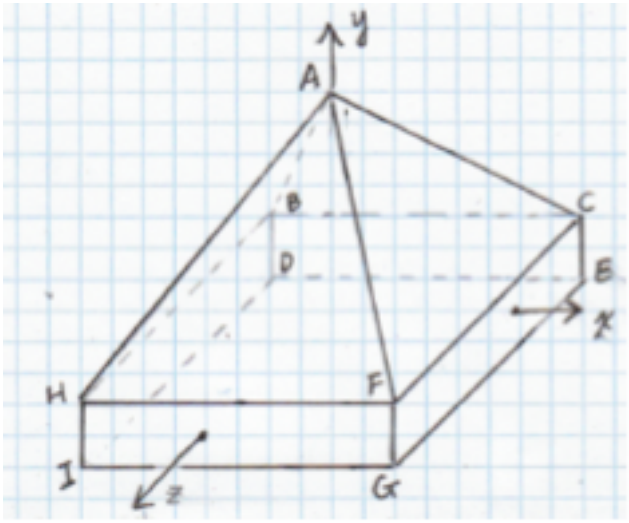
CSCI 4250/5250 solution
Homework 7



A=(0, 6, 0), B=(-4, 1, -1), C=(6, 1, -1), D=(-4, -1, -1), E=(6, -1, -1),
F=(6, 1, 4), G=(6, -1, 4), H=(-4, 1, 4), I=(-4, -1, 4)

1. Given the 3D mesh object in the picture above, show:
    a. The vertex list
    b. The normal list. Compute the normals of the faces using Newell's method. Show computation steps involved.
    c. The face list. Each face should include the vertex (index) list, as well as the normal (index) list.

Solution:

1. The Vertex list

Vertext ID	Coordinates
0	(0, 6, 0)
1	(-4, 1, -1)
2	(6, 1, -1)
3	(-4, -1, -1)
4	(6, -1, -1)
5	(6, 1, 4)
6	(6, -1, 4)
7	(-4, 1, 4)
8	(-4, -1, 4)

2. The normal list

2.1) Face FHIG → normal (0, 0, 1)

2.2) Face CFGE → normal (1, 0, 0)

2.3) Face BDIH → normal (-1, 0, 0)

2.4) Face CEDB → normal (0, 0, -1)

2.5) Face DEBI → normal (0, -1, 0)

2.6) Face AFC: apply Newell's method:

$$n_x = \sum_{i=0}^{N-1}(y_i - y_{ni})(z_i + z_{ni})$$

$$n_y = \sum_{i=0}^{N-1}(z_i - z_{ni})(x_i + x_{ni})$$

$$n_z = \sum_{i=0}^{N-1}(x_i - x_{ni})(y_i + y_{ni})$$

In this face, i=0, 1, 2; corresponding points A, F, C

The next point of A is F, the next point of F is C, and the next point of C is A.

i	(x, y, x)
0	(0, 6, 0)
1	(6, 1, 4)
2	(6, 1, -1)

Apply the Newell's method:

$$n_x = (y_0 - y_1)(z_0 + z_1) + (y_1 - y_2)(z_1 + z_2) + (y_2 - y_0)(z_2 + z_0)$$

$$= (6-1)(0+4) + (1-1)(4-1) + (1-6)(-1+0)$$

$$= 20 + 5 = 25$$

$$n_y = (z_0 - z_1)(x_0 + x_1) + (z_1 - z_2)(x_1 + x_2) + (z_2 - z_0)(x_2 + x_0)$$

$$= (0-4)(0+6) + (4-(-1))(6+6) + (-1-0)(6+0)$$

$$= -24 + 60 - 6 = 30$$

$$n_z = (x_0 - x_1)(y_0 + y_1) + (x_1 - x_2)(y_1 + y_2) + (x_2 - x_0)(y_2 + y_0)$$

$$= (0-6)(6+1) + (6-6)(1+1) + (6-0)(1+6)$$

$$= -42 + 42 = 0$$

Normalize the vector (25, 30, 0), the result is: $(\frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}}, 0)$

2.7) Face AHF:

i	(x, y, x)
0	(0, 6, 0)
1	(6, 1, 4)
2	(6, 1, -1)

Apply Newell's methed:

$$n_x = (y_0 - y_1)(z_0 + z_1) + (y_1 - y_2)(z_1 + z_2) + (y_2 - y_0)(z_2 + z_0)$$
$$= (6-1)(0+4) + (1-1)(4+4) + (1-6)(4+0)$$
$$= 20 - 20 = 0$$
$$n_y = (z_0 - z_1)(x_0 + x_1) + (z_1 - z_2)(x_1 + x_2) + (z_2 - z_0)(x_2 + x_0)$$
$$= (0-4)(0-4) + (4-4)(-4+6) + (4-0)(6+0)$$
$$= 16 + 24 = 40$$
$$n_z = (x_0 - x_1)(y_0 + y_1) + (x_1 - x_2)(y_1 + y_2) + (x_2 - x_0)(y_2 + y_0)$$
$$= (0-(-4))(6+1) + (-4-6)(1+1) + (6-0)(4+0)$$
$$= 28 - 20 + 24 = 32$$

normalize (0, 40, 32), the result is $(0, \frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}})$

2.8 Face ABH

i	(x, y, x)
0	(0, 6, 0)
1	(-4, 1, -1)
2	(-4, 1, 4)

$$n_x = (y_0 - y_1)(z_0 + z_1) + (y_1 - y_2)(z_1 + z_2) + (y_2 - y_0)(z_2 + z_0)$$
$$= (6-1)(0-1) + (1-1)(-1+4) + (1-6)(4+0)$$
$$= -5 - 20 = -25$$
$$n_y = (z_0 - z_1)(x_0 + x_1) + (z_1 - z_2)(x_1 + x_2) + (z_2 - z_0)(x_2 + x_0)$$
$$= (0-(-1))(0-4) + (-1-4)(-4-4) + (4-0)(-4+0)$$
$$= -4 + 40 - 16 = 20$$
$$n_z = (x_0 - x_1)(y_0 + y_1) + (x_1 - x_2)(y_1 + y_2) + (x_2 - x_0)(y_2 + y_0)$$
$$= (0-(-4))(6+1) + (-4-(-4))(1+1) + (-4-0)(1+6)$$
$$= 28 - 28 = 0$$

normalize (25, 20, 0), the result is $(-\frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}}, 0)$

2.9 Face ACB

i	(x, y, x)
0	(0, 6, 0)
1	(6, 1, -1)
2	(-4, 1, -1)

$$n_x = (y_0 - y_1)(z_0 + z_1) + (y_1 - y_2)(z_1 + z_2) + (y_2 - y_0)(z_2 + z_0)$$
$$= (6-1)(0-1) + (1-1)(-1-1) + (1-6)(-1+0)$$
$$= -5 + 5 = 0$$
$$n_y = (z_0 - z_1)(x_0 + x_1) + (z_1 - z_2)(x_1 + x_2) + (z_2 - z_0)(x_2 + x_0)$$
$$= (0-(-1))(0+6) + (-1-(-1))(6-4) + (-1-0)(-4+0)$$
$$= 6 + 4 = 10$$
$$n_z = (x_0 - x_1)(y_0 + y_1) + (x_1 - x_2)(y_1 + y_2) + (x_2 - x_0)(y_2 + y_0)$$
$$= (0-6)(6+1) + (6-(-4))(1+1) + (-4-0)(1+6)$$
$$= -42 + 20 - 28 = -50$$

Normalize (0, 10, -50), the result is $(0, \frac{1}{\sqrt{26}}, -\frac{5}{\sqrt{26}})$

Therefore the normal list is:

ID	Normal
0	(0, 0, 1)
1	(1, 0, 0)
2	(-1, 0, 0)
3	(0, 0, -1)
4	(0, -1, 0)
5	$(\frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}}, 0)$
6	$(0, \frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}})$
7	$(-\frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}}, 0)$
8	$(0, \frac{1}{\sqrt{26}}, -\frac{5}{\sqrt{26}})$

3. The Face list

Face ID	Vertex ID	Normal ID
0	4, 7, 8, 6	0, 0, 0, 0
1	2, 5, 6, 4	1, 1, 1, 1
2	1, 3, 8, 7	2, 2, 2, 2
3	2, 4, 3, 1	3, 3, 3, 3
4	3, 4, 1, 8	4, 4, 4, 4
5	0, 5, 2	5, 5, 5
6	0, 7, 5	6, 6, 6
7	0, 1, 7	7, 7, 7
8	0, 2, 1	8, 8, 8