CSCI 4250/5250 **Homework 3 solution**

1. Given a vector a=(3, - 2, 2), the magnitude of a is ________________; and the normalized vector of a is______________;

   magnitude(a) = $\sqrt{3*3+(-2)*(-2)+2*2}$=$\sqrt{17}$=4.12

   normalized(a)= $\left(\frac{3}{\sqrt{17}}, -\frac{2}{\sqrt{17}}, \frac{2}{\sqrt{17}}\right)$ =(0.72, -0.48, 0.48)

2. Given vector a=(2, 3, 1) and b=(4, 2, 2), the dot product of the two vectors, $aa \cdot bb =$ ____________________; if α is the angle between the two vectors, then cos(α) = ________________________; the cross product of the two vectors, a x b = ____________________.

$$a \cdot b = (2,3,1) \cdot (4,2,2) = 2*4+3*2+1*2 = 16$$

$$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{16}{\sqrt{2*2+3*3+1*1}\sqrt{4*4+2*2+2*2}} = \frac{16}{3.75*4.89} = 0.87$$

$$a \times b = \begin{bmatrix} i & j & k \\ 2 & 3 & 1 \\ 4 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix} i - \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} j + \begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix} k$$

$$= (3*2-2*1)i - (2*2-4*1)j + (2*2-3*4)k = 4i - (0)j + (-8)k$$

$$a \times b = [4, \quad 0, \quad -8]$$

3. Show the parametric form of the line that passes through points A(3, 6, 1) and B(2,10, 5): ____. Show the point that is 1/3 way from A, and 2/3 way from B ______________________________.

$$f(t) = A + t*(B-A) = \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix} + t*\left(\begin{bmatrix} 2 \\ 10 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix} + t*\begin{bmatrix} -1 \\ 4 \\ 4 \end{bmatrix} \quad \text{OR}$$

$$f(t) = (1-t)*A + t*B$$

using the first f(t) definition, substitute t with 1/3, we get:

$$\begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix} + t*\begin{bmatrix} -1 \\ 4 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} x(t=\frac{1}{3}) \\ y(t=\frac{1}{3}) \\ z(t=\frac{1}{3}) \end{bmatrix} = \begin{bmatrix} 3+(\frac{1}{3}*(-1)) \\ 6+(\frac{1}{3}*4) \\ 1+(\frac{1}{3}*4) \end{bmatrix} = \begin{bmatrix} 2\frac{2}{3} \\ 7\frac{1}{3} \\ 2\frac{1}{3} \end{bmatrix}$$

4. Given 3 points: A(2, 1, 1), B(2, 2, 2), and C(4, 2, 2), compute the normal vector for the triangle ABC.

Define the vector from A to B: Vab = B – A = $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

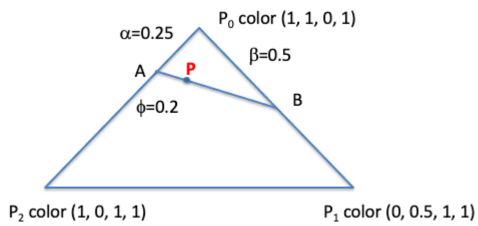
Define the vector from A to C: Vac = C – A = $\begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

The normal vector for the plane with triangle ABC is the cross product of the two vectors Vab and Vac:

$$Vab \times Vac = \begin{bmatrix} i & j & k \\ 0 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} i - \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} j + \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} k = 0i - (-2)j + (-2)k$$

So the normal vector is : $[0, \quad 2, \quad -2]$

5. Given a triangle $P_0P_1P_2$. The color of the three points P_0, P_1, and P_2 are marked on the picture below. The location of the point A is α=0.25 away from P_0, and point B is β= 0.5 away from P_0. Applying Gouraud shading, what is the color of the point at location P? Show computation steps.



A=α*P_0+(1-α)*P_2

$= 0.25 * < 1, 1, 0, 1 > + 0.75 * < 1, 0, 1, 1 > = < 1, 0.25, 0.75, 1 >$

B=β*P_0+(1-β)*P_1

$= 0.5 * < 1, 1, 0, 1 > + 0.5 * < 0, 0.5, 1, 1 > = < 0.5, 0.75, 0.5, 1 >$

P=ϕ*A + (1-ϕ)*B

$= 0.2 * < 1, 0.25, 0.75, 1 > + 0.8 * < 0.5, 0.75, 0.5, 1 >$

=<0.2,0.05,0.15,0.2>+<0.4, 0.6,0.4,0.8>

=<0.6, 0.65, 0.55,1>

6. A ray of light a=<2, 2, 1> from the light source reaches a reflective surface s and bounces off in direction r. The normal vector to this surface is n=<1, 0, 1>. Compute the reflected light vector r. Show computation steps.

$\vec{a} = < 2, 2, 1 >$

$\vec{n} = < 1, 0, 1 >, \hat{n} = < \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} >$

$\vec{r} = \vec{a} - 2\,(\vec{a} \cdot \vec{n}) \cdot \vec{n} \quad = < 2, 2, 1 > -2\left(< 2, 2, 1 > \cdot < \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} >\right) \cdot < \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} >$

$= < 2, 2, 1 > - < 3, 0, 3 >$

=<-1, 2, -2>

7. **Object transformation** problem:

a. Write out the following 4x4 matrices and label each with the following names:
T0: Translate along X-axis by 4 and along Y-axis by 3
R: Rotate about the z-axis by 45 degrees
T1: Translate along X-axis by -4 and along Y-axis by -3
S: Scale along X-axis by a factor of 2 and along Y-axis by a factor of 4 (z is unchanged)

$$T0 = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R = \begin{bmatrix} cos45 & -sin45 & 0 & 0 \\ sin45 & cos45 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T1 = \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, S = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b. Apply the transformation matrix T0 to the point P=(7, 5, 7) to find the transformed point Q by multiply it out.

$$T0 * p = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 7 \\ 5 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 7+4 \\ 5+3 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 11 \\ 8 \\ 7 \\ 1 \end{bmatrix}$$

c. Apply the transformation matrix R to the point P=(7, 5, 7) to find the transformed point Q by multiply it out.

$$R * p = \begin{bmatrix} cos45 & -sin45 & 0 & 0 \\ sin45 & cos45 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 7 \\ 5 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 7cos45 - 5sin45 \\ 7sin45 + 5cos45 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.414 \\ 8.484 \\ 7 \\ 1 \end{bmatrix}$$

d. Suppose two transformations are to be performed in the sequence, first scale an object with S, and then translate the object with T0. Show the combined effect of these two transformations by multiplying out the two matrices.

$$T0 * S = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 4 \\ 0 & 4 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

e. How to apply these transformations to the point P (7, 5, 7)? Write the matrix, matrix, point multiplication. Make sure the two matrices are multiplied to the point in the correct order.

$$P' = \begin{bmatrix} 2 & 0 & 0 & 4 \\ 0 & 4 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 7 \\ 5 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 18 \\ 23 \\ 7 \\ 1 \end{bmatrix}$$