

1. Transformations applied to points

a. Write out the following 4x4 matrices and label each with the following names:

i. T_0 : translate in x by 4 and in y by 3

$$T_0 = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii. R : Rotate about the z axis by $\frac{\pi}{4}$ radians

$$R = \begin{bmatrix} \cos\left(\frac{\pi}{4}\right) & -\sin\left(\frac{\pi}{4}\right) & 0 & 0 \\ \sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii. T_1 : Translate in x by -4 and in y by -3

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iv. S : Scale in x by a factor of 2 and in y by a factor of 4 (z is unchanged)

$$S = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b. Assume you have an object you want to rotate by $\frac{\pi}{4}$ radians around a z-axis centered at (4, 3, 0). Using the matrices defined above, show the correct order of composition of these matrices to perform the desired rotation.

$$T_1 \rightarrow R \rightarrow T_0 \text{ or in matrix product form: } T_0 R T_1$$

c. Find the composite matrix M by multiplying out your answer from question 1.b.

$$M = T_0 R T_1 = \begin{bmatrix} 0.70711 & -0.70711 & 0 & 3.29289 \\ 0.70711 & 0.70711 & 0 & -1.94975 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d. Apply the transformation matrix M to the 3D point $P=(7,5,7)$ to find the transformed point Q .

$$Q = \begin{bmatrix} 4.7071 \\ 6.5355 \\ 7 \\ 1 \end{bmatrix}$$