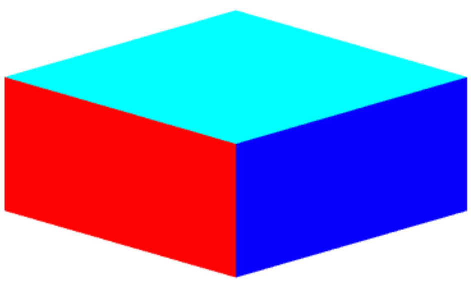
Homework 5 Solution

1) Given the 3D cube example in programs: ortho.js and ortho.html (available on the course web page), if the view position and the orthographic viewing volume is change into each of the following situations, how will the final 2D image change from its original image?

   a. mvMatrix=lookAt(vec3(-4, 0, 0), at, up);   // pMatrix does not change
      pMatrix = ortho(-2, 2, -4, 4, 0, 10); // given
      Shows the yellow rectangle, the left face of the cube. because the eye is now to the left of the cube looking to the center of the cube. The viewing volume is twice as tall (bottom=-4, top=4) as it is wide (left= -2, right=2). The left side of the cube is displayed as a rectangle with width to height ratio of 2:1.

   b. mvMatrix=lookAt(vec3(3, 3, 3), at, up);   // pMatrix does not change
      pMatrix = ortho(-2, 2, -4, 4, 0, 10); // given
      Shows the top, right, and front faces of the cube. Still cube is shown half as tall as it is width and deep, due to the ratio between viewing volume height (8 along y-axis) and width (4 along x-axis).

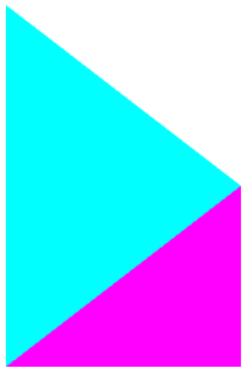


   c. mvMatrix=lookAt(vec3(3, 3, 3, at, up);
      pMatrix=ortho(-3, 3, -3, 3, -1, 1);
      No display, because the cube is outside of the view volume

   d. pMatrix= ortho(-6, 6, -3, 3, 2, 10);   // mvMatrix does not change
      var eye = vec3(4, 4, -4);   // given
      The entire cube is now inside the view volume, between near plane (-2) and far plane (-10). It appears taller. The width of the view volume is doubled and the height of the view volume does not change. Therefore, the height of the cube looks twice as tall as it is wide.

e. pMatrix=ortho(0, 4, 0, 3, 2, 10); // mvMatrix does not change
var eye = vec3(4, 4, -4); // given

Only ¼ of the cube (the upper, right ¼) is inside the view volume and displayed. The width and height ratio of the displayed portion is 3:4 corresponding to the width and height ratio of the viewing volume.



2) Given: mvMatrix=lookAt(vec3(4, 4, - 4), at, up);
pMatrix=ortho(-2, 2, -4, 4, -10, 10);

show:

- the mvMatrix
- the pMatrix
- the coordinates of point F(1, 1, -1) and B(1, 1, 1) when converted into the final clip coordinates. (show intermediate steps in deriving the results)

n=eye-look=[4, 4, -4]
normalized n: [0.577, 0.577, -0.577]
u=upxn = [0, 1, 0] x [0.577, 0.577, -0.577]
normalized u: [-0.707, 0, -0.707]
v=n x u = [0.577, 0.577, -0.577] x [-0.707, 0, -0.707]
normalized v: [-0.408, 0.816, 0.408]

-dot(n, eye) = -6.928
-dot(u, eye) = 0
-dot(v, eye) = 0

$$\text{view matrix} = \begin{bmatrix} -0.707 & 0 & -0.707 & 0 \\ -0.408 & 0.816 & 0.408 & 0 \\ 0.577 & 0.577 & -0.577 & -6.928 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{projection matrix} = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 0 \\ 0 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

M= projection matrix x view matrix= $\begin{bmatrix} -0.35 & 0 & -0.35 & 0 \\ -0.102 & 0.204 & 0.102 & 0 \\ -0.0577 & -0.057 & 0.057 & 0.69 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

F' = M * F = $\begin{bmatrix} -0.35 & 0 & -0.35 & 0 \\ -0.102 & 0.204 & 0.102 & 0 \\ -0.0577 & -0.057 & 0.057 & 0.69 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5.19 \\ 1 \end{bmatrix}$

3) Changing the orthographic viewing volume in problem 2) to a frustum with left=-2, right=2, bottom=-4, top=4 for the near plane, and the near plane at distance 4 and far plane at distance 10 from the eye/camera. How would you call the perspective function to set up the corresponding pMatrix in the .js program?

Convert Frustum(-2, 2, -4, 4, 4, 10) into perspect
Aspect = (right-left)/(top-bottom) = (2-(-2))/(4-(-4)) = 0.5
viewAngle = 2*arctan(1/2*(top-bottom)/N) = 2*arctan(0.5*(4-(-4))/4)=90 degrees

➔ perspect(90, 0.5, 4, 10)

4) With the perspective viewing volume defined in problem 3), what will be the x and y coordinates of points F(1, 1, -1) and B(1, 1, 1) when projected onto the near plane?

F'x = N*Fx/(-Fz) = 4*1/(1)=4
F'y = N*Fy/(-Fz) = 4*1/(1)=4
F': (4, 4)

B'x = N*Bx/(-Bz) = 4*1/(-1)= -4
B'y = N*By/(-Bz) = 4*1/(-1)= -4
B': (-4, -4)