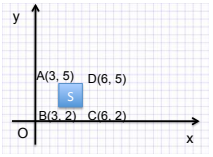
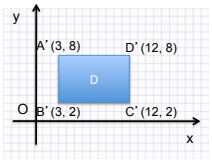
Computer Graphics

Transformation Practices

Translation + Scaling

What is the transformation matrix?
Apply it to point D to see if it arrives at point D' ?





Translation + Scaling

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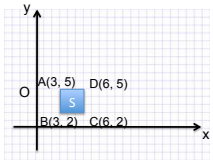
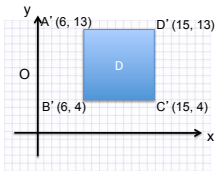




Rotation about a Point

- Point P=(1, 2) rotates about point V=(-2, 3) around the Z-axis for 30 degrees.
  - Compute the Composite Matrix representing this transformation
  - Compute the coordinates for the image point Q

Practice Question

- Point P=(1, 2) rotates about point V=(-2, 3) for 30 degrees.
  - Compute the Composite Matrix representing this transformation
  - Compute the coordinates for the image point Q

$$Q = T_v R T_{-v} = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos 30 & -\sin 30 & 0 \\ \sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.866 & -0.499 & 1.232 \\ 0.499 & 0.866 & 1.402 \\ 0 & 0 & 1 \end{pmatrix}$$

$$Q * P = \begin{pmatrix} 0.866 & -0.499 & -0.098 \\ 0.499 & 0.866 & -1.6339 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.098 \\ 3.634 \\ 1 \end{pmatrix}$$

Composing Affine Transformations (Examples)

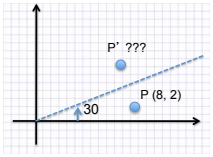
- Reflect across an arbitrary line through the origin O:

  $Q = R(\theta)\, S\, R(-\theta)\, P$

- The rotation transforms the axis to the x-axis, the reflection is a scaling, and the last rotation transforms back to the original axis.

Practice Question

Find the point P(8, 2) reflected across the line from origin that is 30 degree from x-axis

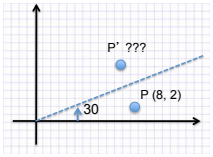


Practice Question

Find the point P(8, 2) reflected across the line from origin that is 30 degree from x-axis

$$Q = R_1 S R_0 = \begin{pmatrix} \cos 30 & \sin 30 & 0 \\ \sin 30 & \cos 30 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(-30) & -\sin(-30) & 0 \\ \sin(-30) & \cos(-30) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.5 & 0.866 & 0 \\ 0.866 & -0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

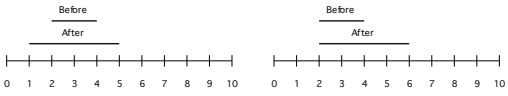
$$P' = Q * P = \begin{pmatrix} 0.5 & 0.866 & 0 \\ 0.866 & -0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 8 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5.73 \\ 5.93 \\ 1 \end{pmatrix}$$



Practice Question

For a line segment A(2, 1) and B(4, 1):

- What transformation matrix is involved in scaling this line by 2 along the X axis?
- What are the resulting coordinates of the two points on the transformed line segment?
- Fixed point scaling, fixed at which point?



Fixed point scaling

Scale by 2 with fixed point = (3,1)

1. Translate the point (3,1) to the origin
2. Scale by 2
3. Translate origin to point (3,1)

$T_v S T_{-v}$

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix}$$

More on Fixed Point Scaling

Scale by 2 with fixed point = (2,1)

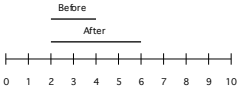
1. Translate the point (2,1) to the origin
2. Scale by 2
3. Translate origin to point (2,1)

3 2 1

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

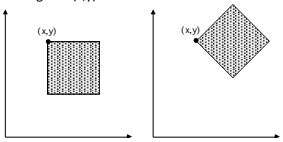
$$\begin{pmatrix} 2 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix}$$



Hint on Rotation about a fixed point

Rotation Of ø Degrees About Point (x,y)

- Translate (x,y) to origin
- Rotate
- Translate origin to (x,y)



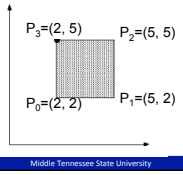
$$\begin{pmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos ø & -\sin ø & 0 \\ \sin ø & \cos ø & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -x \\ 0 & 1 & -y \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos ø & -\sin ø & -x\cos ø + y\sin ø + x \\ \sin ø & \cos ø & -x\sin ø - y\cos ø + y \\ 0 & 0 & 1 \end{pmatrix}$$

You rotate the box by rotating each vertex.

Practice Question

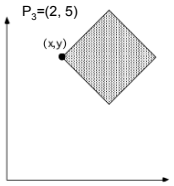
Given the square as shown, show the square after it is rotated 45 degree around P_3:

- What individual transformation matrices are involved ?
- What is the composite transformation matrix?
- What are the coordinates of the 4 points of the square after the transformation?



Practice Question

Show code to draw a square as:



To draw the square as shown, i.e., 45 degree rotated around point P_3 and translated to the final position,

- What are the individual transformation involved ?
- What is the composite transformation matrix?

Practice Question

- Given the unit square centered at the origin point, what does the square look like and where is it located in the coordinate system after the following sequence of transformations (in the order as shown below):
  - Translate the figure along X-axis by 3, along Y-axis by 2
  - Rotate the figure 45 degrees along the Z-axis
  - Scale along X-axis by a factor of 3, along Y-axis by 2
- Compute the composite transformation matrix
- Compute the corner points of the resulting shape