Homework 6 solution

1) Changing the orthographic viewing volume in problem 2) to a frustum with left=-2, right=2, bottom=-4, top=4 for the near plane, and the near plane at distance 4 and far plane at distance 10 from the eye/camera. How would you call the perspective function to set up the corresponding pMatrix in the .js program?

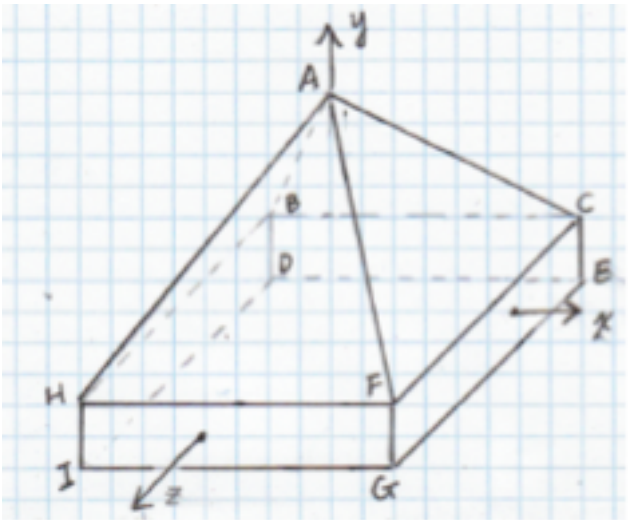
   Convert Frustum(-2, 2, -4, 4, 4, 10) into perspect
   Aspect = (right-left)/(top-bottom) = (2-(-2))/(4-(-4)) = 0.5
   viewAngle = 2*arctan(1/2*(top-bottom)/N) = 2*arctan(0.5*(4-(-4))/4)=90 degrees

   ➔ perspect(90, 0.5, 4, 10)

2) With the perspective viewing volume defined in problem 3), what will be the x and y coordinates of points F(1, 1, -1) and B(1, 1, 1) when projected onto the near plane?

   F'x = N*Fx/(-Fz) = 4*1/(1)=4
   F'y = N*Fy/(-Fz) = 4*1/(1)=4
   F': (4, 4)

   B'x = N*Bx/(-Bz) = 4*1/(-1)= -4
   B'y = N*By/(-Bz) = 4*1/(-1)= -4
   B': (-4, -4)



A=(0, 6, 0), B=(-4, 1, -1), C=(6, 1, -1), D=(-4, -1, -1), E=(6, -1, -1),
F=(6, 1, 4), G=(6, -1, 4), H=(-4, 1, 4), I=(-4, -1, 4)

3) Given the 3D mesh object in the picture above, show:
   a. The vertex list
   b. The normal list. Compute the normals of the faces using Newell's method. Show computation steps involved.
   c. The face list. Each face should include the vertex (index) list, as well as the normal (index) list.

Solution:

a) The Vertex list

Vertext ID	Coordinates
0	(0, 6, 0)
1	(-4, 1, -1)
2	(6, 1, -1)
3	(-4, -1, -1)
4	(6, -1, -1)
5	(6, 1, 4)
6	(6, -1, 4)
7	(-4, 1, 4)
8	(-4, -1, 4)

b) The normal list

2.1) Face FHIG → normal (0, 0, 1)
2.2) Face CFGE → normal (1, 0, 0)
2.3) Face BDIH → normal (-1, 0, 0)
2.4) Face CEDB → normal (0, 0, -1)
2.5) Face DEBI → normal (0, -1, 0)

2.6) Face AFC: apply Newell's method:

$$n_x = \sum_{i=0}^{N-1} (y_i - y_{ni})(z_i + z_{ni})$$

$$n_y = \sum_{i=0}^{N-1} (z_i - z_{ni})(x_i + x_{ni})$$

$$n_z = \sum_{i=0}^{N-1} (x_i - x_{ni})(y_i + y_{ni})$$

In this face, i=0, 1, 2; corresponding points A, F, C

The next point of A is F, the next point of F is C, and the next point of C is A.

i	(x, y, x)
0	(0, 6, 0)
1	(6, 1, 4)
2	(6, 1, -1)

Apply the Newell's method:

$$n_x = (y_0 - y_1)(z_0 + z_1) + (y_1 - y_2)(z_1 + z_2) + (y_2 - y_0)(z_2 + z_0)$$
$$= (6-1)(0+4) + (1-1)(4-1) + (1-6)(-1+0)$$
$$= 20 + 5 = 25$$
$$n_y = (z_0 - z_1)(x_0 + x_1) + (z_1 - z_2)(x_1 + x_2) + (z_2 - z_0)(x_2 + x_0)$$
$$= (0-4)(0+6) + (4-(-1))(6+6) + (-1-0)(6+0)$$
$$= -24 + 60 - 6 = 30$$
$$n_z = (x_0 - x_1)(y_0 + y_1) + (x_1 - x_2)(y_1 + y_2) + (x_2 - x_0)(y_2 + y_0)$$
$$= (0-6)(6+1) + (6-6)(1+1) + (6-0)(1+6)$$
$$= -42 + 42 = 0$$

Normalize the vector (25, 30, 0), the result is: $(\frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}}, 0)$

2.7) Face AHF:

i	(x, y, x)
0	(0, 6, 0)
1	(6, 1, 4)
2	(6, 1, -1)

Apply Newell's methed:

$$n_x = (y_0 - y_1)(z_0 + z_1) + (y_1 - y_2)(z_1 + z_2) + (y_2 - y_0)(z_2 + z_0)$$
$$= (6-1)(0+4) + (1-1)(4+4) + (1-6)(4+0)$$
$$= 20 - 20 = 0$$
$$n_y = (z_0 - z_1)(x_0 + x_1) + (z_1 - z_2)(x_1 + x_2) + (z_2 - z_0)(x_2 + x_0)$$
$$= (0-4)(0-4) + (4-4)(-4+6) + (4-0)(6+0)$$
$$= 16 + 24 = 40$$
$$n_z = (x_0 - x_1)(y_0 + y_1) + (x_1 - x_2)(y_1 + y_2) + (x_2 - x_0)(y_2 + y_0)$$
$$= (0-(-4))(6+1) + (-4-6)(1+1) + (6-0)(4+0)$$
$$= 28 - 20 + 24 = 32$$

normalize (0, 40, 32), the result is $(0, \frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}})$

d. Face ABH

i	(x, y, x)
0	(0, 6, 0)
1	(-4, 1, -1)
2	(-4, 1, 4)

$$n_x = (y_0 - y_1)(z_0 + z_1) + (y_1 - y_2)(z_1 + z_2) + (y_2 - y_0)(z_2 + z_0)$$
$$= (6-1)(0-1) + (1-1)(-1+4) + (1-6)(4+0)$$
$$= -5 - 20 = -25$$
$$n_y = (z_0 - z_1)(x_0 + x_1) + (z_1 - z_2)(x_1 + x_2) + (z_2 - z_0)(x_2 + x_0)$$
$$= (0-(-1))(0-4) + (-1-4)(-4-4) + (4-0)(-4+0)$$
$$= -4 + 40 - 16 = 20$$
$$n_z = (x_0 - x_1)(y_0 + y_1) + (x_1 - x_2)(y_1 + y_2) + (x_2 - x_0)(y_2 + y_0)$$
$$= (0-(-4))(6+1) + (-4-(-4))(1+1) + (-4-0)(1+6)$$
$$= 28 - 28 = 0$$

normalize (25, 20, 0), the result is $(-\frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}}, 0)$

e. Face ACB

i	(x, y, x)
0	(0, 6, 0)
1	(6, 1, -1)

2	(-4, 1, -1)

$$n_x = (y_0 - y_1)(z_0 + z_1) + (y_1 - y_2)(z_1 + z_2) + (y_2 - y_0)(z_2 + z_0)$$
$$= (6-1)(0-1) + (1-1)(-1-1) + (1-6)(-1+0)$$
$$= -5 + 5 = 0$$
$$n_y = (z_0 - z_1)(x_0 + x_1) + (z_1 - z_2)(x_1 + x_2) + (z_2 - z_0)(x_2 + x_0)$$
$$= (0-(-1))(0+6) + (-1-(-1))(6-4) + (-1-0)(-4+0)$$
$$= 6 + 4 = 10$$
$$n_z = (x_0 - x_1)(y_0 + y_1) + (x_1 - x_2)(y_1 + y_2) + (x_2 - x_0)(y_2 + y_0)$$
$$= (0-6)(6+1) + (6-(-4))(1+1) + (-4-0)(1+6)$$
$$= -42 + 20 - 28 = -50$$

Normalize (0, 10, -50), the result is $(0, \frac{1}{\sqrt{26}}, -\frac{5}{\sqrt{26}})$

Therefore the normal list is:

ID	Normal
0	(0, 0, 1)
1	(1, 0, 0)
2	(-1, 0, 0)
3	(0, 0, -1)
4	(0, -1, 0)
5	$(\frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}}, 0)$
6	$(0, \frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}})$
7	$(-\frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}}, 0)$
8	$(0, \frac{1}{\sqrt{26}}, -\frac{5}{\sqrt{26}})$

c) The Face list

Face ID	Vertex ID	Normal ID
0	4, 7, 8, 6	0, 0, 0, 0
1	2, 5, 6, 4	1, 1, 1, 1
2	1, 3, 8, 7	2, 2, 2, 2
3	2, 4, 3, 1	3, 3, 3, 3
4	3, 4, 1, 8	4, 4, 4, 4
5	0, 5, 2	5, 5, 5
6	0, 7, 5	6, 6, 6
7	0, 1, 7	7, 7, 7
8	0, 2, 1	8, 8, 8