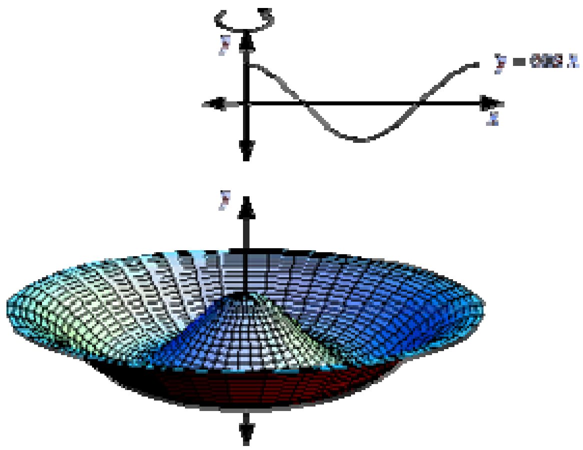
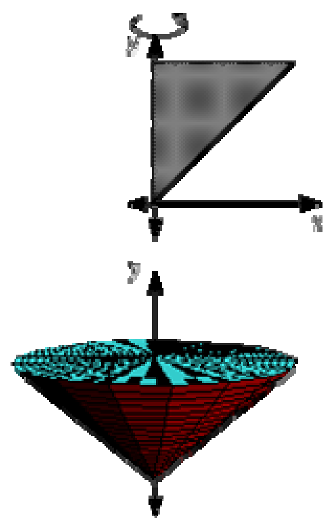
Surface of Revolution

A surface that is obtained by rotating a plane curve in space about an axis coplanar to the curve.

Some pictures of surfaces of revolution appear below.



This surface of revolution is obtained by revolving the graph of $y = \cos x$ about the y-axis



The shaded region is revolved about the y-axis.

This cone, a solid of revolution, is the result.

Each of the figures above were obtained by rotating a 2D curve about the y axis. The process to generate the surface involves:

- Determining points defining the curve in the (x, y) plane say (x_i, y_i, 0) for i=1 to N.
- Then, for each two points, they are rotated about the y axis by a small angle theta.
- This rotation forms a polygon consisting of four points – the original two points and the points after they have been rotated.
- Next, the normal to the polygon is found by using cross products.
- The polygon is then drawn.

- Rotate the two new points about the y-axis by the same amount to create two newer points.
- Form the polygon between the new points and the newer points, determine the normal and draw it.
- Continue this process until a rotation of 360 degrees has been achieved.
- Go to the next two points in the 2D curve and repeat the steps above until all of the points have been used.

To perform a rotation about the y-axis, use the following rotation matrix:

$$\begin{bmatrix} \cos(theta) & 0 & \sin(theta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(theta) & 0 & \cos(theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

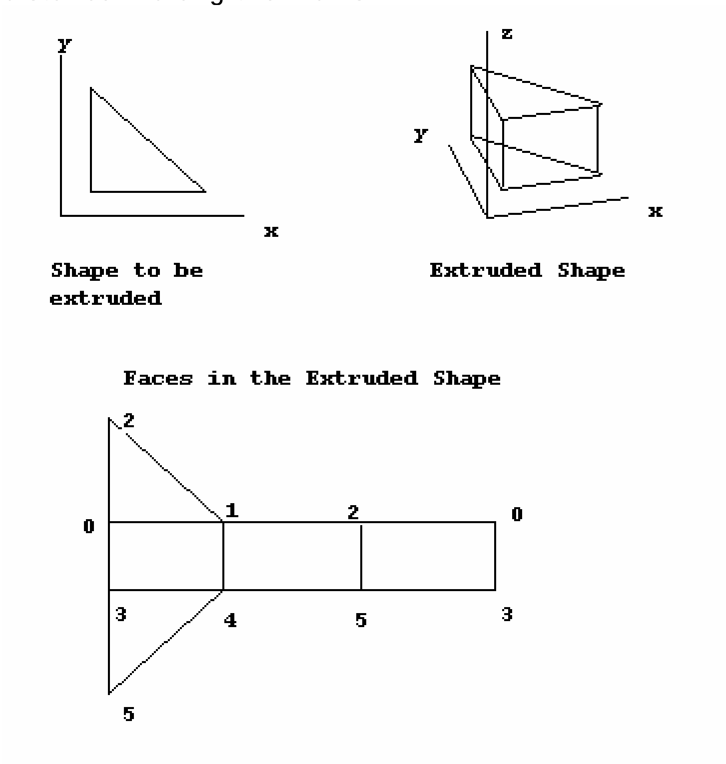
If the matrix is applied to the point (x_i, y_i, 0), the results is ($x_i$cos(theta), y_i, -$x_i$sin(theta)).

Thus the vertices of the polygon formed by rotating the points (x_1, y_1, 0), (x_0, y_0, 0) about the y axis would be (x_1, y_1, 0), (x_0, y_0, 0), ($x_0$cos(theta), y_0, -$x_0$sin(theta)), and ($x_1$cos(theta), y_1, -$x_1$sin(theta)).

See the examples on the web site for creating surfaces of revolution using OpenGL.

Extruded Shapes

A large class of shapes can be generated by **extruding** or **sweeping**, a 2D shape through space. The simplest extruded shape is a prism which is formed by sweeping a 2D polygon in a straight line. The example below shows what would happen if a triangle were extruded by a certain distance H along the z-axis.



The figure at the upper left shows the 2D triangle to be extruded. The figure at the upper right shows what the extruded shape (a prism) would look like after it has been swept by a distance H along the z-axis.

This prism has five faces the bottom, the top and the three sides. To create the polygonal mesh of the prism, the prism is “unfolded” into the model shown in the figure at the bottom to reveal the five faces as seen from the outside. There are three rectangular sides, plus the **base** and the top **cap**.

Because each face is flat, the same normal vector can be associated with each vertex of a face. The cross product of two vectors can be used to determine the normal vectors.

Building a Mesh for the Prism is relatively easy. The prism’s base is a polygon formed by the vertices (x_i, y_i) for i=0, 1, 2. The cap is formed by (x_i, y_i, H) for i=3, 4, 5.

For the jth wall (j = 0, 1, 2), we create a face with the four vertices having indices j, j+3, next(j) + 3, next (j) where next (j) = (j+1)% 2. Thus the first face (j = 0) has vertices 0, 3, 4, 1. The second face (j=1) gives vertices 1, 4, 5, 2 and the last face (j = 2) gives vertices 2, 5, 3, 0.

For the general case, see pages 310-311 in Hill. Also, for an example of creating an extruded shape using OpenGL, see the web site.