

### Clipping Lines

- We want to draw only the parts of lines that are inside the world window.
- To do this, we need to replace line portions outside the window by lines along the window boundaries.
- The process is called clipping the lines.

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### Clipping

- The method we will use is called Cohen-Sutherland line clipping.
- There are 2 trivial cases:
  - a line AB totally inside the window, which we draw all of
  - a line CD totally outside the window, which we do not draw at all.

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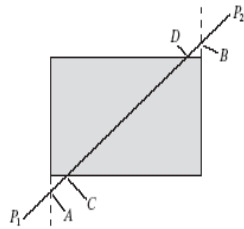
### Clipping

- A line consists of 2 endpoints (vertices),  $P_1$  and  $P_2$ .
- If we do not have a trivial case, we must alter the vertices to the points where the line intersects the window boundaries (replace  $P_1$  by A).

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## A Line Needing 4 Clips

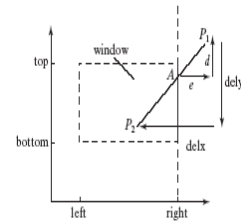
- Clipping order:
  - Clip from the **left**, intersection A
    - $x = W.l$
  - Clip from the **right**, intersection B
    - $x = W.r$
  - Clip from the **bottom**, intersection C
    - $y = W.b$
  - Clip from the **top**, intersection D
    - $y = W.t$



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## Clipping

- In the diagram,  $d/dely = e/delx$  (similar triangles).
- Also,
  - $delx = p_1.x - p_2.x$ ,
  - $dely = p_1.y - p_2.y$  and
  - $e = p_1.x - W.r$ .
- Obviously,  $A.x = W.r$ .
- $A.y = ?$



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## Cohen Sutherland Algorithm

- [www.cs.mtsu.edu/~cen/private/lectures/ClippingAlg.html](http://www.cs.mtsu.edu/~cen/private/lectures/ClippingAlg.html)
- Determine if the line needs clipping
  - Compare TFFT of P1 and P2
- If it does, choose which of the 4 sides to clip against
  - Determine which of the two points need clipping
  - Determine which side does clipping occur
  - Test all 4 sides to make sure the line is fully clipped

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## Clipping by hand...

- Given a window (50,120,0,100)
- Trace the algorithm to clip the following line segments:
  - P1 (50,40), p2 (100,20)
  - P1 (10,170), p2 (100,0)
  - P1 (20,-10), p2 (200,200)
- Give the new endpoints for any clipped segments

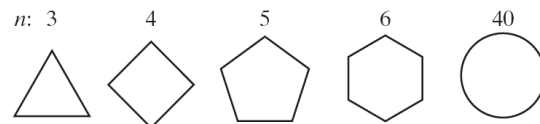
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## Drawing Regular Polygons, Circles, and Arcs

- A polygon is **simple** if no two of its edges cross each other. More precisely, only adjacent edges can touch and only at their shared endpoint.
- A polygon is **regular** if it is simple, if all its sides have equal length, and if adjacent sides meet at equal interior angles.
- We give the name ***n*-gon** to a regular polygon having *n* sides; familiar examples are the 4-gon (a square), an 8-gon (a regular octagon) and so on.

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## Regular Polygons



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## Drawing Circles and Arcs

- Two methods:
  - **The center is given, along with a point on the circle.**
    - Here `drawCircle( IntPoint center, int radius)` can be used as soon as the radius is known.
    - If *c* is the center and *p* is the given point on the circle, the radius is simply the distance from *c* to *p*, found using the usual Pythagorean Theorem.

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## Drawing Circles and Arcs

– **Three points are given through which the circle must pass.**

- It is known that a unique circle passes through any three points that don't lie in a straight line.
- Finding the center and radius of this circle is discussed in Chapter 4.

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## Parametric Curves

- Three forms of equation for a given curve:
  - **Explicit:** e.g.,  $y = m*x + b$
  - **Implicit:**  $F(x, y) = 0$ ; e.g.,  $y - m*x - b = 0$
  - **Parametric:**  $x = x(t)$ ,  $y = y(t)$   
 $t$  is a parameter; frequently,  $0 \leq t \leq 1$ .
- Example: Line:  
 $F(x, y) = A_x + (B_x - A_x) * t$ 
  - A and B are 2D points with  $x$  and  $y$  values. A and B are on the straight line
  - The parametric form is
    - $x(t) = A_x + (B_x - A_x) * t$
    - $y(t) = A_y + (B_y - A_y) * t$

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## Specific Parametric Forms

- line:**

$$x(t) = A_x + (B_x - A_x) * t, \quad t \geq 0$$

$$y(t) = A_y + (B_y - A_y) * t$$
- circle:**

$$x(t) = r * \cos(t), \quad 0 \leq t \leq 2\pi$$

$$y(t) = r * \sin(t)$$
- ellipse:**

$$x(t) = W * \cos(t), \quad 0 \leq t \leq 2\pi$$

$$y(t) = H * \sin(t)$$

—  $W$  and  $H$  are half-width and half-height.

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## Finding Implicit Form from Parametric Form

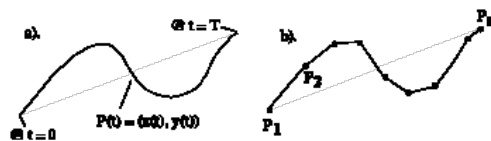
- Combine the  $x(t)$  and  $y(t)$  equations to eliminate  $t$ .
- Example:  
 ellipse:
 
$$x = W * \cos(t)$$

$$y = H * \sin(t)$$
  - $x^2 = W^2 \cos^2(t)$ ,  $y^2 = H^2 \sin^2(t)$
  - Dividing by the  $W$  or  $H$  factors and adding gives  $(x/W)^2 + (y/H)^2 = 1$ , the implicit form.

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## Drawing Parametric Curves

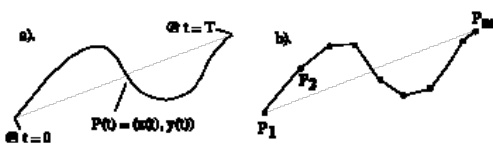
- For a curve  $C$  with the parametric form  $P(t) = (x(t), y(t))$  as  $t$  varies from 0 to  $T$ , we use **samples** of  $P(t)$  at closely spaced instants.



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## Drawing Parametric Curves

- The position  $P_i = P(t_i) = (x(t_i), y(t_i))$  is calculated for a sequence  $\{t_i\}$  of times.
- The curve  $P(t)$  is approximated by the polyline based on this sequence of points  $P_i$ .



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## Drawing Parametric Curves

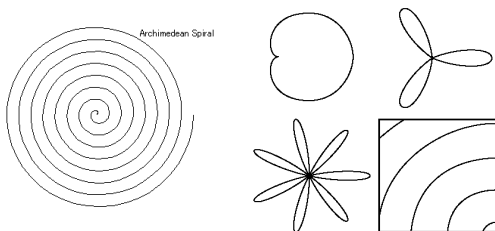
- Code:

```
// draw the curve (x(t), y(t)) using
// the array t[0],...,t[n-1] of sample times
glBegin(GL_LINES);
for(int i = 0; i < n; i++)
    glVertex2f((x(t[i]), y(t[i]));
glEnd();
```

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## Shapes as Parametric Curves

- For drawing purposes, parametric forms circumvent all of the difficulties of implicit and explicit forms.
- Cardioid, 2 rose curves, Archimedean spiral



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## Polar Coordinates Parametric Form

- **General form:**  $x = f(\theta) \cos(\theta)$   
 $y = f(\theta) \sin(\theta)$
- **cardioid:**  $f(\theta) = K(1 + \cos(\theta))$ ,  $0 \leq \theta \leq 2\pi$
- **rose:**  $f(\theta) = K \cos(n\theta)$ ,  $0 \leq \theta \leq 2n\pi$ , where n is number of petals (n odd) or twice the number of petals (n even)
- **spirals:** Archimedean,  $f(\theta) = K\theta$   
Logarithmic,  $f(\theta) = Ke^{a\theta}$
- K is a scale factor for the curves.

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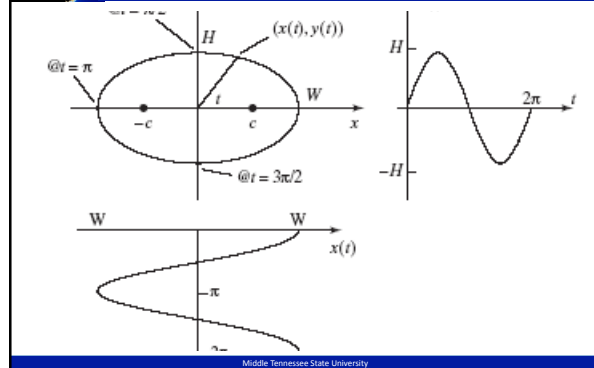
## Polar coordinates Parametric Form

— **conic sections** (ellipse, hyperbola, circle, parabola):  $f(\theta) = (1 \pm e \cos \theta)^{-1}$

- $e$  is eccentricity:
  - 1 : parabola;
  - 0 : circle;
  - between 0 and 1, ellipse;
  - greater than 1, hyperbola

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## Example for Ellipse



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