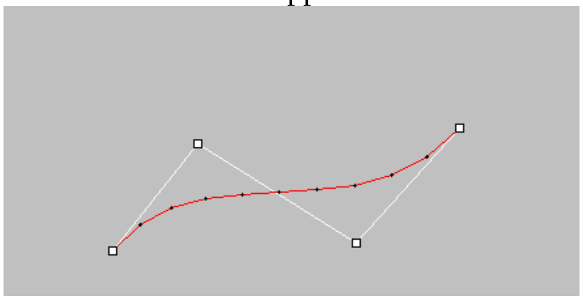
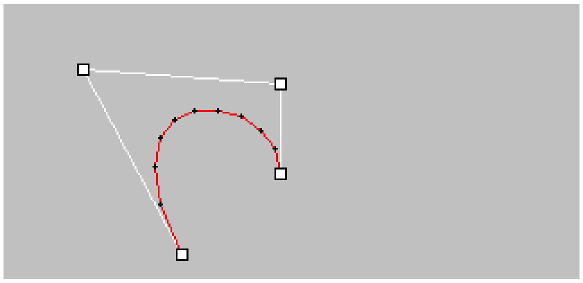
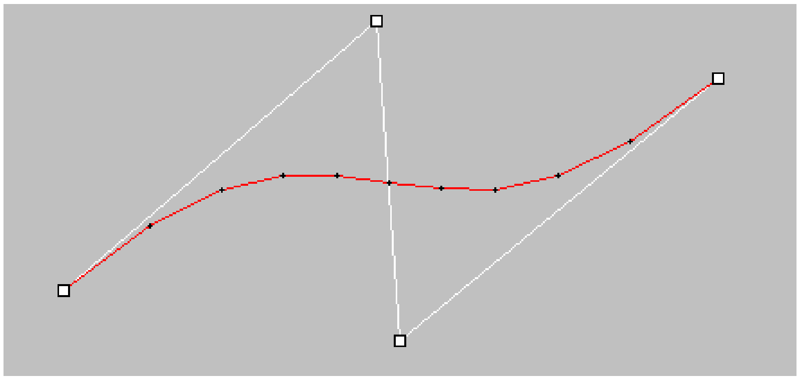
Bezier Curve Notes

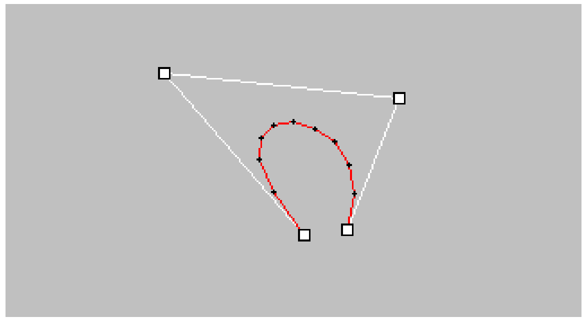
In the middle 1960's, P.E. Bezier of Renault, the major French automobile manufacturer, developed a curve "fitting" method which works for two or three dimensions equally well. The Bezier curve allows the user a feel for the relation between input and output. This enables him/her to use the program as an artist, stylist, or designer, varying curve shape and order by the use of easily controlled parameters until the output matches the desired shape.

A Bezier curve is associated with the "vertices" of a polygon which uniquely define the curve shape. Only the first and last vertices of the polygon actually lie on the curve; however, the other vertices of the polygon define the derivatives, order, and shape of the curve. Some screen shots of Bezier curves appear below.









Since the curve shape tends to follow the polygon shape, changing the vertices of this polygon gives the user a much greater intuitive feel for the input/output relationships. All that is necessary to increase the order of any curve segment is to specify another vertex.

The Bezier polynomial is related to the Bernstein polynomial. Thus the Bezier curve is said to have a Bernstein basis. The basis function is given by

$$J_{N,i}(t) = \binom{N}{i} t^i (1-t)^{N-i}$$

where

$$\binom{N}{i} = \frac{N!}{i!(N-i)!}$$

N is the degree of the polynomial and i is the particular vertex in the ordered set (from 0 to N). The vertices are $P_0, P_1, \ldots, P_N$ where $P_i = (x_i, y_i, z_i)$ or $P_i = (x_i, y_i)$ depending on whether the curve is to be two or three dimensions. The curve points are given by

$$\sum_{i=0}^{N} P_i J_{N,i}(t) \qquad 0 \leq t \leq 1$$