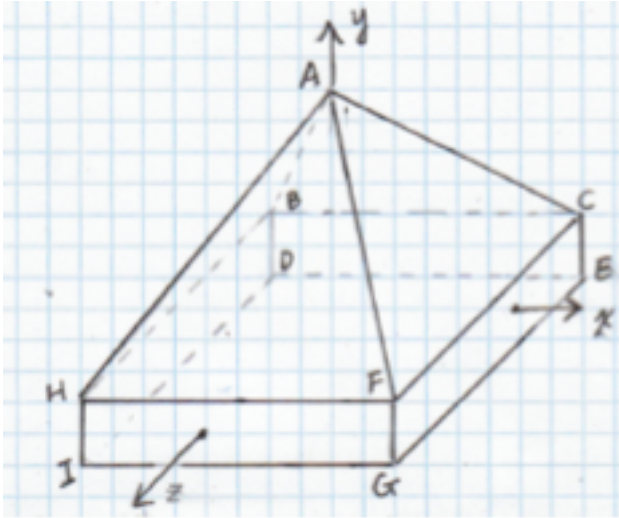


CSCI 4250/5250 solution
Homework 7



$A=(0, 6, 0)$, $B=(-4, 1, -1)$, $C=(6, 1, -1)$, $D=(-4, -1, -1)$, $E=(6, -1, -1)$,
 $F=(6, 1, 4)$, $G=(6, -1, 4)$, $H=(-4, 1, 4)$, $I=(-4, -1, 4)$

1. Given the 3D mesh object in the picture above, show:
 - a. The vertex list
 - b. The normal list. Compute the normals of the faces using Newell's method. Show computation steps involved.
 - c. The face list. Each face should include the vertex (index) list, as well as the normal (index) list.

Solution:

1. The Vertex list

Vertex ID	Coordinates
0	(0, 6, 0)
1	(-4, 1, -1)
2	(6, 1, -1)
3	(-4, -1, -1)
4	(6, -1, -1)
5	(6, 1, 4)
6	(6, -1, 4)
7	(-4, 1, 4)
8	(-4, -1, 4)

2. The normal list

- 2.1) Face FHIG → normal (0, 0, 1)
 2.2) Face CFGE → normal (1, 0, 0)
 2.3) Face BDIH → normal (-1, 0, 0)
 2.4) Face CEDB → normal (0, 0, -1)
 2.5) Face DEBI → normal (0, -1, 0)

2.6) Face AFC: apply Newell's method:

$$n_x = \sum_{i=0}^{N-1} (y_i - y_{ni})(z_i + z_{ni})$$

$$n_y = \sum_{i=0}^{N-1} (z_i - z_{ni})(x_i + x_{ni})$$

$$n_z = \sum_{i=0}^{N-1} (x_i - x_{ni})(y_i + y_{ni})$$

In this face, $i=0, 1, 2$; corresponding points A, F, C

The next point of A is F, the next point of F is C, and the next point of C is A.

i	(x, y, z)
0	(0, 6, 0)
1	(6, 1, 4)
2	(6, 1, -1)

Apply the Newell's method:

$$\begin{aligned} n_x &= (y_0 - y_1)(z_0 + z_1) + (y_1 - y_2)(z_1 + z_2) + (y_2 - y_0)(z_2 + z_0) \\ &= (6 - 1)(0 + 4) + (1 - 1)(4 - 1) + (1 - 6)(-1 + 0) \\ &= 20 + 5 = 25 \end{aligned}$$

$$\begin{aligned} n_y &= (z_0 - z_1)(x_0 + x_1) + (z_1 - z_2)(x_1 + x_2) + (z_2 - z_0)(x_2 + x_0) \\ &= (0 - 4)(0 + 6) + (4 - (-1))(6 + 6) + (-1 - 0)(6 + 0) \\ &= -24 + 60 - 6 = 30 \end{aligned}$$

$$\begin{aligned} n_z &= (x_0 - x_1)(y_0 + y_1) + (x_1 - x_2)(y_1 + y_2) + (x_2 - x_0)(y_2 + y_0) \\ &= (0 - 6)(6 + 1) + (6 - 6)(1 + 1) + (6 - 0)(1 + 6) \\ &= -42 + 42 = 0 \end{aligned}$$

Normalize the vector (25, 30, 0), the result is: $(\frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}}, 0)$

2.7) Face AHF:

i	(x, y, z)
0	(0, 6, 0)
1	(6, 1, 4)
2	(6, 1, -1)

Apply Newell's method:

$$\begin{aligned}
n_x &= (y_0 - y_1)(z_0 + z_1) + (y_1 - y_2)(z_1 + z_2) + (y_2 - y_0)(z_2 + z_0) \\
&= (6 - 1)(0 + 4) + (1 - 1)(4 + 4) + (1 - 6)(4 + 0) \\
&= 20 - 20 = 0
\end{aligned}$$

$$\begin{aligned}
n_y &= (z_0 - z_1)(x_0 + x_1) + (z_1 - z_2)(x_1 + x_2) + (z_2 - z_0)(x_2 + x_0) \\
&= (0 - 4)(0 - 4) + (4 - 4)(-4 + 6) + (4 - 0)(6 + 0) \\
&= 16 + 24 = 40
\end{aligned}$$

$$\begin{aligned}
n_z &= (x_0 - x_1)(y_0 + y_1) + (x_1 - x_2)(y_1 + y_2) + (x_2 - x_0)(y_2 + y_0) \\
&= (0 - (-4))(6 + 1) + (-4 - 6)(1 + 1) + (6 - 0)(4 + 0) \\
&= 28 - 20 + 24 = 32
\end{aligned}$$

normalize (0, 40, 32), the result is $(0, \frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}})$

2.8 Face ABH

i	(x, y, z)
0	(0, 6, 0)
1	(-4, 1, -1)
2	(-4, 1, 4)

$$\begin{aligned}
n_x &= (y_0 - y_1)(z_0 + z_1) + (y_1 - y_2)(z_1 + z_2) + (y_2 - y_0)(z_2 + z_0) \\
&= (6 - 1)(0 - 1) + (1 - 1)(-1 + 4) + (1 - 6)(4 + 0) \\
&= -5 - 20 = -25
\end{aligned}$$

$$\begin{aligned}
n_y &= (z_0 - z_1)(x_0 + x_1) + (z_1 - z_2)(x_1 + x_2) + (z_2 - z_0)(x_2 + x_0) \\
&= (0 - (-1))(0 - 4) + (-1 - 4)(-4 - 4) + (4 - 0)(-4 + 0) \\
&= -4 + 40 - 16 = 20
\end{aligned}$$

$$\begin{aligned}
n_z &= (x_0 - x_1)(y_0 + y_1) + (x_1 - x_2)(y_1 + y_2) + (x_2 - x_0)(y_2 + y_0) \\
&= (0 - (-4))(6 + 1) + (-4 - (-4))(1 + 1) + (-4 - 0)(1 + 6) \\
&= 28 - 28 = 0
\end{aligned}$$

normalize (25, 20, 0), the result is $(-\frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}}, 0)$

2.9 Face ACB

i	(x, y, z)
0	(0, 6, 0)
1	(6, 1, -1)
2	(-4, 1, -1)

$$\begin{aligned}
n_x &= (y_0 - y_1)(z_0 + z_1) + (y_1 - y_2)(z_1 + z_2) + (y_2 - y_0)(z_2 + z_0) \\
&= (6 - 1)(0 - 1) + (1 - 1)(-1 - 1) + (1 - 6)(-1 + 0) \\
&= -5 + 5 = 0
\end{aligned}$$

$$\begin{aligned}
n_y &= (z_0 - z_1)(x_0 + x_1) + (z_1 - z_2)(x_1 + x_2) + (z_2 - z_0)(x_2 + x_0) \\
&= (0 - (-1))(0 + 6) + (-1 - (-1))(6 - 4) + (-1 - 0)(-4 + 0) \\
&= 6 + 4 = 10
\end{aligned}$$

$$\begin{aligned}
n_z &= (x_0 - x_1)(y_0 + y_1) + (x_1 - x_2)(y_1 + y_2) + (x_2 - x_0)(y_2 + y_0) \\
&= (0 - 6)(6 + 1) + (6 - (-4))(1 + 1) + (-4 - 0)(1 + 6) \\
&= -42 + 20 - 28 = -50
\end{aligned}$$

Normalize (0, 10, -50), the result is $(0, \frac{1}{\sqrt{26}}, -\frac{5}{\sqrt{26}})$

Therefore the normal list is:

ID	Normal
0	(0, 0, 1)
1	(1, 0, 0)
2	(-1, 0, 0)
3	(0, 0, -1)
4	(0, -1, 0)
5	$(-\frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}}, 0)$
6	$(0, \frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}})$
7	$(-\frac{5}{\sqrt{41}}, \frac{4}{\sqrt{41}}, 0)$
8	$(0, \frac{1}{\sqrt{26}}, -\frac{5}{\sqrt{26}})$

3. The Face list

Face ID	Vertex ID	Normal ID
0	4, 7, 8, 6	0, 0, 0, 0
1	2, 5, 6, 4	1, 1, 1, 1
2	1, 3, 8, 7	2, 2, 2, 2
3	2, 4, 3, 1	3, 3, 3, 3
4	3, 4, 1, 8	4, 4, 4, 4
5	0, 5, 2	5, 5, 5
6	0, 7, 5	6, 6, 6
7	0, 1, 7	7, 7, 7
8	0, 2, 1	8, 8, 8