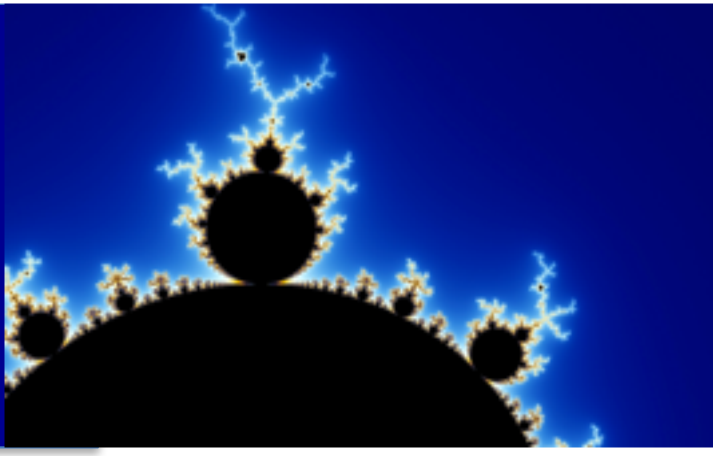
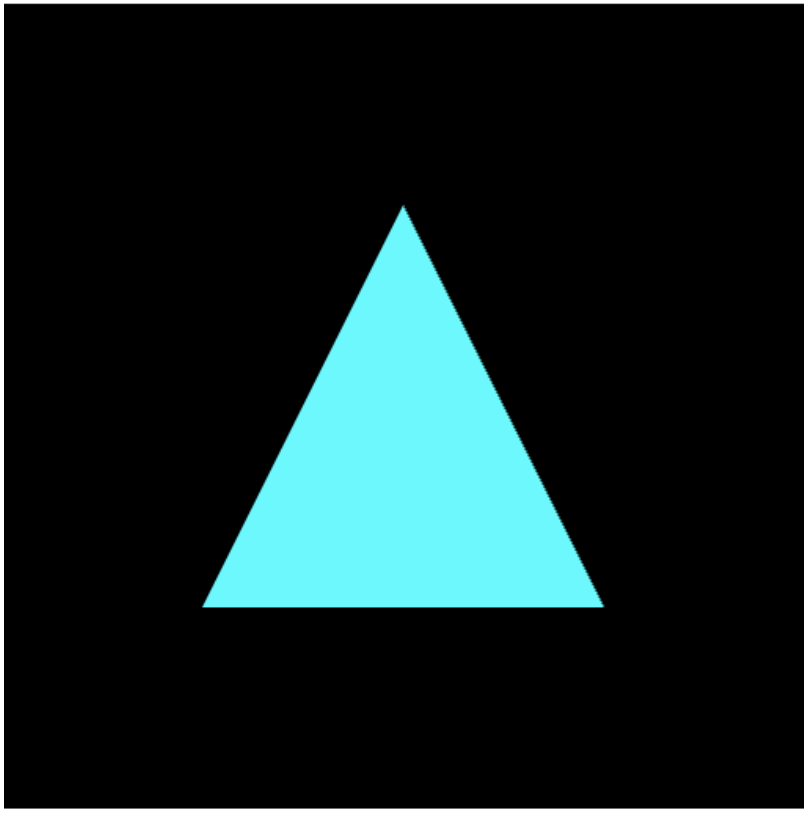
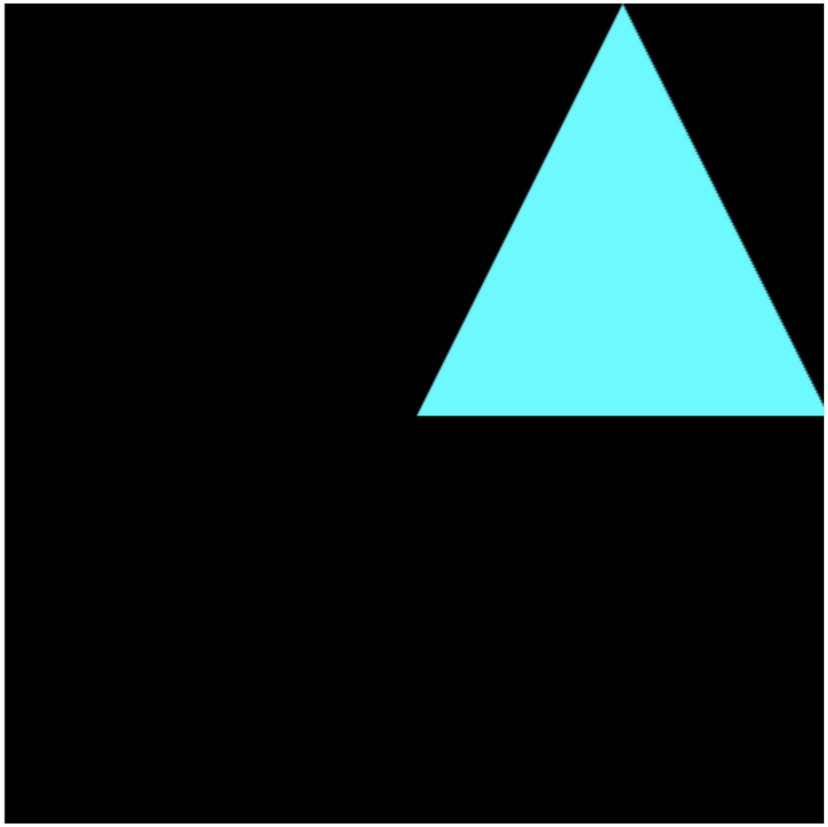
Computer Graphics



A First Look at Transformation

Translate a Triangle



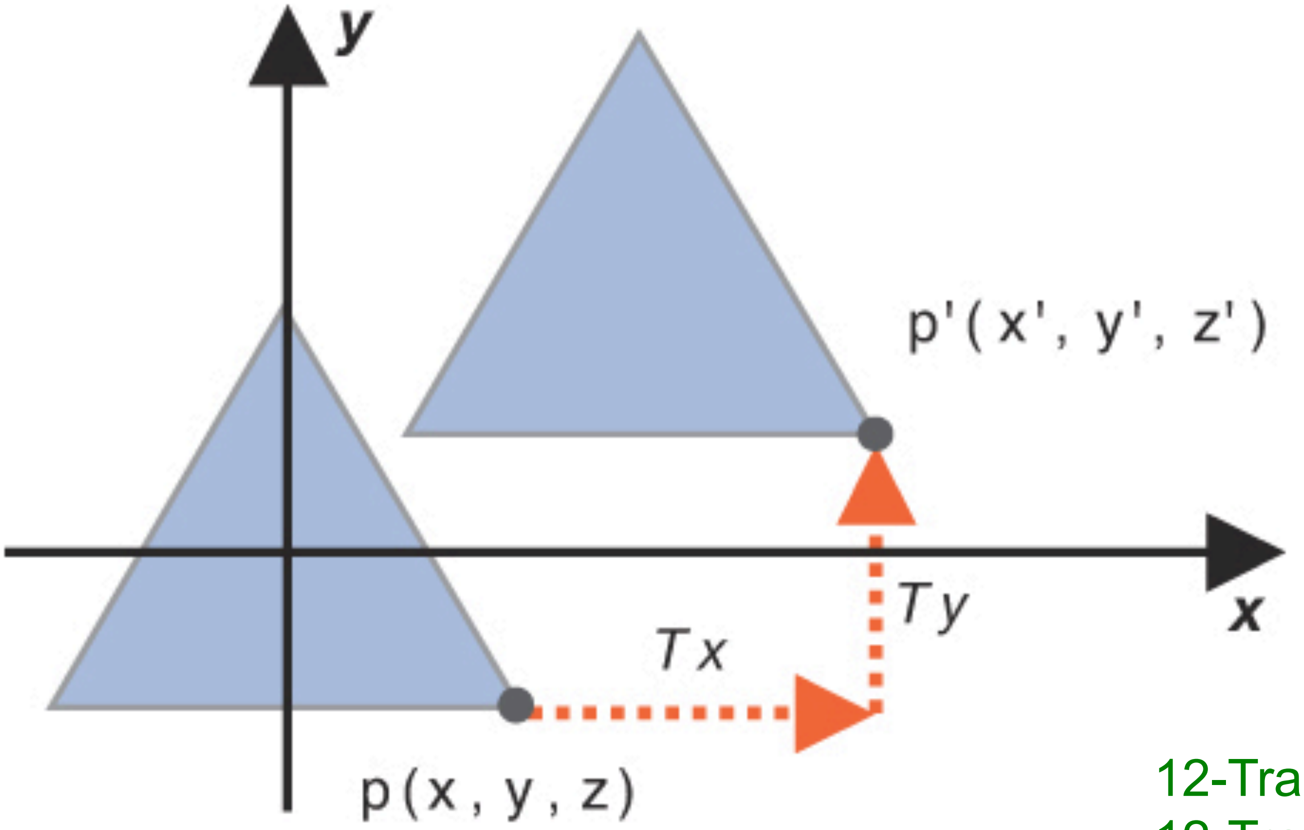


Translate a triangle

$$x' = x + Tx$$

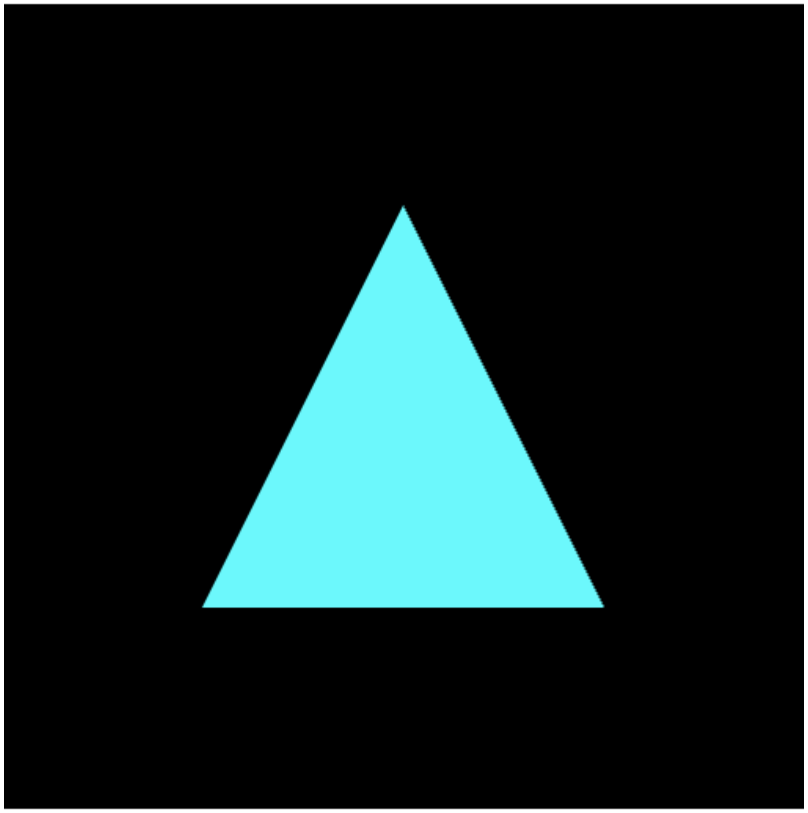
$$y' = y + Ty$$

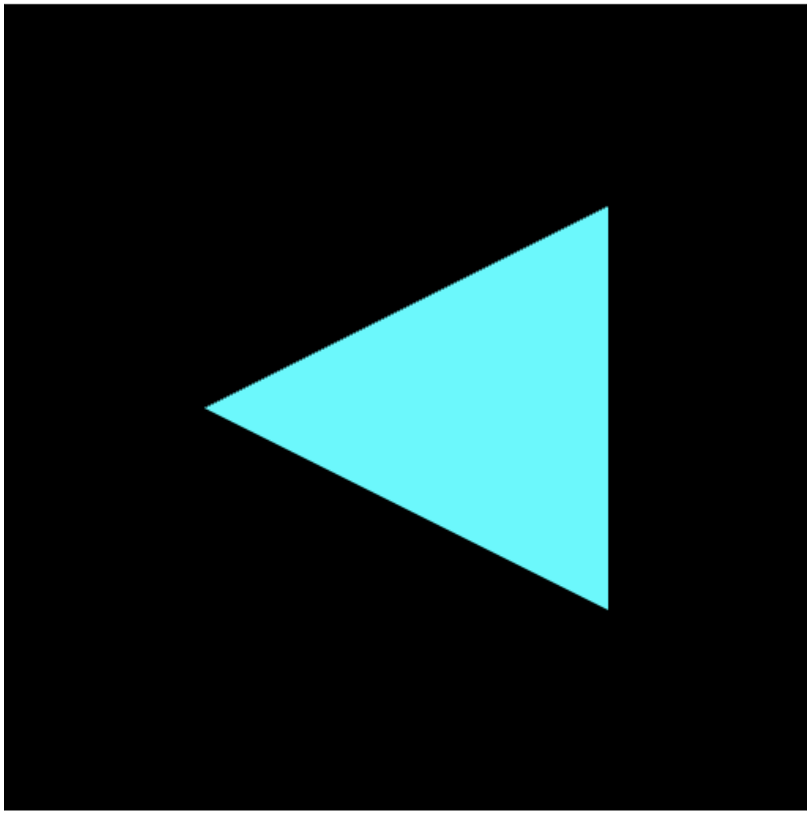
$$z' = z + Tz$$



12-TranslateTriangle.html
12-TranslateTriangle.js

Rotate a Triangle

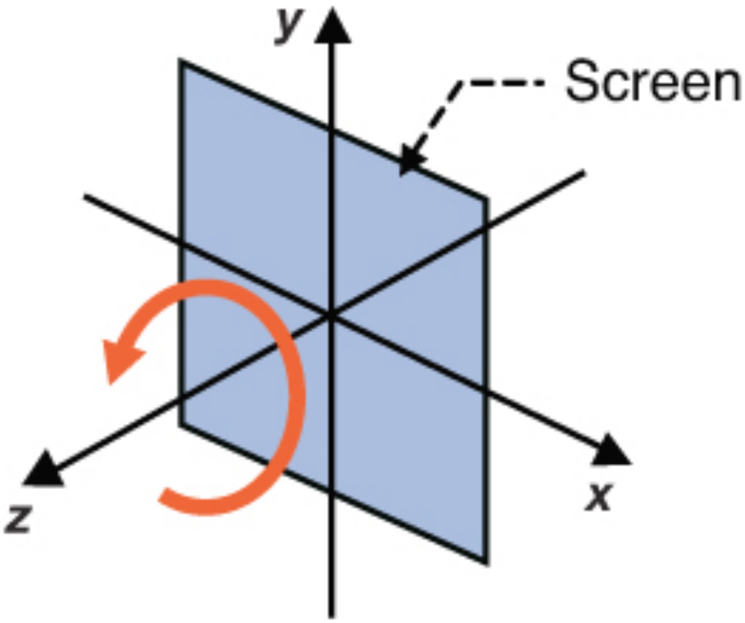


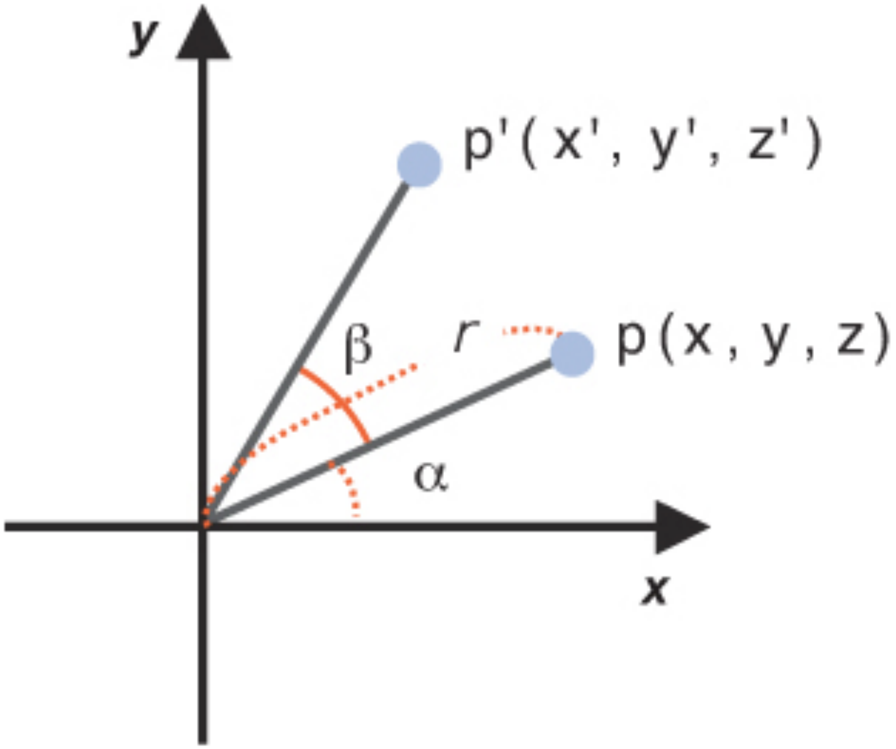


Rotate a Triangle

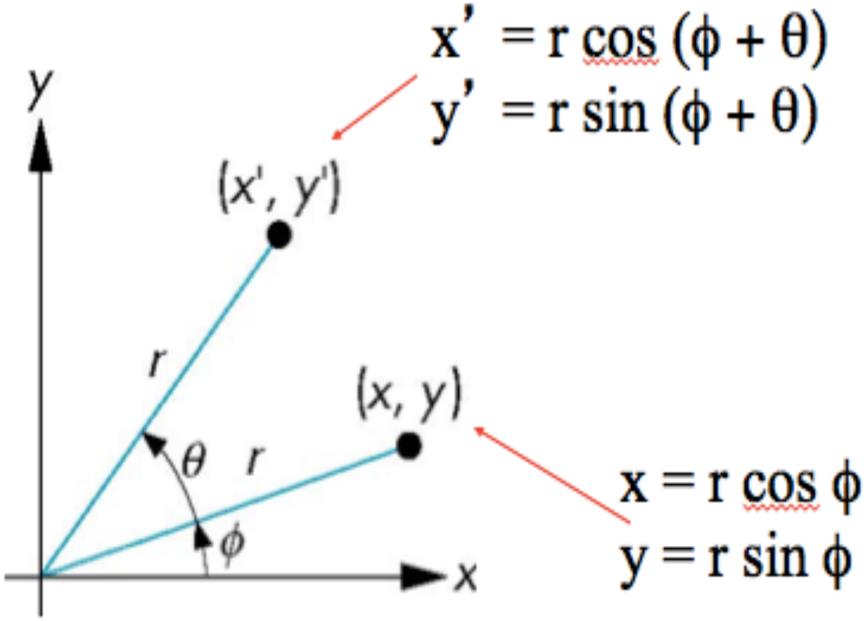
Rotation Angle β:

Positive ←→ Counter Clockwise





Rotate a Triangle



$x' = r\cos(\phi + \theta)$
$y' = r\sin(\phi + \theta)$

$x = r\cos\phi$
$y = r\sin\phi$

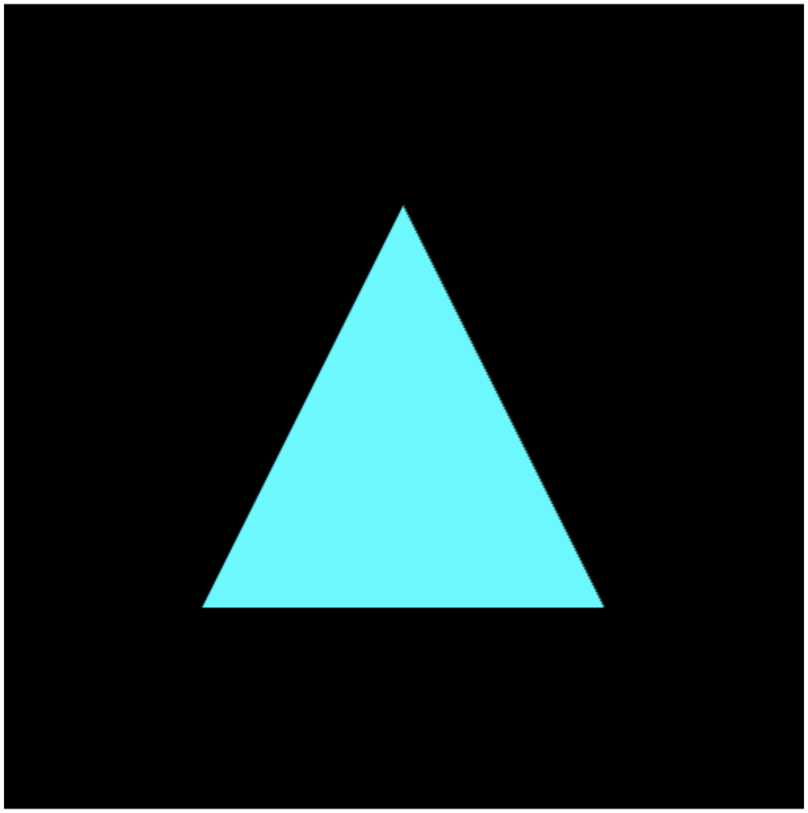
$\cos(\theta + \Phi) = \cos(\theta)\cos(\Phi) - \sin(\theta)\sin(\Phi);$
$\sin(\theta + \Phi) = \sin(\theta)\cos(\Phi) + \cos(\theta)\sin(\Phi).$

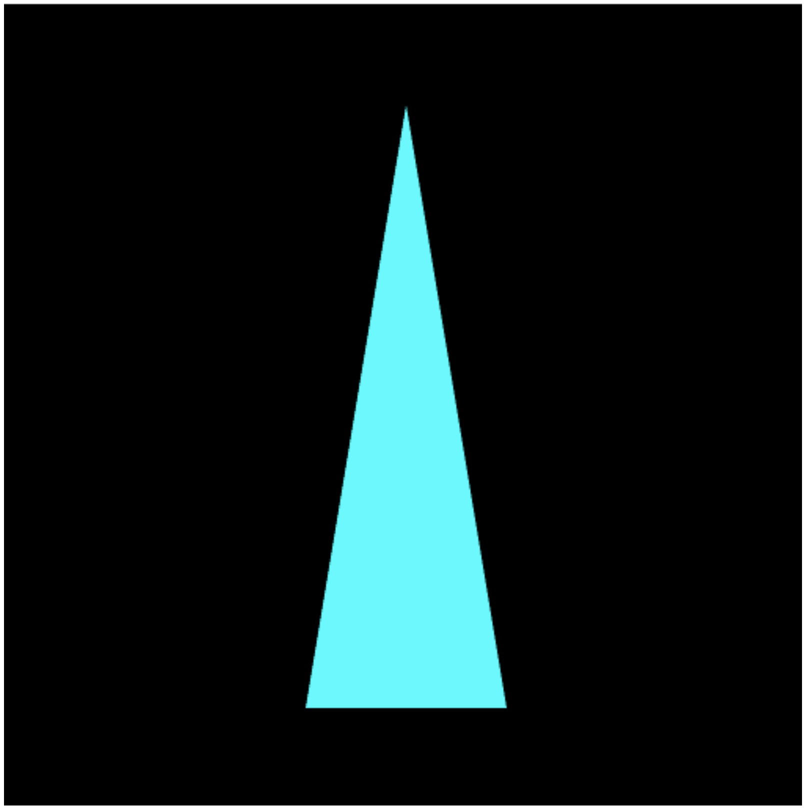
$\Downarrow$

$x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\cos\theta$
$z' = z$

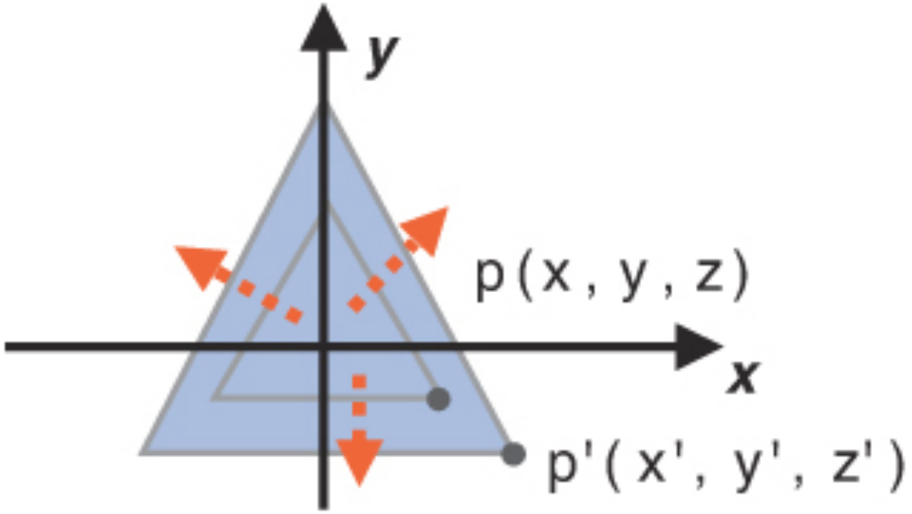
12-RotatedTriangle.html
12-RotatedTriangle.js

Scale a Triangle





Scale a Triangle

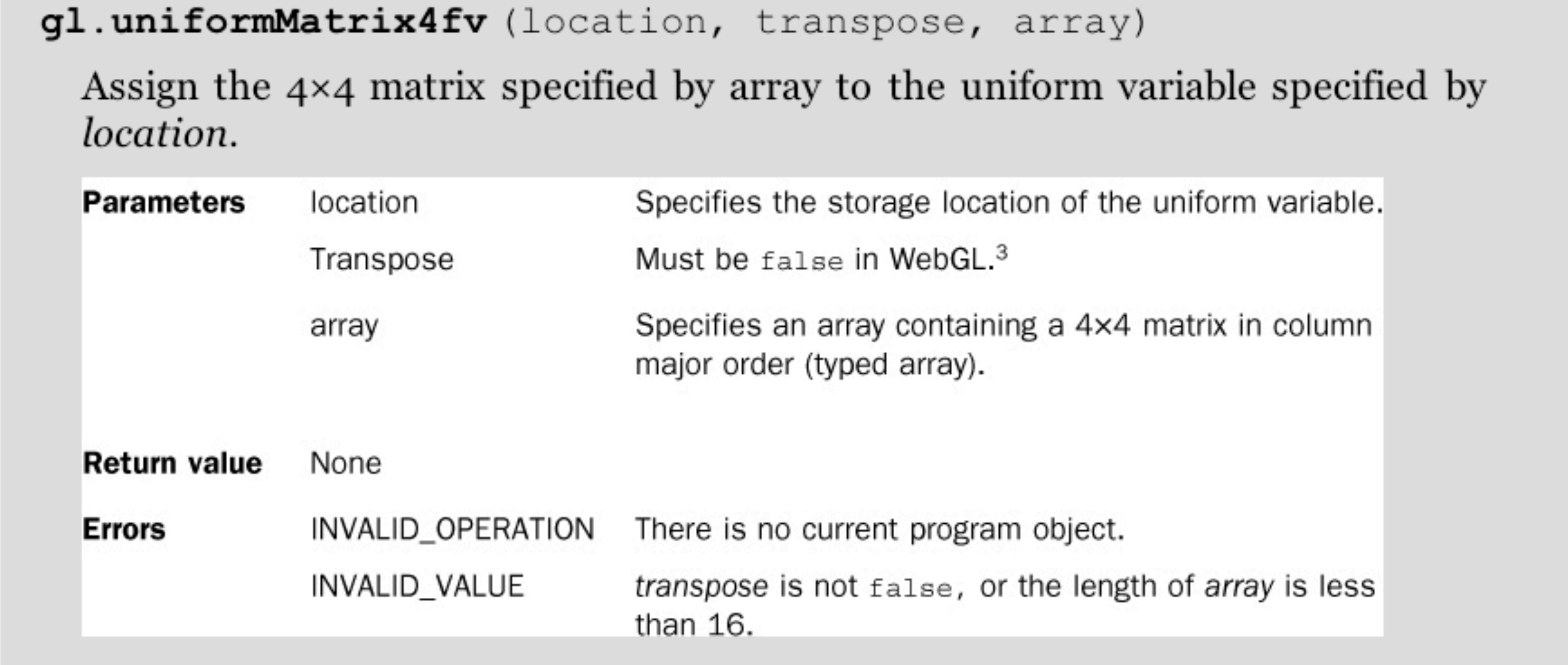


x’ = Sx * x
y’ = Sy * y
z’ = Sz * z

↓

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Scale a Triangle



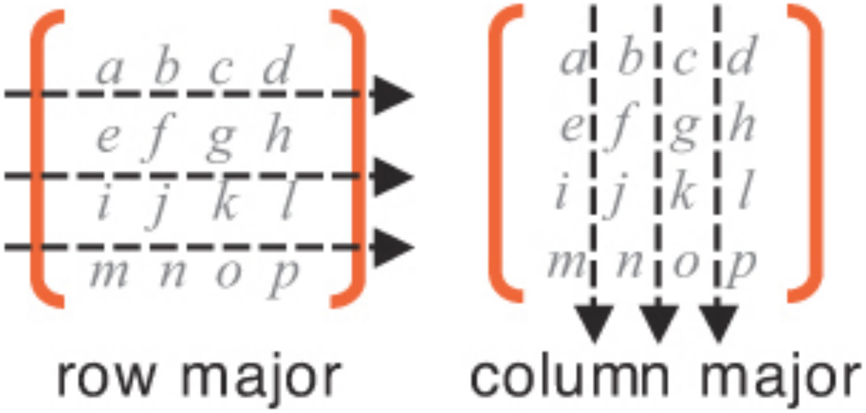
`gl.uniformMatrix4fv` `(location, transpose, array)`

Assign the 4×4 matrix specified by array to the uniform variable specified by *location*.

Parameters	location	Specifies the storage location of the uniform variable.
	Transpose	Must be `false` in WebGL.[3]
	array	Specifies an array containing a 4×4 matrix in column major order (typed array).
Return value	None	
Errors	INVALID_OPERATION	There is no current program object.
	INVALID_VALUE	*transpose* is not `false`, or the length of *array* is less than 16.

12-ScaleTriangle.html
12-ScaleTriangle.js

Row or Column major order?



OpenGL/WebGL
Column major order

$$\begin{bmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row and column major the same

Transformation Matrix for Translation?

Translation Matrix:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & Tx \\ 0 & 1 & 0 & Ty \\ 0 & 0 & 1 & Tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

OpenGL/WebGL
Column major order

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ Tx & Ty & Tz & 1 \end{bmatrix}$$

12-TranslationTriangle2.html
12-TranslationTriangle2.js

Transformation Matrix for Rotation?

Rotation Matrix:

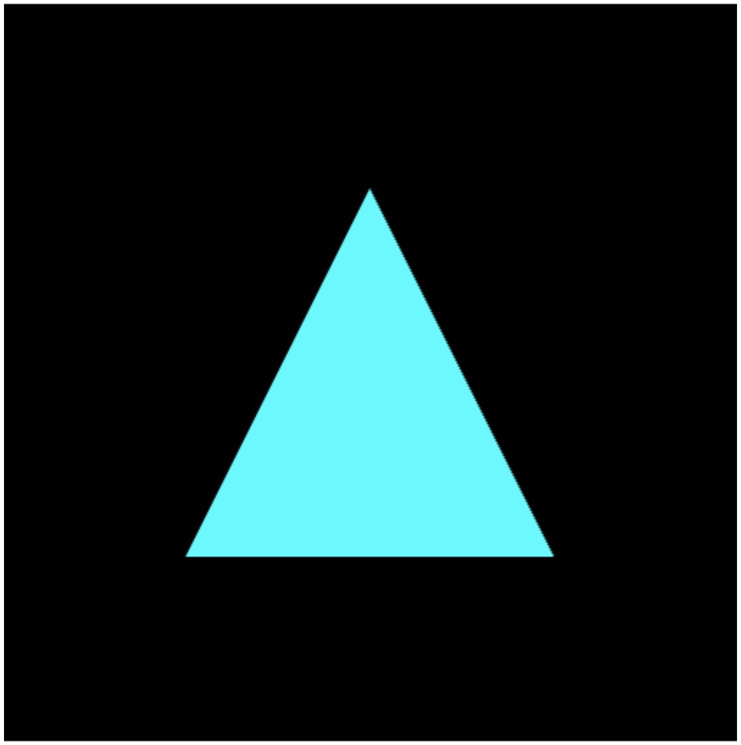
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 & 0 \\ \sin\beta & \cos\beta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

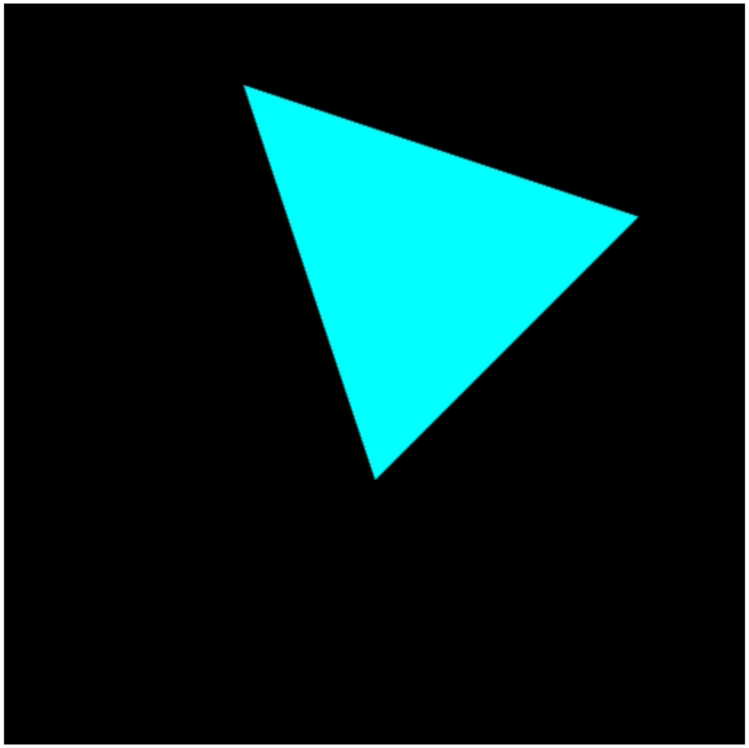
OpenGL/WebGL
Column major order

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

12-RotationTriangle2.html
12-RotationTriangle2.js

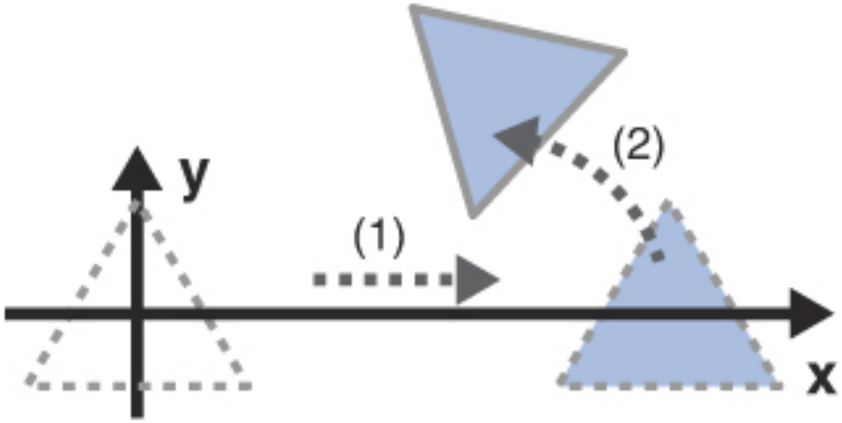
Two Consecutive Transformations





Translate and then Rotate

Two Consecutive Transformations



$$\langle \text{"}\mathit{translated}\text{" } \mathit{coordinates} \rangle = \langle \mathit{translation\ matrix} \rangle \times \langle \mathit{original\ coordinates} \rangle$$

$$\downarrow$$

$$\langle \text{"}\mathit{translated\ and\ then\ rotated}\text{" } \mathit{coordinates} \rangle = \langle \mathit{rotation\ matrix} \rangle \times \langle \mathit{translated\ coordinates} \rangle$$

$$\downarrow$$

$$\langle \text{”}\mathit{translated\ and\ then\ rotated}\text{” } \mathit{coordinates} \rangle = \langle \mathit{rotation\ matrix} \rangle \times (\langle \mathit{translation\ matrix} \rangle \times \langle \mathit{original\ coordinates} \rangle)$$

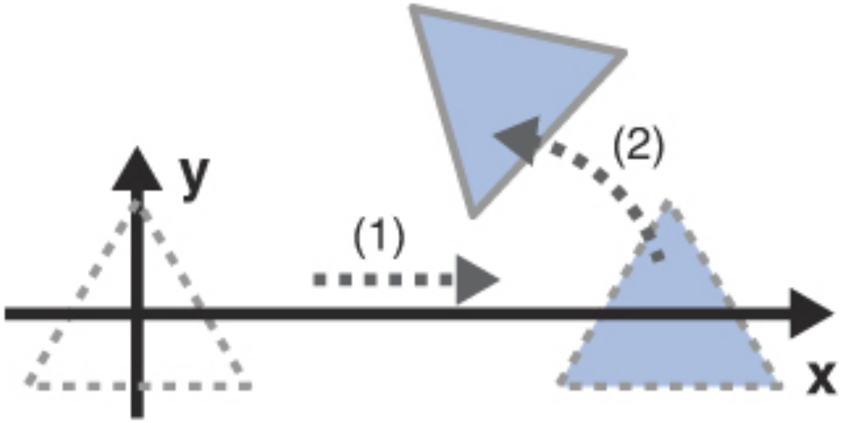
$$\downarrow$$

$$\langle \mathit{rotation\ matrix} \rangle \times (\langle \mathit{translation\ matrix} \rangle \times \langle \mathit{original\ coordinates} \rangle)$$

$$\downarrow$$

$$(\langle \mathit{rotation\ matrix} \rangle \times \langle \mathit{translation\ matrix} \rangle) \times \langle \mathit{original\ coordinates} \rangle$$

Two Consecutive Transformations



$$(\underbrace{\langle rotation\ \ matrix\rangle \times \langle translation\ \ matrix\rangle}) \times \langle original\ \ coordinates\rangle$$

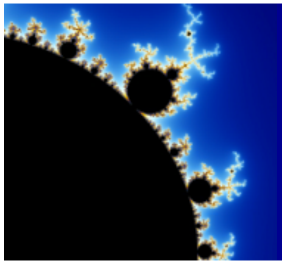
```
// perform translation and then rotation
var modelViewMatrix = mat4();
var r=rotate(ANGLE, 0, 0, 1);
var t=translate(0.3, 0.3, 0);
modelViewMatrix = mult ( mult(modelViewMatrix, r), t);

// send over the modelview transformation matrix to vertex shader
gl.uniformMatrix4fv(modelViewMatrixLoc, false, flatten(modelViewMatrix));

// Draw the rectangle
gl.drawArrays(gl.TRIANGLES, 0, n);
```

Translate and Rotate

r = rotate(45, 0, 0, 1)   t = translate(0.3, 0.3, 0.0)

modelViewMatrix = r * t

$$= \begin{bmatrix} \cos 45 & -\sin 45 & 0 & 0 \\ \sin 45 & \cos 45 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0.3 \\ 0 & 1 & 0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

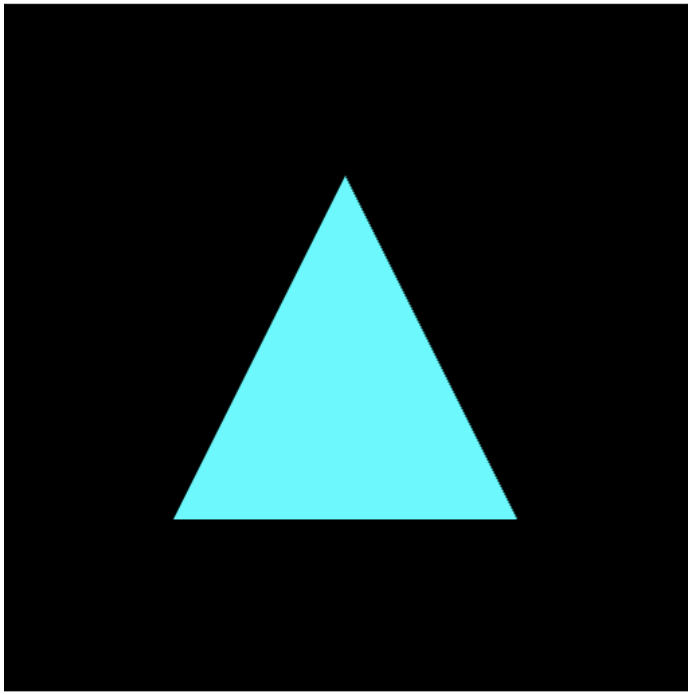


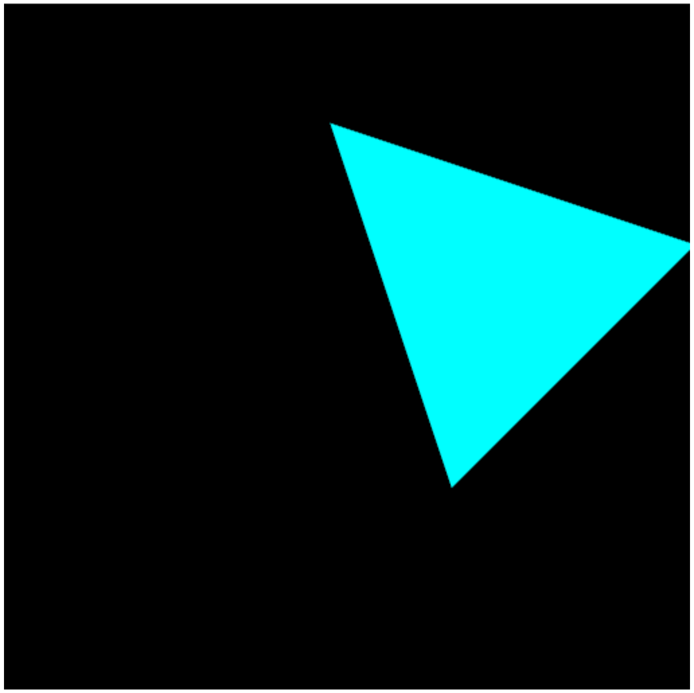
Matrix Multiplication

$$A = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{00} & b_{01} & b_{02} \\ b_{10} & b_{11} & b_{12} \\ b_{20} & b_{21} & b_{22} \end{bmatrix}$$

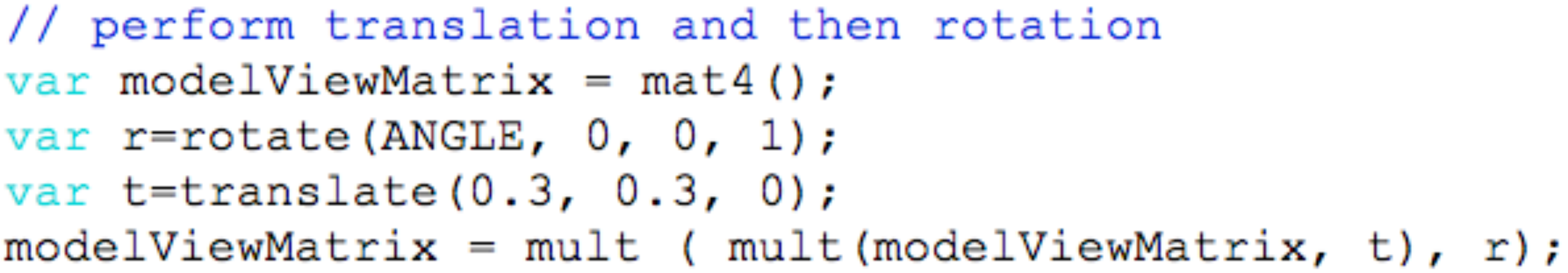
$$\begin{bmatrix} a_{00} \times b_{00} + a_{01} \times b_{10} + a_{02} \times b_{20} & a_{00} \times b_{01} + a_{01} \times b_{11} + a_{02} \times b_{21} & a_{00} \times b_{02} + a_{01} \times b_{12} + a_{02} \times b_{22} \\ a_{10} \times b_{00} + a_{11} \times b_{10} + a_{12} \times b_{20} & a_{10} \times b_{01} + a_{11} \times b_{11} + a_{12} \times b_{21} & a_{10} \times b_{02} + a_{11} \times b_{12} + a_{12} \times b_{22} \\ a_{20} \times b_{00} + a_{21} \times b_{10} + a_{22} \times b_{20} & a_{20} \times b_{01} + a_{21} \times b_{11} + a_{22} \times b_{21} & a_{20} \times b_{02} + a_{21} \times b_{12} + a_{22} \times b_{22} \end{bmatrix}$$

Two Consecutive Transformations





Rotate and then Translate



```
// perform translation and then rotation
var modelViewMatrix = mat4();
var r=rotate(ANGLE, 0, 0, 1);
var t=translate(0.3, 0.3, 0);
modelViewMatrix = mult ( mult(modelViewMatrix, t), r);
```