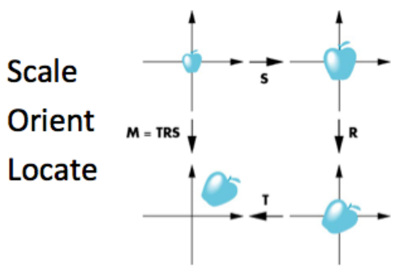
Transformation – Part 2

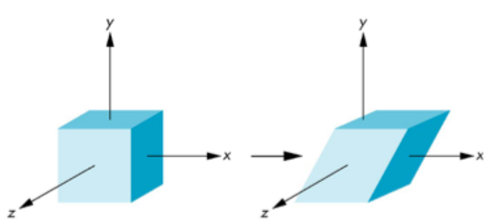
- How to perform the inverse transformation? i.e., undo transformation?
    - Although we could compute inverse matrices by general formulas, we can use simple geometric observations
        - Translation: $\mathbf{T}^{-1}(d_x, d_y, d_z) = \mathbf{T}(-d_x, -d_y, -d_z)$
        - Rotation: $\mathbf{R}^{-1}(\theta) = \mathbf{R}(-\theta)$
        - Scaling: $\mathbf{S}^{-1}(s_x, s_y, s_z) = \mathbf{S}(1/s_x, 1/s_y, 1/s_z)$

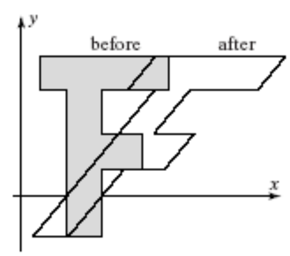
- Forming composite Matrix (concatenate matrices) -- We can form arbitrary affine transformation matrices by multiplying together rotation, translation, and scaling matrices
    - Does the order of the matrices matter?
    - How to form the order of the matrices?
        - right to left, rightmost the first operation applied
        - $Q=MP = T(p_f)\, R(q)\, T(-p_f)P$ // rotation about a fixed point
        - gl_Position= translation * rz * ry * rx * scale * vPosition; // in vertex shader
    - In modeling, we often start with a simple object centered at the origin, oriented with the axis, and at a standard size. We apply an ***instance transformation*** to its vertices:



- Practice Exercise: Given a unit square centered at the origin, what does the square look like after the following sequence of transformations have been applied?
    - Translate along X-axis by 3 units, along Y-axis by 2 units
    - Rotate 45 degrees along the Z-axis about the origin
    - Scale it along X-axis by 3, along Y-axis by 2, about the origin
- Order of the operations in: gl_Position= translation * rz * ry * rx * scale * vPosition; // in vertex shader?

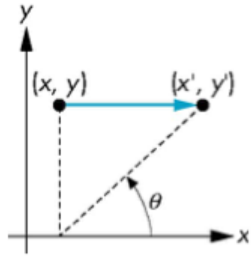
- **Shear** -- Equivalent to pulling faces in opposite directions
    - Shear is the translation along an axis (say, X axis) by an amount that increases linearly with another axis (Y).





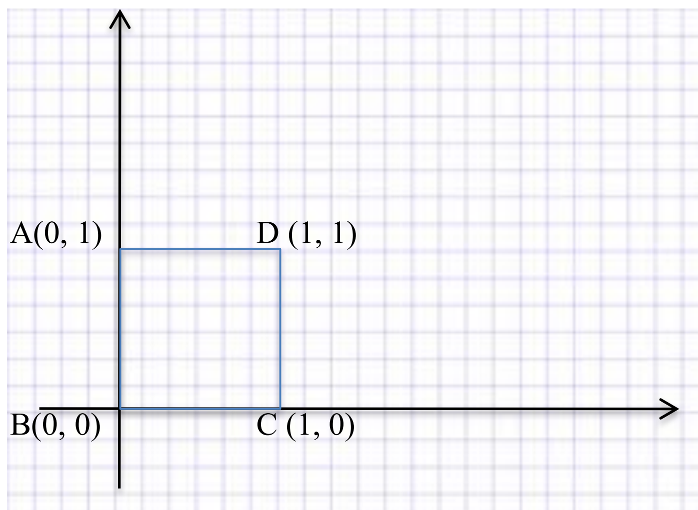
- Matrix for this Shear transformation is : $T=\begin{bmatrix} 1 & \cot\theta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$x' = x + y \cot \theta$$
$$y' = y$$
$$z' = z$$



- Matrix for a shear transformation along both x and y: T= $\begin{bmatrix} 1 & h & 0 & 0 \\ g & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
- Practice Exercise:
  - Given a unit square shown, what is the square after the following shear transformation?

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



- **Perform transformations in WebGL using the provided library functions**
  - Library functions:

```
var r = rotate(theta, vx, vy, vz);
var t = translate(tx, ty, tz);
var s = scale(sx, sy, sz);
var m = mat4()      → identity matrix
var m = mult(m1, m2)   → multiple 2 4x4 matrices
```

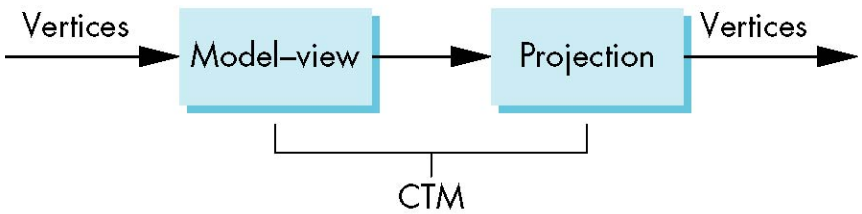
  - Example 1: Perform rotation about z axis by 30 degrees about a fixed point P: (1.0, 2.0, 3.0)

```
var t1=translate(1.0, 2.0, 3.0);
var t2= translate(-1.0, -2.0, -3.0));
var r=rotate(30.0, 0.0, 0.0, 1.0));
var m = mult(t1, r);
m = mult(m, t2);
// now send the matrix m to the vertex shader
```

(note: the last matrix specified in the program is the first applied)

- Practice Exercise: Given a unit square centered at the origin, what is the WebGL .js code for the following transformations?
  - Translate along X-axis by 3 units, along Y-axis by 2 units, and along Z-axis by 5 units
  - Rotate 45 degrees along the Z-axis, about the origin
  - Scale it to be 3 times as wide along X-axis, two times as tall along Y-axis, flipped across the Y-axis, no change along the Z-axis.

- Current Transformation Matrix (CMT) in WebGL
  - The *current transformation matrix* (CTM) is a 4 x 4 homogeneous coordinate matrix
  - It is part of the state and is applied to all vertices that pass down the pipeline

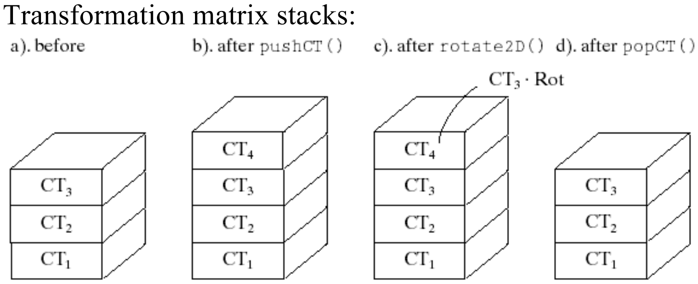


  - In WebGL, the model-view matrix is used to
    - Build models of objects (model M)
    - Position the camera (view V)
      - Can be done by rotations and translations but is often easier to use the lookAt function (discussed later) in MV.js
  - The projection matrix (P) is used to define the view volume and to select a camera lens
  - position' = P*MV*position

- **Matrix Stacks**
  - In many situations we want to save transformation matrices for use later. In JS, it can be achieved by:

```
var stack = [ ]
stack.push(modelViewMatrix);
modelViewMatrix = stack.pop();
```

  - Transformation matrix stacks:



- Practice Exercise: how to draw the following figures using a single branch for the snowflake and using the vertices of a single dinosaur?