SGN-11006, Basic Course in Signal Processing

Exercise 10 (Test Exam).

All problems should be solved and returned before the deadline: **28.11 at 4pm**. Submit your solutions either through Moodle or in the postbox #527 next to the room TC421.

28.11 – 02.12.2016

1. (3 points) Given a discrete-time system, where $y[n]$ and $x[n]$ are the output and the input sequences, respectively:

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

(a) Is the system linear? (b) Is it time invariant? (c) Is it stable? (d) Is it causal? (e) Can the system be represented by its impulse response? **Justify your answers.**

2. (4 points) Consider the interconnection of LTI system shown on Figure 1.

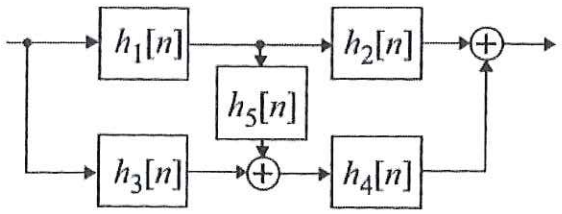

Figure 1: Interconnection of LTI system

(a) Express the impulse response of the overall system $h[n]$ in terms of the frequency responses of the subsystems $h_1[n]$, $h_2[n]$, $h_3[n]$, $h_4[n]$ and $h_5[n]$.

(b) Determine the impulse response of the overall system if:

$$h_1[n] = 2\delta[n-2]$$
$$h_2[n] = 3\delta[n] + \delta[n-1]$$
$$h_3[n] = -2\delta[n+1]$$
$$h_4[n] = -\delta[n] + \delta[n+2]$$
$$h_5[n] = \delta[n+3]$$

(c) Determine the frequency response $H(e^{j\omega})$ of the overall system.

(d) What is the output $y[n]$ of the system for the input $x[n] = \mu[n-2] - \mu[n+1]$.

3. (3 points) A continuous-time signal $x_a(t)$ is to be filtered to remove frequency components in the range $f \geq 10$kHz. The maximum frequency represented in $x_a(t)$ is 20kHz. The filtering is to be done by sampling $x_a(t)$, filtering the sampled signal, and reconstructing a continuous signal using an ideal D/C converter. Find the minimum sampling frequency that may be used to avoid aliasing. For this minimum sampling frequency, sketch the magnitude characteristics of the ideal digital filter $H(e^{j\omega})$ that will remove the desired frequencies from $x_a(t)$.

4. (3 points) Consider a length 8 sequence $x[n]$ defined for $0 \leq n \leq 7$
$\{x[n]\} = \{-4.3, 5.2, 2.7, -3.4, 0, -2, 3.9, 4\}$ with an 8-point DFT given by $X[k]$, $0 \leq k \leq 7$.

Evaluate the following expressions without computing the DFT:

(a) $X[0]$

(b) $X[4]$

(c) $\sum_{k=0}^{7} X[k]$

(d) $\sum_{k=0}^{7} e^{-j(3\pi k/4)} X[k]$

(e) $\sum_{k=0}^{7} |X[k]|^2$

5. (4 points) A signal $x[n]$ has been passed through a causal LTI system given by the following difference equation

$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + x[n] - x[n-1]$$

(a) Find the system transfer function, $H(z)$.

(b) Plot the poles and zeros of $H(z)$ and indicate the region of convergence.

(c) Find the impulse response $h[n]$.

(d) Is the system stable? Is it minimum phase? **Justify your answers.**

6. (3 points) Find the inverse of the following Z-transforms:

(a) $X(z) = 4 + 3(z^2 + z^{-2})$, $0 < |z| < \infty$

(b) $X(z) = \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{3}{1-\frac{1}{3}z^{-1}}$, $1/3 < |z| < 1/2$

(c) $X(z) = \frac{1}{1+3z^{-1}+2z^{-2}}$, $1 < |z| < 2$