SGN-11006, Basic Course in Signal Processing.

Exercise #8. Solutions.

I. $H(z) = \frac{z^{-1}-1}{1-\frac{1}{4}z^{-1}}$; $y[n] = 2\left(\frac{3}{2}\right)^n \mu[-n-1] - \frac{1}{2}\left(\frac{1}{4}\right)^n \mu[n+1] =$

$$= 2\left(\frac{3}{2}\right)^n \mu[-n-1] - 2\left(\frac{1}{4}\right)^{n+1} \mu[n+1]$$

$$Y(z) = -\frac{2}{1-\frac{3}{2}z^{-1}} - \frac{2z}{1-\frac{1}{4}z^{-1}} = \frac{-2+\frac{1}{2}z^{-1}-2z+3}{(1-\frac{3}{2}z^{-1})(1-\frac{1}{4}z^{-1})} = \frac{\frac{1}{2}z^{-1}+1-2z}{(1-\frac{3}{2}z^{-1})(1-\frac{1}{4}z^{-1})}$$

$ROC \equiv \frac{1}{4} < |z| < \frac{3}{2}$

$$Y(z) = H(z)X(z) \Rightarrow \underline{\underline{X(z)}} = \frac{Y(z)}{H(z)} = \frac{(\frac{1}{2}z^{-1}+1-2z)\cancel{(1-\frac{1}{4}z^{-1})}}{(1-\frac{3}{2}z^{-1})\cancel{(1-\frac{1}{4}z^{-1})}(z^{-1}-1)}$$

$ROC \equiv \cancel{\frac{1}{4}} < |z| < 1$ or $1 < |z| < \frac{3}{2}$

{as $\frac{1}{z^{-1}-1}$ can be transformed either to $-\mu[n]$ or $\mu[-n-1]$}

II.

$x_1[n] = 2\delta[n] \xRightarrow{z} X_1(z) = 2$

$x_2[n] = \delta[-n] + \delta[n-1] \xRightarrow{z} X_2(z) = 1 + z^{-1}$, $ROC \equiv z \neq 0$

$x_3[n] = 3^n \mu[n+1] = \frac{1}{3} 3^{n+1} \mu[n+1] \xRightarrow{z} X_3(z) = \frac{z}{3(1-3z^{-1})}$ $ROC \equiv |z| > 3$

$$X(z) = X_1(z)X_2(z)X_3(z) = \frac{2z + 2\cancel{z^2}}{3(1-3z^{-1})} = \frac{2z}{3(1-3z^{-1})} + \frac{2\cancel{z^2}}{3(1-3z^{-1})} \xRightarrow{z}$$

$$\Rightarrow x[n] = \frac{2}{3} 3^{n+1} \mu[n+1] + \frac{2}{3} 3^{n\cancel{+2}} \mu[n\cancel{+2}].$$

III. a) $x_a[n] = n \cdot 2^n \mu[n+1] = \sum_{k=-1}^{\infty} k \cdot 2^k \delta[n-k] = -2^{-1}\delta[n+1] + \sum_{k=0}^{\infty} k \cdot 2^k \delta[n-k] =$

$$= -\frac{1}{2}\delta[n+1] + n \cdot 2^n \mu[n] \xRightarrow{z} \underline{-\frac{z}{2} + \frac{2z^{-1}}{(1-2z^{-1})^2}} \quad \left\{ n\alpha^n \mu[n] \xRightarrow{z} \frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2} \right\}$$

$\underline{ROC \equiv |z| > 2.}$

b) $x_b[n] = \begin{cases} 1, & 1 \le n \le 3 \\ \delta[n], & \text{otherwise} \end{cases} \Rightarrow x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] \xRightarrow{z}$

$$\xRightarrow{z} \underline{X(z) = 1 + z^{-1} + z^{-2} + z^{-3} \quad ROC \equiv z \neq 0}$$

c) $x_c[n] = \mu[n] \circledast \mu[n-1]$

$$X(z) = \frac{1}{1-z^{-1}} \cdot \frac{z^{-1}}{1-z^{-1}} = \underline{\frac{z^{-1}}{(1-z^{-1})^2} \quad ROC \equiv |z| > 1.}$$