SGN-11006, Basic Course in Signal Processing

Exercise 7.

The first 4 problems should be solved and returned before the deadline: **31.10 at 4pm**. Submit your solutions either through Moodle or in the postbox #527 next to the room TC421. Matlab part is checked during the exercise sessions.

31.10 – 04.11.2016

Problem 1: Given the following system function of a causal system:

$$H(z) = \frac{-6z - 3 + 2z^{-1}}{1 - 3z^{-1}} \qquad \text{ROC} \equiv |z| > 3 \tag{1}$$

Find the impulse response $h[n]$ of the system. (2 points)

Problem 2: Evaluate the convolution of the two sequences $h[n] = 2^n \mu[-n]$ and $x[n] = (-0.5)^n \mu[n]$ by using the Z-transform. (2 points)

Problem 3: Find the inverse Z-transform (3 points):

$$X(z) = \frac{1 - 5z^{-3}}{(1 - 2z^{-1})(1 + 0.4z^{-1})} \qquad \text{ROC} \equiv 0.4 < |z| < 2$$

Problem 4: A signal $x[n]$ has been passed through a causal LTI system given by the following difference equation:

$$y[n] - y[n-1] + \frac{1}{4}y[n-2] = 3x[n] - 3x[n-1] - \frac{9}{4}x[n-2] + 2x[n-3] - \frac{1}{4}x[n-4]$$

Find the impulse response of the system. Is the system stable? (3 points)

Hint: $$G(z) = \frac{\frac{1}{2}z^{-1}}{(1 - \frac{1}{2}z^{-1})^2} \Rightarrow g[n] = n\left(\frac{1}{2}\right)^n \mu[n]$$

Problem 5: (Matlab) The discrete-time system is characterized by the following transfer function:

$$H(z) = \frac{1}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}. \tag{2}$$

Find with pen and paper the impulse response using a partial fraction expansion. Investigate what `residuez` shows when applying to the given transfer function. Then use (help `impz`) to compute the impulse response. Plot your result. (4 points)

Problem 6: (Matlab) A digital elliptic low-pass filter is designed giving the following transfer function:

$$H(z) = \frac{0.0798(1 + z^{-1} + z^{-2} + z^{-3})}{1 - 1.556z^{-1} + 1.272z^{-2} - 0.398z^{-3}}. \tag{3}$$

1. Find with pen and paper the recursive difference equation based on the given transfer function.
2. Obtain the system's poles and zeros (help `roots`). Visualize them on the pole-zero plot (help `zplane`).
3. Compute and plot the frequency response of the filter, including magnitude and phase. Demonstrate two plots of the magnitude: on a logarithmic (decibel scale) and on a linear scale. Rationalize the behavior of the magnitude plot in terms of the pole-zero plot. (6 points)