SGN-11006, Basic Course in Signal Processing

Exercise 6.

The first 3 problems should be solved and returned before the deadline: **24.10 at 2pm**. Submit your solutions either through Moodle or in the postbox #527 next to the room TC421. Matlab part is checked during the exercise sessions.

24 – 28.10.2016

Problem 1: A continuous-time signal $x_a(t)$ has a band-limited spectrum $X_a(j\Omega)$, as indicated in Figure 1. Determine the smallest sampling frequency F_T that can be employed to sample $x_a(t)$ so that it can be fully recovered from its sampled version $x[n]$ for each of the following sets of values of the bandedges Ω_1 and Ω_2. **Sketch the DTFT of the sampled version $x[n]$ obtained by sampling $x_a(t)$ at the smallest sampling rate F_T and the frequency response of the ideal reconstruction filter needed to fully recover $x_a(t)$** for each case: (4 points)

a) $\Omega_1 = 128\pi, \Omega_2 = 164\pi$;

b) $\Omega_1 = 140\pi, \Omega_2 = 210\pi$;

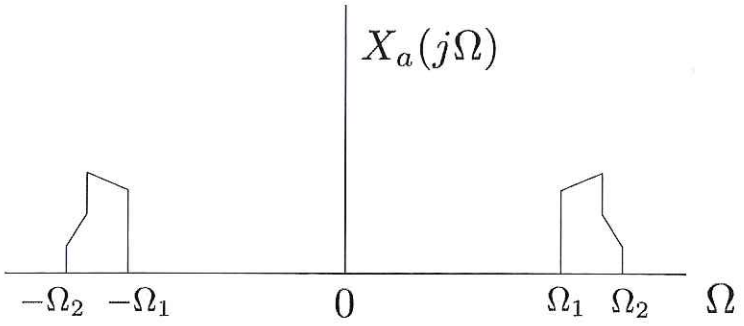

Figure 1: To problem 1.

Problem 2: A discrete-time signal $x[n]$ is obtained by sampling a band-limited, continuous-time signal $x_a(t)$ so that there is no aliasing. Prove that the energy of $x_a(t)$ is equal to the energy of $x[n]$ multiplied by the sampling period. (3 points)

Problem 3: Suppose we want to process the continuous-time signal

$$x_a(t) = 3\cos(2\pi 1000t) + 7\sin(2\pi 1000t)$$

using a discrete-time system. The sampling frequency used is 4000 samples per second. The discrete-time processing carried out on the signal samples $x(n)$ is described by the following difference equation

$$y(n) = x(n) + x(n-2)$$

After the processing, the samples of the output $y(n)$ are converted back to continuous-time form using *sinc* interpolation. (3 points)

a) Give a closed-form expression for the processed continuous-time signal $y_a(t)$;

b) Interpret the effect this processing has on the input signal.

Problem 4: (Matlab) Suppose the signal $x(t) = 5\cos(2\pi 5t) + 2\cos(2\pi 50t)$, sampled with $F_s = 1000$ samples per second is corrupted by a small amount of noise. Try to filter the signal by moving average filter. Simulate the following (5 points):

- Generate 1 second of signal;
- Add white Gaussian noise by `randn`;
- Generate impulse response of `N`-length moving average filter: `ones(1,N)/N`;
- Filter noisy signal by `filter` command;
- Vary `N`, length of the moving average filter, and find dependence on the result.

Problem 5: (Matlab) Write a Matlab program for filtering long data sequences using Overlap-Save Method. Using this program, determine the convolution of the following pair:

$g[n] = \{5, -2, 2, 0, 4, 3, 4, 0, 2, -1, 3\}$,

$h[n] = \{0.1, 0.2, 0.4, 0.2, 0.1\}$;

Verify your result using the function `conv`. Explain difference between Overlap-Save and Overlap-Add methods. (5 points)

Note: here you will need to use one cycle to divide sequence into blocks and no other cycles are needed.