SGN-11006 Basic Course in Signal Processing.

Exercise #2. Solutions.

I. $x(t) = 5\cos(50\pi t + 1)$ $T_s = 0.03.$

General form: $x(t) = A\cos(2\pi f_0 t + \varphi) = A\cos(\omega t + \varphi)$
where f_0 - frequency (cycles/sec)
ω_0 - angular frequency (rad/sec)
Sampling frequency $f_s \overset{def}{=} 1/T_s$ (samples/sec).
Normalized frequency and normalized angular frequency are: $\frac{f}{f_s}$ (cycles/sample) and $\frac{\omega}{f_s}$ (rad/sample) respectively.

a) $f = \frac{50\pi}{2\pi} = 25\ Hz$; $f_s = \frac{1}{T_s} = \frac{1}{0.03} \approx 33.33$; $\omega_n = 50\pi \cdot T_s = 1.5\pi \frac{rad}{sample}$

b) $x[n] = x(nT_s) = 5\cos(50\pi T_s n + 1) = 5\cos(\omega_n n + 1)$

c) $x[n] = 5\cos(\omega_n \cdot n + 1) = 5\cos(\omega_n n + 1 + 2\pi) =$
$= 5\cos((\omega_n + 2\pi)n + 1) = 5\cos\left(\left(\frac{2\pi f}{f_s} + 2\pi\right)n + 1\right) = 5\cos\left(2\pi\left(\frac{f+f_s}{f_s}\right)n + 1\right) =$
$= 5\cos(2\pi(f+f_s)T_s n + 1) \Rightarrow \{x[n] = x(nT_s)\} \Rightarrow$
$\Rightarrow x'(t) = 5\cos(2\pi(f+f_s)t + 1)$ where $f + f_s > f$.

II. $y[n] = \frac{(n+1)^2}{x[n-1]}$

Linearity:

$x[n] = \alpha x_1[n] + \beta x_2[n]$; $y[n] = \frac{(n+1)^2}{\alpha x_1[n-1] + \beta x_2[n-1]}$ (1)

$\alpha y_1[n] + \beta y_2[n] = \frac{\alpha(n+1)^2}{x_1[n-1]} + \frac{\beta(n+1)^2}{x_2[n-1]}$ (2)

(1) $\neq$ (2) $\Rightarrow$ not linear.

Time-invariance:

$x_1[n] = x[n-n_0]$; $y_1[n] = \frac{(n+1)^2}{x[n-1-n_0]}$ (1)

$y[n-n_0] = \frac{(n-n_0+1)^2}{x[n-n_0-1]}$ (2)

(1) $\neq$ (2) $\Rightarrow$ not time-invariant.

<u>Causality</u>: yes, as doesn't depend on future input

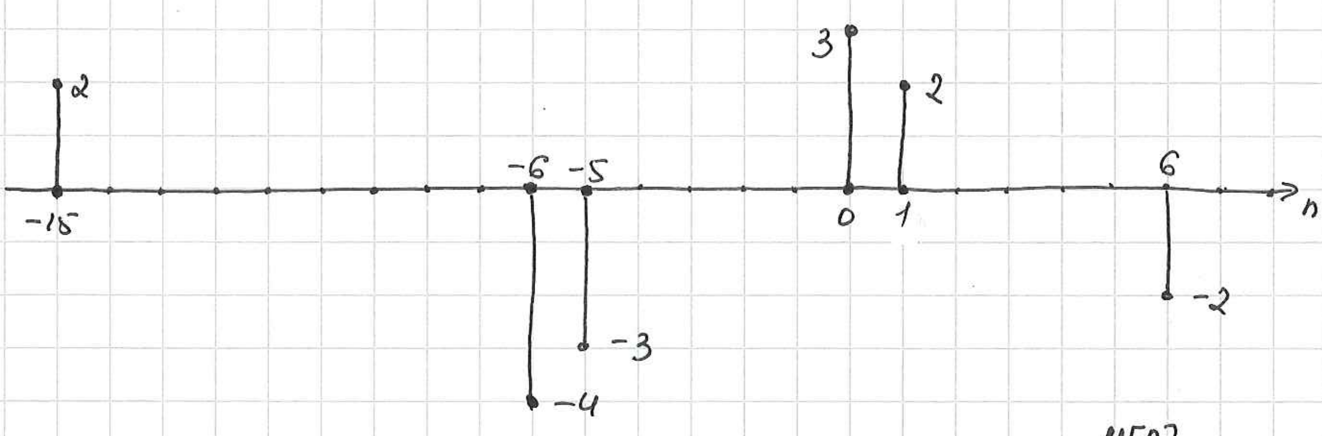
<u>Stability</u>: $|x[n]| \le B_x$

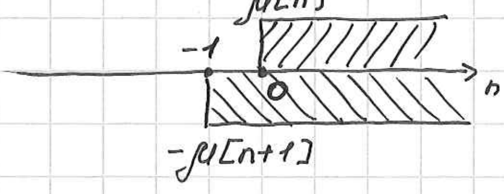
$$|y[n]| = \left| \frac{(n+1)^2}{x[n-1]} \right| = \frac{(n+1)^2}{|x[n-1]|} \xrightarrow[x[n]\to 0]{} \infty \Rightarrow \text{not stable.}$$

<u>Impulse response</u>: system is not LTI, so the output can not be determined using only impulse response.

(III) { Two or more LTI systems in a cascade can be replaced by a single system which impulse response can be founded by convolving the impulse responses of initial systems. }

$$\begin{aligned}
h[n] &= h_1[n] \circledast h_3[n] + h_2[n] \circledast h_4[n] = \\
&= h_1[n] \circledast (-2\delta[n+8]) + h_2[n] \circledast (3\delta[n] - 2\delta[n-6]) = \\
&= -2h_1[n+8] + 3h_2[n] - 2h_2[n-6] = \\
&= -4\delta[n+6] + 2\delta[n+15] + 3\delta[n] - 3\delta[n+5] - 2\delta[n-6] + 2\delta[n-1] = \\
&= 2\delta[n+15] - 4\delta[n+6] - 3\delta[n+5] + 3\delta[n] + 2\delta[n-1] - 2\delta[n-6]
\end{aligned}$$





$$\begin{aligned}
x[n] &= u[n] - u[n+1] = -\delta[n+1] \\
y[n] &= x[n] \circledast h[n] = -h[n+1] = \\
&= -2\delta[n+16] + 4\delta[n+7] + 3\delta[n+6] - \\
&\quad - 3\delta[n+1] - 2\delta[n] + 2\delta[n-5].
\end{aligned}$$