SGN-11006, Basic Course in Signal Processing

Exercise 5.

The first 4 problems should be solved and returned before the deadline: **10.10 at 2pm**. Submit your solutions either through Moodle or in the postbox #527 next to the room TC421. Matlab part is checked during the exercise sessions.

10 – 14.10.2016

Problem 1: Let $x[n]$, $0 \leq n \leq N-1$ be a length-N sequence with an N-point DFT $X[k]$, $0 \leq k \leq N-1$. Determine the N-point DFTs of the following length-N sequences in terms of $X[k]$ (2 points):

(a) $w[n] = \alpha x[\langle n - m_1 \rangle_N] + \beta x[\langle n - m_2 \rangle_N]$, where m_1 and m_2 are positive integers less than N.

(b) $g[n] = \begin{cases} x[n] & \text{for } n \text{ even} \\ -x[n] & n \text{ odd} \end{cases}$

(c) $y[n] = x[n]x[n]$.

Problem 2: A 130-point DFT $X[k]$ of a real-valued sequence $x[n]$ has the following DFT samples: $X[0] = 8.8 + j\alpha$, $X[13] = -3.7 + j2.2$, $X[k_1] = 2.1 - j4.5$, $X[k_2] = 3.6 + j1.3$, $X[55] = 5 - j7.1$, $X[65] = 18 + j\beta$, $X[k_3] = \gamma + j7.1$, $X[79] = 3.6 + j\delta$, $X[108] = \epsilon + j4.5$, $X[k_4] = -3.7 - j2.2$. The remaining DFT samples are assumed to be equal to zero. (3 points)

(a) Determine the values of the indices k_1, k_2, k_3 and k_4.

(b) Determine the values of α, β, γ, δ, and ϵ.

(c) What is the energy of $\{x[n]\}$?

Problem 3: Consider the length-8 sequence, defined for $0 \leq n \leq 7$,

$$\{x[n]\} = \{6.5, 8, -7.5, 8.5, 2.5, -8, -4.5, 1\},$$

with a 8-point DFT given by $X[k]$, $0 \leq k \leq 7$. Evaluate the following functions of $X[k]$ without computing the full DFT:

(a) $X[0]$,

(b) $X[4]$,

(c) $\sum_{k=0}^{7} X[k]$,

(d) $\sum_{k=0}^{7} e^{-11j\,\pi k/4} X[k]$,

(e) $\sum_{k=0}^{7} |X[k]|^2$.

Note: $x[n]$ is a real-valued sequences, so you don't need to operate with complex numbers. (3 points)

Problem 4: Compute the 4-point DFTs of the two real sequences:

- $x_1 = \{-2,\ 0,\ 1,\ 3\}$
- $x_2 = \{1,\ 4,\ -2,\ 0\}$

using ONLY a single 4-point DFT. Homework #4 task 1 might help you. You are not allowed to use MATLAB for this problem (but you may use it to check that your result is correct). (2 points)

Problem 5: (Matlab) A 10 kHz sinusoidal signal is sampled at 80 KHz. A total of 64 samples are collected and used to compute the DFT of the signal. At what DFT indices, $k = 0, 1, ..., N-1$ would you expect to see any peaks in the DFT? Verify it in Matlab. (3 points)

Problem 6: (Matlab) Write a Matlab program to compute the circular convolution of two length -N sequences via the DFT-based approach. Using this program, determine the circular convolution of the following pairs of sequences:

a) $g[n] = \{5, -2, 2, 0, 4, 3\}$, $h[n] = \{3, 1, -2, 2, -4, 4\}$;

b) $x[n] = \cos(\pi n/2)$, $y[n] = 3^n$, $0 \leq n \leq 4$.

Verify your result using the function `cconv`. (3 points)

Problem 7: (Matlab) The noisy signal is defined in Matlab in the following way:

```
t=[0:.01:10.23];
f=exp(-t).*sin(10*t);
noise=rand(1,1024);
signal=f+noise;
```

Construct the plots of the original and noisy signals in time domain and plot their DFT magnitudes. Try to filter the noisy signal in the best way you can by shaping its spectrum using the `fft` function. You can solve this task by zeroing parts of the fft and computing `ifft` to get back to time domain. Provide the plots of the filtered and original signals. (4 points)