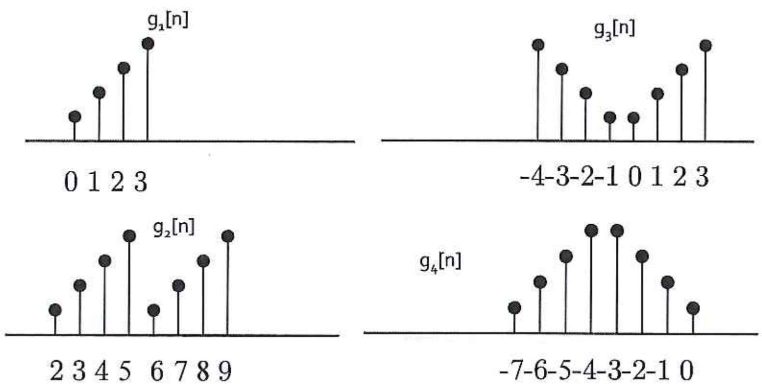
SGN-11006, Basic Course in Signal Processing

Exercise 3.

The first 3 problems should be solved and returned before the deadline: **26.09 at 2pm**. Submit your solutions either through Moodle or in the post box #527 next to the room TC421. Matlab part is checked during the exercise sessions.

26-30.09.2016

Problem 1: Let $G_1(e^{j\omega})$ denote the discrete-time Fourier transform of the sequence $g_1[n]$ shown in the figure below. Express the DTFTs of $g_2[n]$, $g_3[n]$ and $g_4[n]$ in terms of $G_1(e^{j\omega})$. Do not evaluate $G_1(e^{j\omega})$. (2 points)



Problem 2: Determine the DTFT of each of the following sequences: (4 points)

- $x_1[n] = \alpha^n \mu[n-1], \quad |\alpha| < 1;$
- $x_2[n] = n\alpha^n \mu[n{+}1] \; \alpha| < 1;$
- $x_3[n] = \alpha^n \mu[-n] \quad , \quad |\alpha| > 1.$

Problem 3. Consider the interconnection of linear shift-invariant systems in the figure below (such interconnection is often called a Feedback System): (4 points)

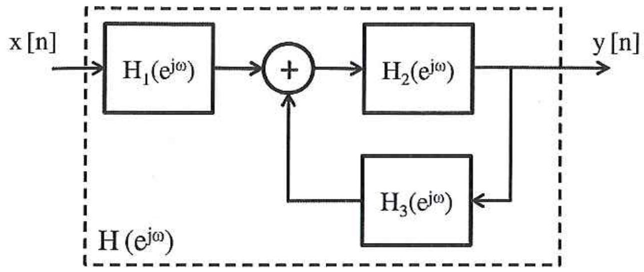


Figure 1: To problem 3

(a) Express the frequency response of the overall system $H(e^{j\omega})$ in terms of the frequency responses of the subsystems $H_1(e^{j\omega})$, $H_2(e^{j\omega})$ and $H_3(e^{j\omega})$.

(b) Determine the frequency response $H(e^{j\omega})$ of the overall system if:

$$
\begin{aligned}
h_1[n] &= \frac{\sin(\pi n)}{\pi n} \\
h_2[n] &= (\ \ 3)^n \mu[\text{-}n] \\
h_3[n] &= n\delta[n-2]
\end{aligned}
$$

Hint: y[n] = h[n]*x[n] => $Y(e^{j\omega}) = H(e^{j\omega})\cdot X(e^{j\omega})$
Hint: $\sin(\pi n)/\pi n$ is sinc function

Problem 4. (Matlab) The signal

$$
x(n) = \begin{cases} n+1, & 0 \le n \le 2 \\ 0, & \text{otherwise,} \end{cases}
$$

is passed through two systems as shown in Figure 2.
The unit sample responses of the systems are given by

$$
h_1(n) = \delta(n) + \delta(n-1) + \delta(n-2),
$$

$$
h_2(n) = \begin{cases} 0.9^n, & 0 \le n \le 10 \\ 0, & \text{otherwise} \end{cases}
$$

(a) Compute and sketch the signal $y_1(n)$.
(b) Use Matlab to compute and plot the signal $y_2(n)$.

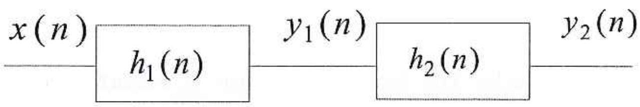


Figure 2: Filtering procedure.

(c) What is the relationship between signal lengths at the input and the output of the two systems?

(d) Use Matlab to compute and plot the signals $y_1(n)$ and $y_2(n)$ when the order of the two systems in Figure 2 is interchanged. Compare with the plots in (a) and (b) and comment. (3 points)

Problem 5. (Matlab) Calculate the discrete Fourier transform of the vector $[-1, 2, 3, 1]$, plot its absolute values and arguments. (`help fft; help abs; help angle`) (2 points)

Problem 6. (Matlab) Use the `filter`-command to filter the input signal $x[n]$ with the systems $y_1[n]$, $y_2[n]$ *(Try what happens, if you filter an input vector that has only non-zero values of x. After that add zeroes in the end of the input vector.)* (3 points)

$$x(n) = \begin{cases} 3-n, & 0 < n < 3 \\ 0, & \text{otherwise,} \end{cases}$$

$$y_1(n) = 3x(n) + 2x(n-1) - 3x(n-2) + x(n-4),$$
$$y_2(n) = -0.9y_2(n-1) + x(n).$$

Problem 7. (Matlab) Use the `impz`-command to get the impulse response of the following filters. Which of these filters seem to be unstable? (2 points)

a) $y(n) = -0.9y(n-1) + x(n)$

b) $y(n) = -0.9y(n-1) - 0.2y(n-2) + x(n)$

c) $y(n) = y(n-1) + x(n-1)$

d) $y(n) = 2y(n-1) - 2x(n-1)$