SGN-11006, Basic Course in Signal Processing

Exercise # 4, Solutions.

(I.) $y[n] = x[n] - ju[-n] \xrightarrow{DTFT} Y(e^{j\omega})$

$Re(y[n]) = x[n] \xrightarrow{DTFT} \underbrace{X(e^{j\omega}) = \frac{1}{2}\left(Y(e^{j\omega}) + Y^*(e^{-j\omega})\right)}$

$Im(y[n]) = -u[-n] \xrightarrow{DTFT} \frac{1}{2j}\left(Y(e^{j\omega}) - Y^*(e^{-j\omega})\right)$

$\Downarrow$

$u[n] \xrightarrow{DTFT} \underbrace{U(e^{j\omega}) = -\frac{1}{2j}\left(Y(e^{-j\omega}) - Y^*(e^{j\omega})\right)}$

(II.) $x[n] \xrightarrow{DTFT} X(e^{j\omega})$

$y[n] = x[-n-1] \circledast x^*[n+1] \xrightarrow{DTFT} ?$

Let $x_1[n] = x[n-1] \xrightarrow{DTFT} X_1(e^{j\omega}) = X(e^{j\omega})e^{-j\omega}$

Let $x_2[n] = x_1[-n] = x[-n-1] \xrightarrow{DTFT} X_2(e^{j\omega}) = X_1(e^{-j\omega}) = X(e^{-j\omega})e^{+j\omega}$

$x_1^*[n] = x^*[n+1] \xrightarrow{DTFT} X_1^*(e^{-j\omega}) = \left(X(e^{-j\omega})e^{-j\omega}\right)^* =$
$= X^*(e^{-j\omega})(e^{-j\omega})^* = X^*(e^{-j\omega})e^{j\omega}.$

Then $y[n] = x_2[n] \circledast x_1^*[n] \xrightarrow{DTFT} X(e^{-j\omega})e^{+j\omega} \cdot X^*(e^{-j\omega}) \cdot e^{j\omega} =$

$= X(e^{-j\omega}) \cdot X^*(e^{-j\omega}) \cdot e^{2j\omega} = |X(e^{-j\omega})|^2 \cdot e^{2j\omega} \Rightarrow \cancel{Y(e^{j\omega}) \in \mathbb{R}}$

in this case it can not be proven that sequence is real-valued.

(III.) a) $h[n] = \delta[n]/3 + \delta[n-1]/3 + \delta[n-2]/3$

$h[n] \xrightarrow{DTFT} \underline{H(e^{j\omega}) = \frac{1}{3}\left(1 + e^{-j\omega} + e^{-2j\omega}\right)} =$

$= \frac{1}{3}e^{-j\omega}\left(e^{j\omega} + 1 + e^{-j\omega}\right) = \frac{1}{3}e^{-j\omega}(\cos\omega + j\sin\omega + 1 + \cos\omega - j\sin\omega) =$

$= \frac{1}{3}e^{-j\omega}(2\cos\omega + 1)$

{as $e^{j\varphi}$ is a unit complex number}

$\underbrace{|H(e^{j\omega})| = \frac{1}{3}\underbrace{|e^{-j\omega}|}_{=1}|2\cos\omega + 1| = \frac{1}{3}|2\cos\omega + 1|}.$

If $2\cos\omega + 1 > 0 \Rightarrow |\omega| < 2\pi/3$

$\Rightarrow \underline{\varphi(\omega)} = \mathrm{arctg}\frac{Im(e^{-j\omega})}{Re(e^{-j\omega})} =$

$= \mathrm{arctg}\frac{-\sin\omega}{\cos\omega} = \underbrace{-\omega, |\omega| < \frac{2\pi}{3}}$

If $2\cos\omega + 1 < 0 \Rightarrow \frac{2\pi}{3} < |\omega| < \pi$

$\Rightarrow \varphi(\omega) = \mathrm{arctg}\frac{Im(-e^{-j\omega})}{Re(-e^{-j\omega})} =$

$= \{-e^{-j\omega} = e^{-j\omega} \cdot e^{\pm j\pi}\} = \underbrace{-\omega \pm \pi, \frac{2\pi}{3} < |\omega| < \pi}$