SGN-11006, Basic Course in Signal Processing

Exercise # 4, Solutions.

(I.) $y[n] = x[n] - ju[n] \xrightarrow{DTFT} Y(e^{j\omega})$

$$Re(y[n]) = x[n] \xrightarrow{DTFT} \underbrace{X(e^{j\omega}) = \frac{1}{2}\left(Y(e^{j\omega}) + Y^*(e^{-j\omega})\right)}$$

$$Im(y[n]) = -u[-n] \xrightarrow{DTFT} \frac{1}{2j}\left(Y(e^{j\omega}) - Y^*(e^{-j\omega})\right)$$

$$\Downarrow$$

$$u[n] \xrightarrow{DTFT} \underbrace{U(e^{j\omega}) = -\frac{1}{2j}\left(Y(e^{-j\omega}) - Y^*(e^{j\omega})\right)}$$

(II.) $x[n] \xrightarrow{DTFT} X(e^{j\omega})$

$y[n] = x[-n-1] \circledast x^*[n+1] \xrightarrow{DTFT} ?$

Let $x_1[n] = x[n+1] \xrightarrow{DTFT} X_1(e^{j\omega}) = X(e^{j\omega})e^{j\omega}$

Let $x_2[n] = x_1[-n] = x[-(n+1)] \xrightarrow{DTFT} X_2(e^{j\omega}) = X_1(e^{-j\omega}) = X(e^{-j\omega})e^{-j\omega}$

$$x_1^*[n] = x^*[n+1] \xrightarrow{DTFT} X_1^*(e^{-j\omega}) = \left(X(e^{-j\omega})e^{-j\omega}\right)^* =$$
$$= X^*(e^{-j\omega})(e^{-j\omega})^* = X^*(e^{-j\omega})e^{j\omega}.$$

Then $y[n] = x_2[n] \circledast x_1^*[n] \xrightarrow{DTFT} X(e^{-j\omega})e^{-j\omega} \cdot X^*(e^{-j\omega}) \cdot e^{j\omega} =$

$$= X(e^{-j\omega}) \cdot X^*(e^{-j\omega}) = |X(e^{-j\omega})|^2 \Rightarrow Y(e^{j\omega}) \in \mathbb{R}$$

(III.) a) $h[n] = \delta[n]/3 + \delta[n-1]/3 + \delta[n-2]/3$

$$h[n] \xrightarrow{DTFT} \underbrace{H(e^{j\omega}) = \frac{1}{3}\left(1 + e^{-j\omega} + e^{-2j\omega}\right)} =$$

$$= \frac{1}{3}e^{-j\omega}\left(e^{j\omega} + 1 + e^{-j\omega}\right) = \frac{1}{3}e^{-j\omega}\left(\cos\omega + j\sin\omega + 1 + \cos\omega - j\sin\omega\right) =$$

$$= \frac{1}{3}e^{-j\omega}\left(2\cos\omega + 1\right)$$

{as $e^{j\varphi}$ is a unit complex number}

$$\underbrace{|H(e^{j\omega})| = \frac{1}{3}\underbrace{|e^{-j\omega}|}_{=1}\,|2\cos\omega + 1| = \frac{1}{3}|2\cos\omega + 1|}$$

If $2\cos\omega + 1 > 0 \Rightarrow |\omega| < 2\pi/3$

$$\Rightarrow \underline{\varphi(\omega)} = \operatorname{arctg}\frac{Im(e^{-j\omega})}{Re(e^{-j\omega})} =$$

$$= \operatorname{arctg}\frac{-\sin\omega}{\cos\omega} = \underbrace{-\omega,\ |\omega| < \frac{2\pi}{3}}$$

If $2\cos\omega + 1 < 0 \Rightarrow \frac{2\pi}{3} < |\omega| < \pi$

$$\Rightarrow \varphi(\omega) = \operatorname{arctg}\frac{Im(-e^{-j\omega})}{Re(-e^{-j\omega})} =$$

$$= \{-e^{-j\omega} = e^{-j\omega} \cdot e^{\pm j\pi}\} = \underbrace{-\omega \pm \pi,\ \frac{2\pi}{3} < |\omega| < \pi}$$

b). $h[n] = \delta[n - n_0]$

$h[n] \xrightarrow{DTFT} \underline{H(e^{j\omega}) = e^{-n_0 j\omega}} = \cos(n_0\omega) - j\sin(n_0\omega)$

$\underline{|H(e^{j\omega})| = |e^{-n_0 j\omega}| = 1.}$

$\underline{\varphi(\omega)} = \operatorname{arctg} \frac{Im(e^{-n_0 j\omega})}{Re(e^{-n_0 j\omega})} = \operatorname{arctg} \underbrace{\frac{-\sin(n_0\omega)}{\cos(n_0\omega)}}_{} = \underline{-n_0\omega}$

" $tg(n_0\omega)$