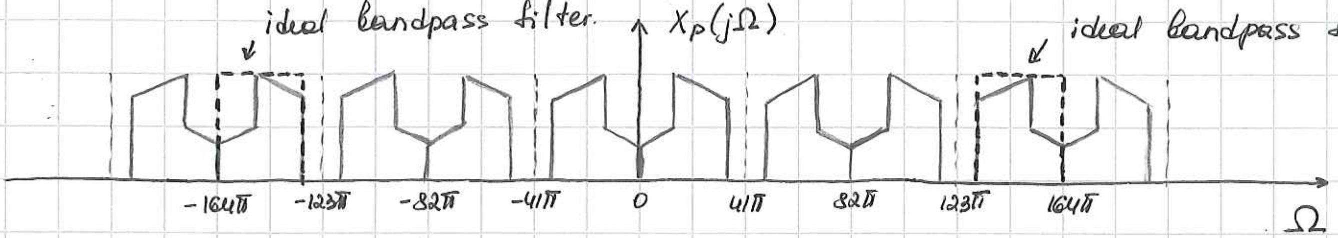
SGN-11006, Basic Course in Signal Processing

Exercise #6. Solutions

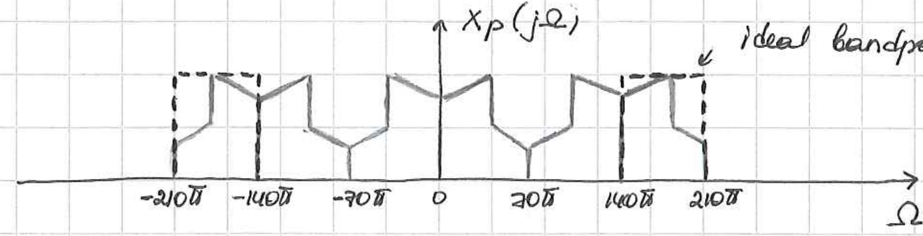
(I) As $x_a(t)$ is a bandpass signal, it can be sampled at rate below Nyquist rate and still be fully recovered from its sampled version if sampling rate satisfies following condition: $\frac{2\Omega_2}{n} \le F_s \le \frac{2\Omega_1}{n-1}$, for integer n such as $1 \le n \le \left\lfloor \frac{\Omega_2}{\Omega_2-\Omega_1} \right\rfloor$

The lowest possible sampling rate corresponds to the highest n.

a) $\Omega_1 = 128\pi$, $\Omega_2 = 164\pi$, $n = \left\lfloor \frac{164\pi}{164\pi - 128\pi} \right\rfloor = 4$, $F_s = \frac{2 \cdot 164\pi}{4} = 82\pi$



b) $\Omega_1 = 140\pi$, $\Omega_2 = 210\pi$, $n = \left\lfloor \frac{210\pi}{210\pi - 140\pi} \right\rfloor = 3$, $F_s = \frac{2 \cdot 210\pi}{3} = 140\pi$



$x_a(t)$ can be recovered from $x[n]$ by passing it through an ideal bandpass filter with the passband $\Omega_1 \le |\Omega| \le \Omega_2$

(II) Let Ω_0 be the band limit of signal $x_a(t)$, then

$$E[x_a(t)] = \int_{-\infty}^{\infty} |x_a(t)|^2 dt \quad \text{and} \quad E[x[n]] = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

Using Perseval's relation: $\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$ and

$$\int_{-\infty}^{\infty} |x_a(t)|^2 dt = \frac{1}{2\pi}\int_{-\Omega_0}^{\Omega_0} |X(e^{j\Omega})|^2 d\Omega = \left\{\omega = \frac{\Omega}{F_s} = \Omega T_s\right\} =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\frac{\omega}{T_s} = \frac{1}{2\pi T_s}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega \Rightarrow E[x_a(t)] = T_s E[x[n]].$$

III) a) The sampling rate is higher than Nyquist rate $\Rightarrow$ continuous-time signal can be perfectly reconstructed from discrete-time signal. The sampling period is $T_s = 1/4000$, so

$$y[n] = x[n] + x[n-2] \xrightarrow[\text{interp.}]{\text{sinc}} y_a(t) = x_a(t) + x_a\left(t - \frac{2}{4000}\right)$$

b) $y[n] = x[n] + x[n-2] = 3\cos\left(2\pi \frac{1000n}{4000}\right) + 7\sin\left(2\pi \frac{1000n}{4000}\right) +$

$+ 3\cos\left(2\pi \frac{1000(n-2)}{4000}\right) + 7\sin\left(2\pi \frac{1000(n-2)}{4000}\right) =$

$= 3\cos\left(\frac{\pi n}{2}\right) + 7\sin\left(\frac{\pi n}{2}\right) + 3\cos\left(\frac{\pi n}{2} - \pi\right) + 7\sin\left(\frac{\pi n}{2} - \pi\right) =$

$= 3\cos\left(\frac{\pi n}{2}\right) + 7\sin\left(\frac{\pi n}{2}\right) - 3\cos\left(\frac{\pi n}{2}\right) - 7\sin\left(\frac{\pi n}{2}\right) = \emptyset.$

$\Rightarrow$ input signal is completely suppressed.