SGN-11006, Basic Course in Signal Processing

Exercise 4.

The first 3 problems should be solved and returned before the deadline:

03.10 at 2pm.

Submit your solutions either through Moodle or in the postbox #527 next to the room TC421. Matlab part is checked during the exercise sessions.

03–07.10.2016

Problem 1: Let $x[n]$ and $u[n]$ be two real-valued sequences with DTFT given by $X(e^{jw})$ and $U(e^{jw})$, respectively. Define a complex-valued $y[n]$ formed as follows: $y[n] = x[n] - ju[-n]$. Express $X(e^{jw})$ and $U(e^{jw})$ in terms of the DTFT $Y(e^{jw})$. (3 points)

Hint: use DTFT properties from Lecture 3.

Problem 2: Let $X(e^{j\omega})$ denote the DTFT of a sequence $x[n]$. Determine the DTFT $Y(e^{j\omega})$ of the sequence $y[n] = x[-n-1]\,(*)\,x^*[n+1]$ in terms of $X(e^{j\omega})$, and show that it is a real-valued function of ω. (3 points)

Problem 3: Find the frequency response, magnitude and phase of a system with impulse response (4 points):

a) $h[n] = \delta[n]/3 + \delta[n-1]/3 + \delta[n-2]/3$;

b) $h[n] = \delta[n - n_0]$.

Hint: $\arg(H(e^{j\omega})) = \arctan(H_{Im}(e^{j\omega})/H_{Re}(e^{j\omega}))$.

Problem 4: (Matlab) Using Matlab verify the following general properties of the DTFT : (a) linearity, (b) time-shifting. Consider an exponential N-length sequence $g[n] = e^{-0.5n}$ and a 1 Hz sinusoidal sequence $h[n] = sin(2\pi n/(N/2))$ to obtain your results. Use freqz command (help freqz) to compute the DTFT. (2 points)

Problem 5: (Matlab) Load corrupted version of "handel" test signal (www.cs.tut.fi/courses/SGN-1157/groupA/corrupt.mat). If we assume that the corruption process has been linear (i.e. it can be presented with convolution $z[n] = y[n](*)h[n]$, where $h[n]$ is the impulse response of the corrupting filter and y[n] is the original signal), we can solve this process with DFT (help fft). Since DFT transforms convolution in to a multiplication, we can present the corrupting process Z(n) = Y(n)H(n). Solve H(n), take inverse Fourier transform (help ifft) and plot the first 10 values of it. (4 points)

Problem 6: (Matlab) Consider the frequency response of an ideal low-pass filter in Figure 1 (with normalized edge frequency f_c). Calculate its inverse discrete-time Fourier transform

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X\left(e^{i\omega}\right) e^{i\omega n}\, d\omega, \qquad \omega = 2\pi f.$$

Convolve a part of it (11 samples, so that the index n runs from -5 to 5) with the `handel` test sample in Matlab (now you may choose f_c arbitrarily). Plot the spectrogram. Increase the length of the filter (take more samples from the inverse transform, note that you should use an odd number, e.g. 21) a couple of times, and examine how this affects the filter's performance (use `freqz`-command). Explain what the term "windowing" means in filter design. (4 points)

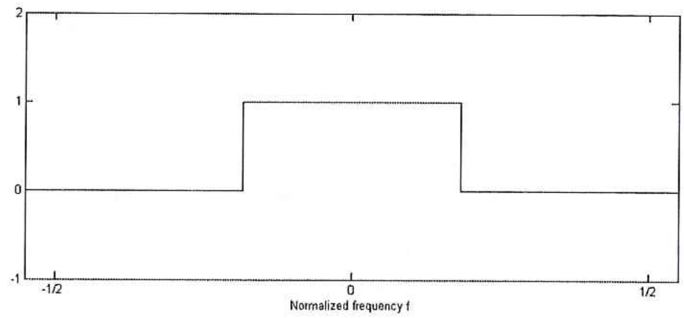

Figure 1: Filtering procedure.