

# A Neural Network Approach for Reconstructing Surface Shape from Shading \*

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## Abstract

*In this work, a framework for the reconstruction of smooth surface shapes from shading images is presented. The method is based on using a backpropagation based neural network for learning brightness patterns and associating them with range data. The network is designed to reconstruct surface range from localized intensity patches of  $7 \times 7$  pixels. Two methods for training the network are investigated, one based on a novel weight diffusion process which enforces a local smoothness constraint and the other using the eigen coefficients of the input and output patterns which make the training computationally efficient. An elegant and simple method for integrating reconstructed surface patches by minimizing the sum squared error in overlapped areas is derived. Results are shown for reconstruction of simple shapes like cylinders, hyperboloids and paraboloids as well as complex shapes like facial structure from intensity images.*

## 1. Introduction

Shading is a unique cue for reconstruction of surface shape, primarily because of its omni-presence under all illumination conditions. Unlike stereo imaging, in which the disparity falls off with distance or motion parallax which is obviously not present for still imagery, shading cues are used by the human visual system under all conditions to estimate the shapes of smooth surfaces.

In previous works, the reconstruction of shape from shading was based on satisfying the image irradiance equation at each imaged point. Horn used a variational approach to satisfy the equation and reconstruct the surface normal [1]. Most methods since have been based on modifications and enhancements of this approach using various constraints. Constraints can be added to the primary functional using Lagrange multipliers. This include the meth-

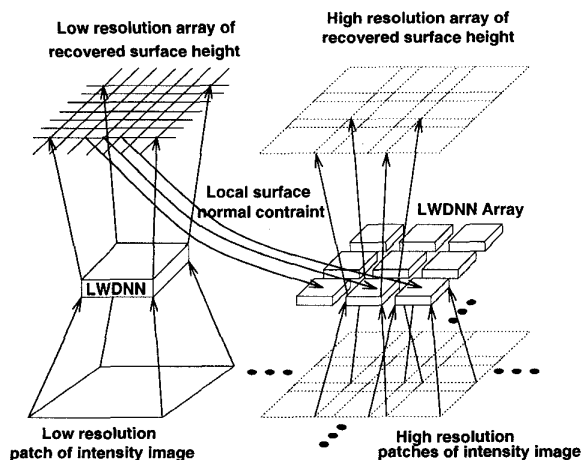
ods of photometric stereo [11], modeling diffuse illumination by inter-reflections [10], direct reconstruction of height by enforcing differential smoothness constraints [3], using line drawing interpretation to reconstruct piecewise smooth surfaces [5] or using only local differential conditions [7]. Each method has some shortcomings. For example, in [7] it is assumed that the surface is locally spherical. Solving the variational equations for a general reflectance map requires successive integration along paths whose loci are determined only as the integration proceeds [6, pp. 248]. Thus, using variational methods, an efficient parallel solution appears quite impossible without simplifications and further constraints.

We propose a novel approach to shape from shading based on learning spatially localized brightness patterns. A neural network based on the backpropagation algorithm is trained to associate intensity input patterns with the corresponding range data, given the illumination direction. We significantly improve the efficiency of learning by using a Karhunen-Loeve compressed representation for the brightness and the surface height. Only few approaches using neural networks for the shape from shading problem have been proposed. This includes the work of Lehky and Sejnowski [4] who showed that it is possible to reconstruct the approximate surface normals of ellipsoidal shapes by training a network with intensity patterns. Also, Kim et al. [2] modeled the reconstruction of surface normals of a hybrid reflectance surface using a photometric stereo approach using known multiple illuminations.

## 2. Shape Reconstruction Neural Network

In this work, we present a novel approach for *learning brightness patches* using a *backpropagation based neural network*. The patches are localized brightness patterns and arise due to changes in surface orientation relative to the illumination. The underlying assumption made is that the surface is smooth. Here, we consider Lambertian surfaces under a single illumination with varying source direction. Two different approaches are used in the training process: 1) A novel method of smoothing localized weights

\*This work was supported by the National Science Foundation under Grant No. IRI-9623966, and Grant No. IRI-97-11925.



**Figure 1.** Shown are two layers of the Multiresolution Neural Network for Shape from Shading. The low resolution layer reconstructs a low resolution height map using the Local Weight-Diffusion Neural Network (LWDNN). These values are used as constraints for the next high resolution layer. Each LWDNN reconstruct heights in patches of the local surface. A layer on top of the LWDNN array uses the overlapping LWDNN output patches and the heights and surface normals obtained from the lower resolution to reconstruct the high resolution surface shape.

called weight diffusion. 2) Using Karhunen-Loeve transform (KLT) coefficients for efficiency. The learned patches can then be associated by the network with the corresponding surface range patch. Such localized range patches are then integrated by using a local error constraint to reconstruct the entire shape. This approach presents an elegant and efficient parallel approach to shape reconstruction from shading.

In Fig. 1, we show a block diagram of our overall approach. The approach is top-down with lower resolution layers first used to extract approximate shape from the low resolution intensity patches. Fig. 1 shows the relation between two resolution layers on the network. Each layer is composed of arrays of neural networks with local and overlapping receptive fields (LWDNN). These are trained with our novel variation of the back-propagation algorithm called *weight diffusion* and learn local surface intensity patterns from different viewpoints and illumination conditions to directly generate the local shape of the imaged surface (described in Section 3). Height and surface normal at each low-resolution grid point is used to guide more detailed extraction of surface shape of corresponding image patches in parallel at higher resolution. Only the lowest resolution layer does not use these constraints.

Currently, we have implemented a single stage of the overall multi-resolution network, by using only one layer. This is similar to an intermediate layer, but without the local

surface normal constraints. A simple error measure based on the sum squared error of the overlapping areas of neighboring patches is used as explained in Section 4. The network has been successfully used to reconstruct various surfaces like cylinders and ellipsoids. We have also used it successfully to reconstruct facial surface structure from an image of a illuminated face as explained in Section 5.

### 3. Learning Brightness Patterns

We now describe a feed-forward neural network that can learn the many-to-one mapping to reconstruct local surface shape from the local image shading pattern. The neural model is a three layer structure with input, hidden and output layers. The input array has a local receptive field in the image domain. The array of output units defines the reconstructed surface function corresponding to the local receptive field.

The learning problem is quite complex as it is expected that the network should be able to estimate relative surface heights of neighboring pixels under varying conditions of illumination as well as variations of the surface type from being concave or convex, or ellipsoids, cylinders or hyperboloids. We attempted two novel techniques to ease the computational burden on the network. In the first case, a smoothness constraint is incorporated in the training stage to force neighboring points in the network output to have similar range values and is enforced by locally smoothing the weight vectors of the output layers. This ensures that noisy training samples do not cause outliers in the training. The implicit assumption is that the surfaces being trained are locally smooth. In another variation, which is computationally more efficient, the eigen space of the input and output brightness patterns are created using the KLT. The eigen coefficients of the illumination patterns and range data are then used in training.

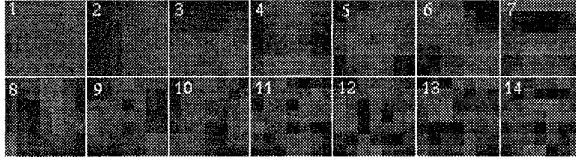
The input consists of an intensity image patch of  $7 \times 7$  with known illumination vector given by the direction cosines. Thus a total of 52 inputs are used. The output is the lexicographically ordered height matrix corresponding to the same  $7 \times 7$  imaged are. The range of curvatures for the trained samples and the input images is assumed to be within a range  $[\kappa_{min}, \kappa_{max}]$ . Variations in viewpoint and illumination directions are incorporated in the training set. Both are allowed to vary  $360^\circ$  in azimuth and  $90^\circ$  in elevation to generate a 4-D space of possible illumination-viewpoint directions. In addition variations in the curvatures are in 3 possible directions and allow another 3 degrees of freedom. This results in a 7-D parameter space which has to be learned. Neighboring points in the 7-D curvature-viewpoint-illumination parameter space vary smoothly in both input intensity and output range values. The input intensities and output heights are normalized such that the center grid point is shifted to 0. The input intensity

and output height range is then scaled to be in the range  $[-1, 1]$  corresponding to the actual heights for the curvature extrema  $\kappa_{max}$  and  $\kappa_{min}$  over the  $7 \times 7$  patch. The network output is an array of floating surface heights with the center pixel height at 0.

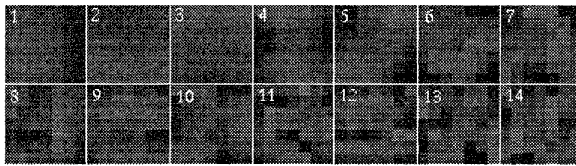
In our first approach, backpropagation training [9] is used to implement a novel process called *weight diffusion* which enforces local smoothness in the output space of the network. After each error backpropagation iteration, the weight vectors for the nodes of the output layer are *diffused* into a  $3 \times 3$  neighborhood corresponding to the spatial ordering of the output range. The process is given by

$$\mathbf{w}_n(x, y) = \mathbf{w}(x, y) + \alpha \sum_{i=-1}^{+1} \sum_{j=-1}^{+1} \lambda(i, j) \mathbf{w}(x+i, y+j) \quad (1)$$

where  $\mathbf{w}_n(x, y)$  is the updated weight vector,  $\mathbf{w}(x, y)$  is the original weight vector,  $\alpha$  is a proportionality constant supplied during training,  $\lambda$  is a  $3 \times 3$  Laplacian mask. The effect of weight diffusion causes neighboring pixel weight vectors to be similar, ensuring surface smoothness. Hence we call it the Local Weight Diffusion Neural Network (LWDNN). Here the intensity patterns with the illumination direction are used to form 51 dimensional input vectors, and the normalized range data is used to form 49 dimensional output vectors. The hidden layer consists of 30 nodes.



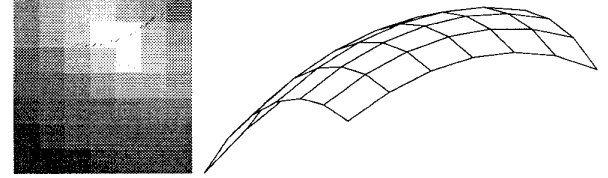
**Figure 2.** First 14 eigen bases of intensity patches. The coefficients of these bases are used in training our network.



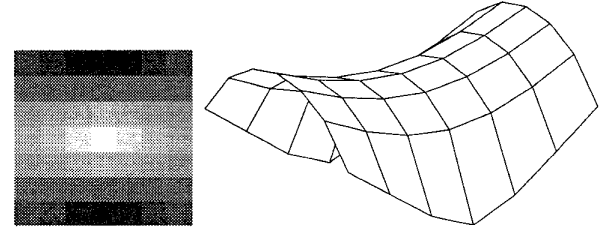
**Figure 3.** First 14 eigen bases of range patches. First 5 coefficients are used in training.

The second approach is to train the network with the principal components of the distribution of input image patches and output image ranges. The use of the principal components decreases computational complexity for the training stage, primarily because a smaller number of inputs and outputs are required. For  $7 \times 7$  patches, the first 14 eigen vector coefficients for the input space and the first 5 eigen vector coefficients for the output space are used in training.

These account for greater than 95% of the energy in the distribution as given by  $E_p = \sum_{i=1}^{P \ll N} \lambda_i$  [8]. The first 14 eigen-images of the inputs and outputs are shown in Fig. 2 and Fig. 3. Here also, the illumination direction coded as direction cosines are included in the input along with the first 14 coefficients. 30 hidden nodes are used. Training with eigen coefficients reduces the training time considerably with only minimal increase in error. In fact, all the results shown in this paper have been duplicated with both the LWDNN and the training using the eigen coefficients.



**Figure 4.** Shown are normalized intensity and reconstructed normalized range ( $7 \times 7$  patches) for a convex ellipsoidal surface with high curvature.



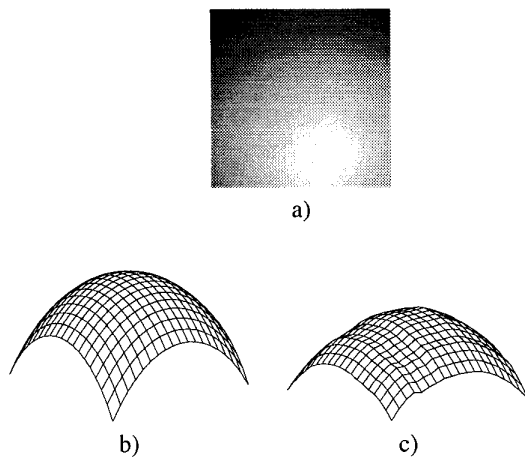
**Figure 5.** Here we see normalized intensity and reconstructed normalized range ( $7 \times 7$  patches) for a hyperboloid (saddle) surface with high curvature.

Two of examples demonstrating the intensity information that is input and the range information that is extracted are shown in Fig. 4 and Fig. 5. The two surfaces are ellipsoid and hyperboloid respectively and have different illumination directions. The intensities and the illumination source vector is input to the network. The surfaces were reconstructed with average error of about 0.5%/pixel.

#### 4. Integrating Surface Patches

The reconstructed patches at a particular resolution are integrated together in a smooth manner by a mechanism which minimizes the difference in heights in the overlapping pixels of neighboring patches. Each patch  $P_{i,j}$  output by the LWDNN array has a floating height  $h_{i,j}$  because of the normalization described above. The relative height  $\Delta h$  between two overlapping patches  $P_{i,j}$  and  $P_{i+1,j+1}$  is given by minimizing the sum squared error  $E$  in heights on the overlapping area grid points indexed by  $\{k, l\}$ .

$$E = \sum_x \sum_y^{overlap} \|h_{i,j}(x, y) - h_{i+1,j+1}(x, y) + \Delta h\|^2 \quad (2)$$



**Figure 6.** a) The intensity pattern of a spherical ellipsoid with a radius of 16 pixels and illuminated from an azimuth of  $20^\circ$  and elevation of  $25^\circ$  with respect to the viewing direction. b) The original shape. c) The shape reconstructed by integrating overlapping patches of the reconstructed local shapes as explained in Section 4. As can be seen, the reconstructions are quite accurate.

Minimizing  $E$  by setting  $\frac{\partial E}{\partial(\Delta h)} = 0$  gives

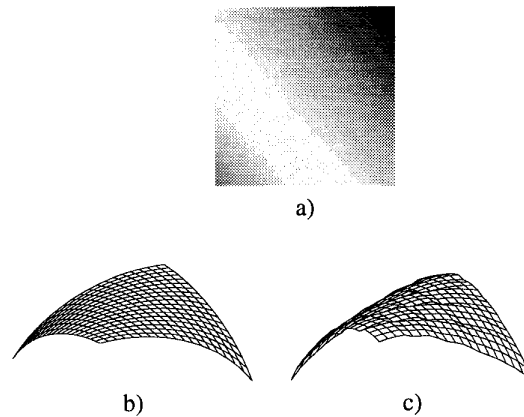
$$\Delta h = \sum_x \sum_y^{overlap} (h_{i,j}(x,y) - h_{i+1,j+1}(x,y)) \quad (3)$$

This relative height correction is applied to every pair of overlapping patches in local operations. Now overlapping area grid points have multiple heights with minimum sum squared errors. They are averaged to generate the height estimate for that point. This results in complete shape reconstruction. Reconstructed surface patches like the ones shown in Fig. 4 and Fig. 5 can be integrated together to form larger contiguous patches.

## 5. Results

We now present some preliminary results of the approach used for surface shape reconstruction. In Fig. 6(a), is shown the intensity pattern due to a ellipsoidal surface patch. The ellipsoid is a sphere of radius 16 pixels, with a Lambertian surface reflectance and illuminated from a direction to the right bottom of the image (azimuth:  $20^\circ$ , elevation:  $25^\circ$ ). Also shown are the original surface and the reconstructed surface. As we can see, the reconstruction and integration of the overlapping surface patches are quite accurate. An overlap of 4 pixels in both the  $x$  and  $y$  directions was used. Similarly, in Fig. 7, a reconstructed cylindrical surface is shown. The cylinder has its axis in the left bottom (azimuth:  $-30^\circ$ , elevation:  $20^\circ$ ).

Finally in Fig. 8, we show the intensity image of a partial face image. The image is  $43 \times 31$  pixels. The neural



**Figure 7.** a) The intensity pattern of a convex cylinder with a radius of 15 pixels and illuminated from an azimuth of  $-30^\circ$  and elevation of  $20^\circ$  with respect to the viewing direction. The viewing direction is such that the cylinder is approximately diagonal across the image. b) The original shape. c) The shape reconstructed by integrating overlapping patches of the reconstructed local shapes as explained in Section 4. As can be seen, the reconstructions are quite accurate.

network used for this was also trained with a multiplicity of surface types under different illuminations. The input image is due to an illumination with which the network is not trained. The original (Fig. 9(a-1) and Fig. 9(a-2)) and the reconstruction (Fig. 9(b-1) and Fig. 9(b-2)) are each displayed in two views. As can be seen, the reconstructed surface is remarkably accurate as compared to the original indicating that both the local patch reconstruction and the minimum sum squared error integration techniques are quite robust.

## 6. Conclusions

We have proposed a multiresolution framework for the reconstruction of smooth surface shapes from shading images. The method is based on using a neural network for learning brightness patterns and associating them with range data. We investigated two methods for training the network, one based on our novel weight diffusion process which enforces a local smoothness constraint and the other using the eigen coefficients of the input and output patterns which make the training computationally efficient. We also proposed an elegant and simple method for integrating reconstructed surface patches by minimizing the sum squared error in overlapped areas.

In our preliminary results, we have implemented one stage of the multiresolution framework. Thus, we have successfully reconstructed a variety of simple shapes like ellipsoids and cylinders as well as complex shapes like facial structure. In future, it is intended to develop the multiresolution framework in its entirety and incorporate local surface normal constraints as shown in Fig. 1. This could

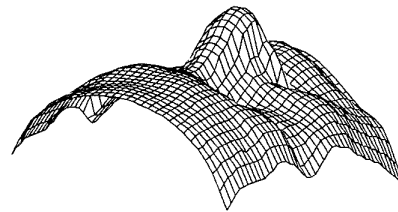
further improve the results of surface reconstruction.



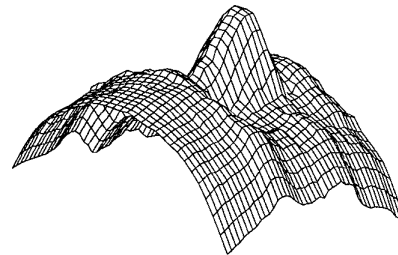
**Figure 8.** A partial face image illuminated frontally.

## References

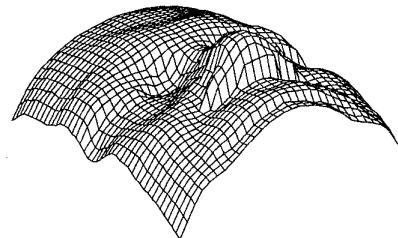
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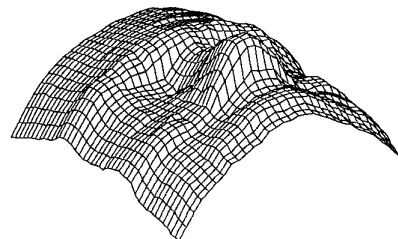
a-1) original



b-1) reconstruction



a-2) original - second view



b-2) reconstruction - second view

**Figure 9.** a-1) The original face shape. b-1) The reconstructed facial structure obtained by integrating overlapping patches of the reconstructed local shapes. a-2) The original face and b-2) the reconstructed face viewed from another angle. The reconstruction is quite accurate indicating that both the local patch reconstruction and the minimum sum squared error integration techniques are quite robust.