

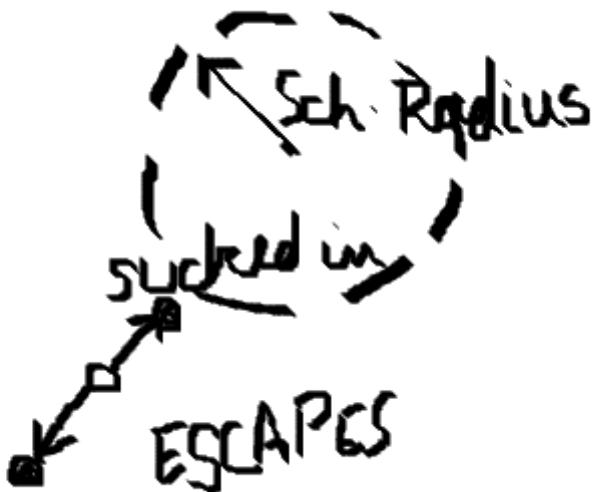
## Black hole radiation

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Fri, Aug 17, 2018 at 3:37 AM

Sorry, I was very very occupied.

So how does one particle able to escape into space when the other gets sucked into the black hole?  
The point you raised is that the particles should feel identical force at the same distance from the black hole centre, because they have the same mass too. That is correct. But the thing which allows one particle to escape is the **direction** of the velocity. Because the particle and the particle have the same mass, the conservation of momentum means that they must have **equal speeds in opposite directions**.



Now, the effect of gravity on each of the particles will be different because of the direction of their initial velocities. This is very obvious: a ball thrown **up** in the air at some initial speed decelerates, whereas a ball of the same mass, and thrown with the same initial speed but **towards** the Earth will accelerate. Then, when we take the initial speed  $\geq$  the escape velocity (speed) of the Earth, the first ball will escape into space, while the second falls to the surface. A similar thing happens with the two particles above- the first particle has initial velocity away from the centre and escapes, whereas the second particle has a velocity towards the centre and gets sucked in.

Now, this explanation is not obviously correct or complete. The problems with it can all be solved as below.

1. Shouldn't the 1st particle travel at or above the speed of light to escape a black hole?

No. The particles are not **at** the Schwarzschild radius distance from the centre. The escape velocity formula depends on the distance 'r' from the centre as the inverse square root. Now, if the point at which the particles are pair produced  $r = R \cdot d$ , where  $R$  is the Schwarzschild radius, then the escape velocity will become:

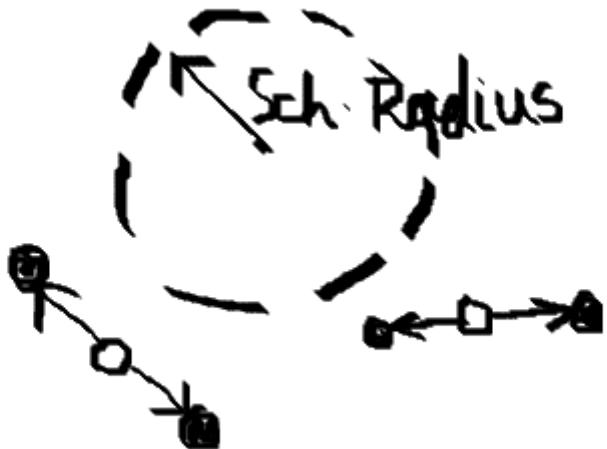
$$\sqrt{\frac{2GM}{r}} = \sqrt{\frac{2GM}{R \cdot d}} = \frac{c}{d^{1/2}}$$

and since  $d > 1$  for  $r > R$ ,

$$\frac{c}{d^{1/2}} < c$$

So they will **not** need to travel above  $c$ .

2. What if the particle-antiparticle pair produced does not appear in the above orientation? I.E., What if it is like this below?



The answer is: the **component** of the motion along the line joining the escaping particle and the centre i(the radial line) is the same as before. The p.v. is to be added to the component along the line perpendicular to this to get the full trajectory. If we work out the maths, we can see that this just means that the **component** of the initial velocity along the radial line must be  $\geq$  the escape velocity ( i.e.  $c/\sqrt{d}$ ), and the perpendicular component can be anything as long as the absolute value of the velocity remains under  $c$ . Under such conditions, one particle **will** escape and one **will** fall into the black hole.

Since the particles produced in a pair production will have different initial velocities in different productions, naturally, not all will have one particle flying away into space. Only those with a radial velocity above  $c/\sqrt{d}$  will escape. It is this fraction that constitutes the radiation.

3. All this is classical. Shouldn't we use QM to study the particles? And GR for the gravitation?

I don't know about the GR part, but I know that a spherically symmetric system in GR and Newtonian's gravity gives nearly identical results on all regular (not too small) scales. This is a spherically symmetric system. Also the Schwarzschild radius is defined using Newtonian gravity. Further, S. Chandrasekhar gives a lecture in which he treats black holes almost completely in the Newtonian style.

The QM part: the only thing that matters is if the **momentum** has some meaning. It does, the wavevector of the wavefunction varies in the same way as the classical particles' momentum.

Furthermore, we don't have to feel guilty about using a classical explanation at all.

Victor Weisskopf once gave a series of lectures at the then-newly inaugurated CERN. They were called "Modern Physics from an Elementary point of View". In that he gives completely different explanations of many modern phenomena using just our intuition and assumes that we are ignorant of the modern methods. In this way he even shows us that our guesses can lead to accurate predictions of experimental data, like stellar radiation, with minimal knowledge, and basic arithmetic. The calculation techniques in that tiny book are fantastic, and it is unbelievable how our own explanations and guesses are equivalent to the complex theoretical ones. An Indian Scientist, Dr. G. Venkataraman found a rare copy of that book, and decided to make it more accessible to Indian students. So he copied most of the book into a commentary called "Why are things the way they are?", which has added background material and diagrams. I think it is in the Matscience library, but I also have a copy here.

A few notes:

1. Interestingly, in school we only learn how about the trajectory of an object in a Newtonian gravitational field in few different conditions: under some conditions it is either an ellipse, or parabola (near the Earth/any relatively biiig body). But when we try to derive it for the general case, it shows us that the trajectory can be, for different initial conditions and masses involved, an ellipse, hyperbola, parabola or a circle. Newton simply proved straight away that, for any inverse square force like gravity, the object describes a **conic section**. The story is that Edmund Haley was at a meeting of the Royal Society where someone suggested that the Sun was **pulling** the Earth towards it and this was what caused the orbit. Halley thought this made no sense, because any such force will just pull the Earth closer- and into- the Sun. He had not realised the significance of **initial conditions**. (See Feynman's nice explanation of this at: [http://www.feynmanlectures.caltech.edu/I\\_07.html](http://www.feynmanlectures.caltech.edu/I_07.html).) When he angrily left and went to tell Newton what he had heard, he found out that Newton had already proven this rigorously months ago, and had also identified the force as gravity. When he showed Halley his notes, Halley persuaded him to publish them and this led to the publication of the "Principia". The modern type of proof using solving one ODE and then complex integration is very simple (and interesting), and I did it only recently for the first time. I have not found it in any school book so far, but I think Goldstein has it as a problem or something. I can send it to you if you want.

2. The thing about accelerating in one direction or decelerating in the opposite is a **defining property** of **conservative forces**. You can even define conservative forces as ones which will accelerate bodies with some initial

velocity and decelerate bodies with opposite and equal initial velocity. As you can see, "friction" does not fit this- it can decelerate objects with any initial velocity.

Anyway, hope you're doing good.

Tell me what you think,

Pitambar Sai Goyal