# Clarifications and Corrections

### 1. $f$: R→R

The author has used "**R**" in many places where "(0,1)" was correct.

Further, when all the numbers we are dealing with are *bounded*, we can *normalise* them to lie in (0,1). This was the idea behind the writing of $g(x)$ as $\max(g) \times h(\text{something})$ in section (5),"Finally"; $h$ is the same as $g$ normalised to lie in (0,1). For more abstract purposes, one can also completely encode the real numbers with a bijection (0,1)→**R** by either performing the mapping: $x \mapsto \tan \pi\left(x + \frac{1}{2}\right)$, or by using the digit "2" to mark the radix-point of a base-2 real number and interpret a sequence of 0's and 1's with one "2" after the radix point as a real in (0,1) written out in base $\geq 3$.

### 2. Error in Fourier coefficients' expressions

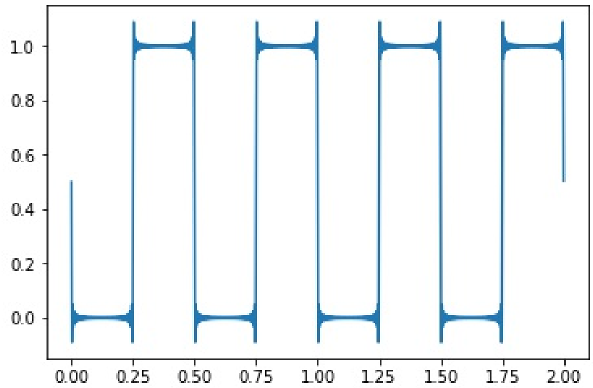
There was an arithmetic error in Fourier expanding the function that yields the $N^{\text{th}}$ digit of $x \in (0,1)$.

Solving te problem directly, you get:

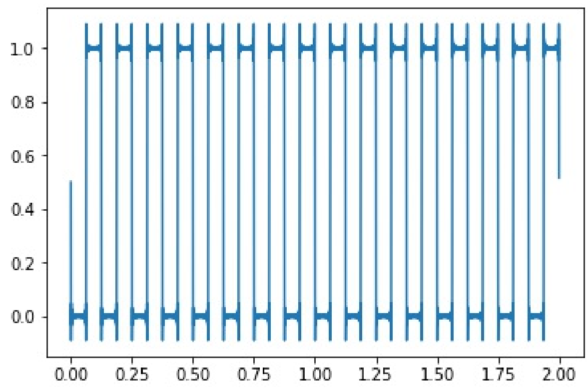
$$N^{\text{th}}\,\text{digit of}\; x = \frac{1}{2} + \Sigma_{\nu=1}^{\infty} \frac{-2(\nu \bmod 2)}{\pi\nu} \sin 2^N \pi \nu x$$

This was further verified by the author numerically. Here is what you get on plotting this function of $x$ for $N = 2$ and $N = 4$, with the summation being performed upto 100 times:

$N = 2$




$N = 4$



Furthermore, this requires the computations in the last section to be corrected as below:

$$g(x_1, ...., x_n) = \max(g) \times h\left( \left\{ \frac{1}{2} + \Sigma_{\nu=1}^{\infty} \frac{-2(\nu \bmod 2)}{\pi\nu} \sin 2^N \pi\nu x_i \right\}_{i=1,....,n; N=1,...\infty} \right)$$

and for Cantor's map:

$$g(x_1, ...., x_n) = \Sigma_{j=1}^{n} \Sigma_{m=1}^{\infty} 2^{-j \times m} \left( \frac{1}{2} + \Sigma_{\nu=1}^{\infty} a_\nu \sin 2^m \pi\nu x_j \right)$$

with:

$$\frac{\partial g}{\partial x_k} = \Sigma_{m=1}^{\infty} 2^{-k \times m} \Sigma_{\nu=1}^{\infty} \frac{a_\nu}{2^m \pi\nu} \cos 2^m \pi\nu x_k$$

where $a_\nu = \frac{-2(\nu \bmod 2)}{\pi\nu}$.

From this, it should be easy to guess all the related corrections.

**3. About the "full form"**

In section (3), "Our solution", under "How, then, do we explicitly represent this class?", the variables $p_I$ and $q_I$ are actually supposed to be functions of $x$, which only need to be of the form:

$$\eta_I 2^{-[x] + P_I} \text{and } \mu_I 2^{-[x] + Q_I}$$

respectively.

**4. The point of it all**

By repeating the word "differentiation", the author has drawn away from the original purpose- to make functions of time, which have somewhat strange structure- nowhere continuous- amenable to traditional (or quasi-traditional) analysis. Perhaps the author would have done better to, after estimating the velocity of the particle traveling along the path suggested by Cantor's map, included an example to support this. (We may study "strange(r)" potentials, driving forces etc. in this way, in addition to paths.)

Anyway, few are given below.

**In dynamics.**

We take:

$$\frac{d^2 x}{dt^2} = \Sigma_{j=1}^{\infty} 2^{-2j} \left( \frac{1}{2} + \Sigma_{\nu=1}^{\infty} a_\nu \sin 2^j \pi\nu x \right)$$

$$\Rightarrow x(t) = \frac{t^2}{6} + c_1 t + c_0 + \Sigma_{\nu=1}^{\infty} \frac{-a_\nu}{(2^j \pi\nu)^2} \sin 2^j \pi\nu x$$

The force here takes the coordinate $x$, removes its integer part, and inserts zeroes inbetween every consecutive digit (base 2). It is a periodic force field.

**Regarding canonical quantisation-** a minor curiosity that someone brought up earlier.

How do we quantise the quantity:

$$F(\boldsymbol{x}, \boldsymbol{p}) = \Sigma_{j=1}^{\infty} 2^{-2j} \left( \frac{1}{2} + \Sigma_{\nu=1}^{\infty} a_\nu \sin 2^j \pi\nu \boldsymbol{p}.\hat{\boldsymbol{i}} \right)$$

where, similarly, we insert a zero in between all consecutive digits of the $x$-coordinate of momentum (in base 2).

We proceed as:

$$\hat{F} = \Sigma_{j=1}^{\infty} 2^{-2j}\left(\frac{1}{2} + \Sigma_{\nu=1}^{\infty} a_\nu \sin 2^j \pi\nu \hat{p_x}\right) = \frac{I}{6} + \Sigma_{\nu=1}^{\infty} a_\nu \Sigma_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)!} \hat{p_x}^{2m-1}$$

$$= \frac{I}{6} - \Sigma_{\nu=1}^{\infty} a_\nu \Sigma_{m=1}^{\infty} \frac{i h^{2m-1}}{(2m-1)!} \frac{\partial^{2m-1}}{\partial x^{2m-1}}$$

You will also notice that we are leaving all analyses of convergence for this. As is known, the non-convergence of Fourier series' expressions as power series is most common, but this does not detract anything from its correctness.

### 5. "Class of functions much larger than $C^1(\boldsymbol{R})$"

The document stated, multiple times, that continuous analysis ("differentiability") will be extended to a larger class than $C^1(\boldsymbol{R})$ with the given construction. But it appears that there is no explicit representation of that class anywhere in it. There is, for the same reason, a suggestion that the writer has erred in estimating the size of this class. All clarifications below.

The class which we discussed extensively, i.e., the class of all *finitely expressible* functions on the reals, is $\aleph_0$-cardinal, which makes it *smaller* than $C^1(\boldsymbol{R})$- even smaller, in fact, than the set of all piecewise functions $f: \boldsymbol{R} \to \boldsymbol{R}$. However, it is not *contained* in either of those classes.

The generalisation, therefore, has been made to *include* all finitely expressible real-valued functions on the reals, therefore making the functions in:

$$C^1(\boldsymbol{R}) \cup \text{The set of all finitely expressible functions } f: \boldsymbol{R} \to \boldsymbol{R}$$

amenable to quasi-traditional analysis.

Indeed, by a very similar construction we can represent all functions in $\boldsymbol{R}^{\boldsymbol{R}}$, but that generalisation will rarely be more useful than the highly unspecified expression

$$h\left(\left\{\frac{1}{2} + \Sigma_{\nu=1}^{\infty} \frac{-2(\nu \bmod 2)}{\pi\nu} \sin 2^N \pi\nu x_i\right\}_{N=1}^{\infty}\right)$$

used in the document. This is generally far more agreeable.

### 6. Floor function

In section (3)-"Our solution", under "How to represent this class?", it is written as a footnote to the definition of the "[ ]" function:

"The floor function was originally considered, but was ruled out after the considerations in section 3, later."

But no explanation was given. The reason the floor function is not employed is - it does *not* coincide with the "$[x]$" operation for negative $x$.

### 7. Real numbers

There was no intention on the author's part to write an exact paper with the given document- hence the ostensible terseness of some of the argument. Indeed, as can be seen, this list of corrections, in order to keep the combination of the original document and the corrections lightweight, is also not comprehensive enough.

The abstractions in "Just an Idea" seem to boil down to the fact that we are able to comfortably tackle certain "strange" functions, because of a certain generalisation. And, further, such a generalisation is necessary because real numbers are ultimately infinite sequences of digits, and the ability to make the fullest possible use of a real number warrants the capability of arguing every single digit.