

## Lesson 4

### The sequences

A sequence is a list (a set) of numbers called terms, generated by a rule (a pattern).

$u_n$  is called the  $n$ th term of the sequence.  $n$  is the rank or the index

A sequence can be given by two different ways

- sequences with general formula (explicit sequence)  
ex :  $a_n = 2^n - 3n$  (a sub  $n$  (or a  $n$ ) is 2 to the power of  $n$  minus 3  $n$ ) (explicit formula)  
to calculate  $a_{25}$  you plug in 25 whenever  $n$  is, you put 25 instead of  $n$
- recurrent sequences  
ex :  $a_n = 3a_{n-1}$  or  $u_n = u_{n-1} + u_{n-2}$  (Fibonacci sequence) (recursive formula)  
to work out  $a_{25}$  you need to know  $a_{24}$  ..

### the arithmetic sequences

In an arithmetic sequence, each term is the result of adding the same number to the previous term ; this number is called the common difference.

Ex : 10, 5, 0, -5, -10 ... this is an arithmetic sequence with 10 as first term and a common difference of 5.

If  $a_0, a_1, a_2, a_3, \dots, a_n$  is an arithmetic sequence with  $d$  as common difference then

$$\text{the } n\text{th term is : } a_n = a_0 + n \times d$$

ex: let  $(a_n)$  be an arithmetic sequence with  $a_1 = 3$  and  $d = 5$ , write its explicit formula

Gauss was the first to discover the formula to find the sum , he was 9 y.o !!

$$1+2+3+4+\dots+n = \frac{n(n+1)}{2}$$

More generally, the sum of the  $n$  first terms of an arithmetic sequence is given by the formula :

$$S = \frac{\text{number of terms}(\text{1st one} + \text{last one})}{2}$$

### the geometric sequences

With a geometric sequence, each term is the result of multiplying the same number to the previous term ; this number is called the common ratio.

Ex :  $\frac{3}{2}, \frac{-1}{3}, \frac{2}{27}, \frac{-4}{243}$  ... this is a geometric sequence with  $\frac{3}{2}$  as first term and a ratio of  $\frac{-2}{9}$

If  $a_0, a_1, a_2, a_3, \dots, a_n$  is a geometric sequence that has a ratio of  $r$  then

$$\text{the } n\text{th term is : } a_n = a_0 \cdot r^n$$

The sum of the  $n$  first terms of a geometric sequence is given by the formula :

$$S = \text{1st term} \times \left( \frac{1 - \text{ratio}^{\text{number of term} - 1}}{1 - \text{ratio}} \right)$$