Lesson 2

Functions
Quadratic functions

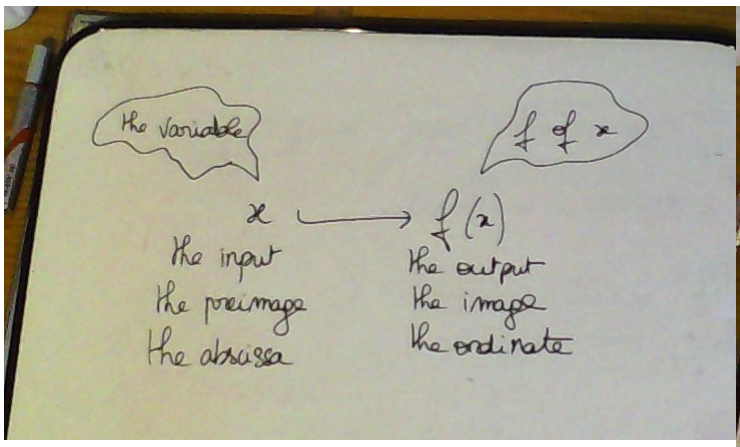
Part 1 : What is a function ?

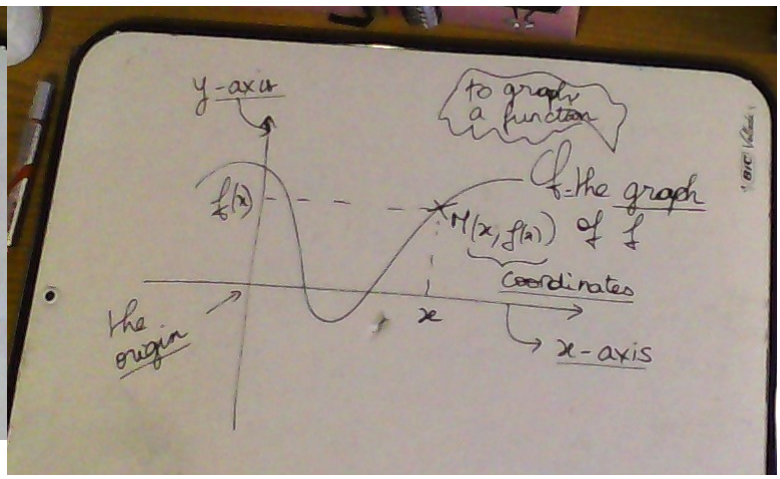
A **function**= a process, a relationship, where each input number has a single output.
The **Domain** of a function= part of the set of real where the function can be defined
example :

if $f(x)=x^2-3x+2$, the domain of f is $\mathbb{R}$

if $f(x)=\frac{1}{x-3}$ the domain is $\mathbb{R}\setminus\{3\}=\ ]-\infty;3[\cup]3;+\infty[$ because dividing by 0 is forbidden

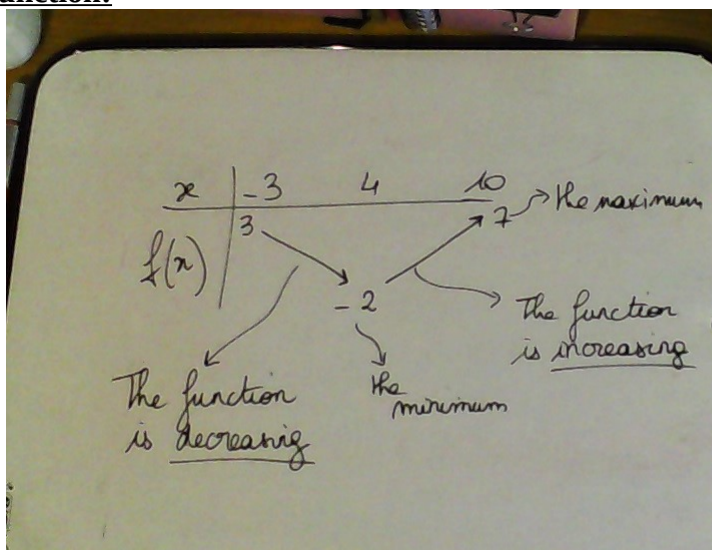
https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:functions/x2f8bb11595b61c86:evaluating-functions/v/what-is-a-function





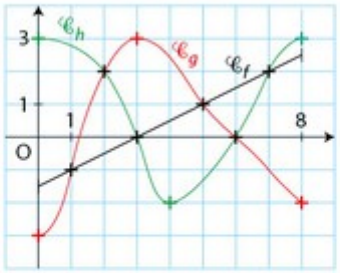
different sorts of functions=
linear function: functions with f(x)=mx+p as formula. Their graph is a straight line
square function: f(x)=x²
quadratic functions: functions with f(x)=ax²+bx+c. Their graph is a parabola
cube function: $f(x)=x^3$, its graph is a hyperbola

Variation table of a function:



Exercises:

In this coordinate system, $\mathscr{C}_f$, $\mathscr{C}_g$ and $\mathscr{C}_h$ are graphs of the functions f, g and h for the interval $[0\,;\,8]$.



In each case, find the solution set with a graphic reading.

a) $f(x) \leqslant g(x) \leqslant h(x)$

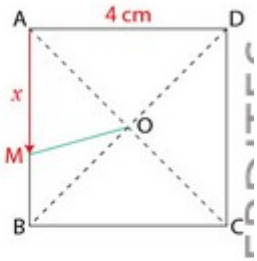
b) $h(x) \leqslant g(x) \leqslant f(x)$

c) $f(x) \leqslant h(x) \leqslant g(x)$

Square ABCD has a side of 4 cm and a centre O.

The point M moves around the perimeter of the square in alphabetical order starting from A.

x is the distance travelled by M from A and f the function giving the straight line distance OM, depending on x.



Do not try to find a general formula for $f(x)$.

1. a) Explain why f is defined on $[0\,;16]$.

b) Compute $f(x)$ when x is equal to 0; 2; 4; 6; 8.

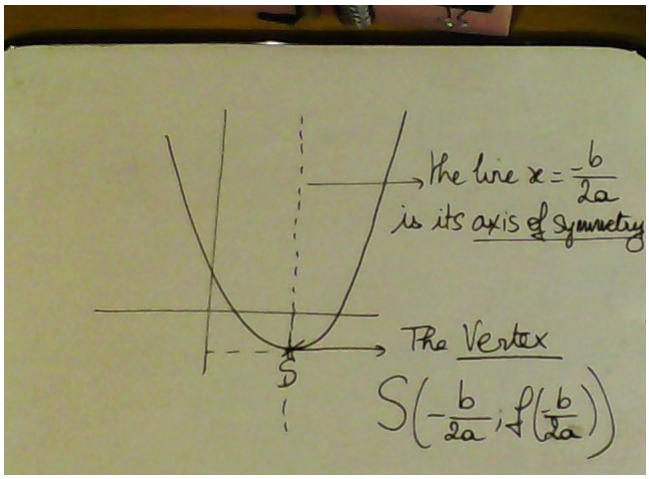
2. a) Describe the variations of f.

b) Give the extremes of function f on $[0\,;16]$. For what values of x are they reached?

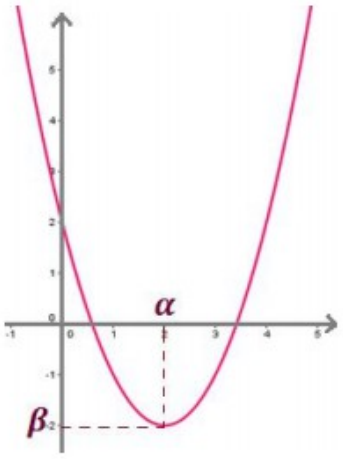
Part 2: the quadratic functions

$$f(x)=a^2x^2+bx+c\ ,\ a\neq 0$$

the graph of a quadratic function is **a parabola**

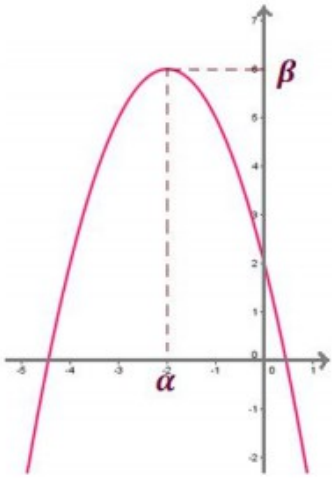


If $a > 0$

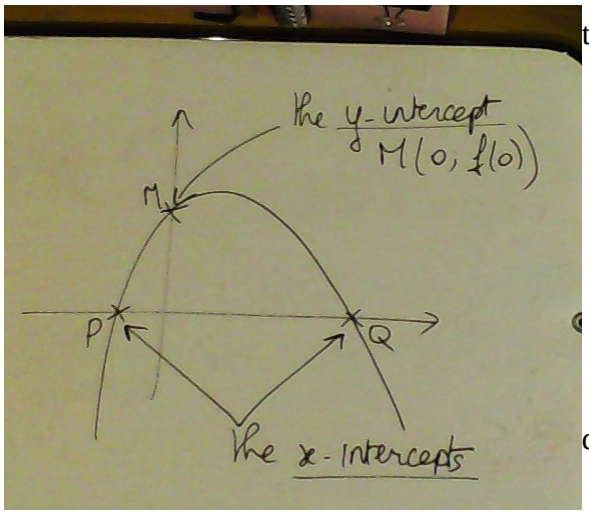


The curve opens up.

If $a < 0$



The curve opens down.



the number of x-intercepts depends on the value of $\Delta = b^2 - 4ac$ named **the discriminant:**

- if %DELTA<0, there is no x-intercept
- if %DELTA=0, there is only one x-intercept, which is also the vertex of the parabola S
- if %DELTA>0, there are 2 x-intercepts P and Q:

$$P\left(\frac{-b-\sqrt{\Delta}}{2a};0\right) \text{ and } Q\left(\frac{-b+\sqrt{\Delta}}{2a};0\right)$$

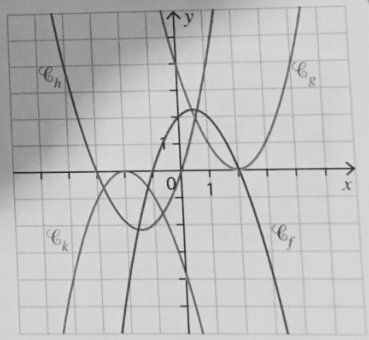
$x_1 = \frac{-b-\sqrt{\Delta}}{2a}$ and $x_2 = \frac{-b+\sqrt{\Delta}}{2a}$ are the **roots** of the quadratic equation $ax^2+bx+c=0$

Exercises:

exercise 1:

2 Matching

For each parabola give the sign of a and the number of roots.



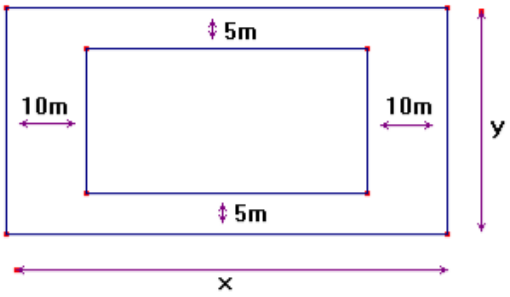
Exercise 2:

The flag of England is a rectangle with 2 m for width and 3 m for length. The background is white croissed with a red cross.

The cross is made with two rectangles with the same width. Which width should we choose for the cross if we want that red and white part to have the same area?

Exercise 3:

A fish-farmer wants to use 1,800 m² of field to dig a new rectangular tank surrounded by a terrace, as shown below :



1. Express y in terms of x.

2. What are the values taken by x ?

3. Express the area of the tank $A(x)$ in terms of x.

4. The farmer wants to breed as many fish as possible : find the value of x that makes the area of the tank maximum and give the dimensions of the tank.