Hassling Add on

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1 Introduction to the Model

This model aims to formalize some of the points made in Gannon et al., specifically, that gray zone conflict may represent a state's "optimal strategy," or gray zone conflict may be the best the state can do in the face of deterrence (i.e. the existence gray zone conflict is "the result of deterrence success").

Referring to the existence of gray zone conflict as an "optimal strategy" or "the result of deterrence success" is, admittedly, partially misleading. If gray zone conflict is occurring, then the state using gray zone conflict is choosing that strategy over going to war, thus making it optimal even in the case when gray zone conflict is deterrence success. A way of distinguishing these two points is as follows. If gray zone conflict represents a state's "optimal strategy," then a reduction in the targeted state's wartime capabilities would not alter the selected level of gray zone conflict. If gray zone conflict is "the result of deterrence success," then a reducing in the targeted state's wartime capabilities would change the level of of gray zone conflict.

Put another way. Russia's use of gray zone conflict in Ukraine may be interpreted as deterrence success. In this interpretation, the threat of NATO's involvement is a binding constraint, and Russia is tailoring it's involvement so that its level of gray zone conflict is on "the knife edge of war" (something like this is a Mazaar quote). In a different interpretation, Russia may be using gray zone conflict because this is the best strategy they have for achieving their objectives in Ukraine. In this interpretation, Russia may be selecting a level of conflict that falls below the "knife edge of war" because using gray zone conflict up to this knife edge may be suboptimal.

This is a point also made by Frank Hoffman. He points out "[Cases of gray zone conflict] clearly demonstrate that states that lack the capability to gain their strategic objectives with conventional means can find ways to erode the international order or to paralyze responses by other states through ambiguously aggressive actions. They also demonstrate that states that do possess the necessary conventional means may determine that their objectives can be achieved without resorting to conventional war and that this "gray zone" of war may actually suit their purposes better."

Modeling gray zone conflict is helpful for two reasons. First, the discussion above has been done without considering any kind of diplomacy used to prevent gray zone conflict. How does the dynamic described above hold when the targeted state can make political concessions? Second, a formal model identifies the relevant parameters for distinguishing between the two cases.

This model considers one form of gray zone conflict, hassling, which is one-sided gray zone conflict (used by one state against a targeted state). This paper will assume that hassling (a) creates costs for the targeted state and the state conducting the hassling, and (b) has some ability to shift political outcomes.

2 Game Form

Consider a modification of the bargaining model in **?** without uncertainty. This is a one-period model where one state makes an offer, then the other state can accept the offer, go to war, or hassle as a way to achieve a greater portion of the offer. If hassling occurs, the target state has the opportunity to escalate to war.

2.1 Actors

There are two actors in the game, State A and State D. State D possesses the entirety of an asset valued at 1. At times, these states will be referred to as A and D.

2.2 Game Order

At the start of the game, D makes an offer of $x \in [0, 1]$ to A or goes to war. Notably, if any actor chooses to go to war at any point, the game terminates and states receive their expected war utility for the remaining periods.

Second, A can accept the offer, go to war, or hassle. The degree of hassling is denoted by $h \in [0, 1]$. Higher values of h indicate greater degrees of harassment, and $h = 0$ indicates A accepting the offer. If A sets $h_t = 0$, the game terminates and utilities are realized. If A selects war, then the game terminates. If A selects some $h \in (0, 1]$, then the game enters the third stage.

Third, D can respond to by either going to war (denoted "war") or accepting the hassling (denoted "accept"). After D's selection, the game terminates.

2.3 Per-Period Utility Functions

$\mathbb{E}U_A$ and $\mathbb{E}U_D$ denote A's and D's expected utilities. When D makes offer x and A selects $h_t = 0$, the expected utilities are

$$\mathbb{E}U_A = x,$$

$$\mathbb{E}U_D = 1 - x.$$

The expected utilities from A or D going to war are

$$\begin{aligned}\mathbb{E}U_A =& p - c_A, \\ \mathbb{E}U_D =& 1 - p - c_D.\end{aligned}$$

The variable p is the probability A wins in war. $c_A \geq 0$ and $c_D \geq 0$ denote each state's costs of war.

The expected utilities from hassling are

$$\begin{aligned}\mathbb{E}U_A =& x + (1-x)\alpha h - K_A(h), \\ \mathbb{E}U_D =& (1-x)(1-\alpha h) - K_D(h).\end{aligned}$$

Hassling is a costly technique that transfers a portion of the D's share of the asset to A. State A derives positive utility from the offer x, as well as any portion of D's share captured through hassling $\alpha h(1-x)$. Hassling leaves D with share $(1-x)(1-\alpha h)$.

Hassling is also costly, which is formalized in the $K_A(h)$ and $K_D(h)$ terms. Both functions are assumed to be increasing, differentiable, and weakly convex, and take values $K_A(0) = 0$ and $K_D(0) = 0$.

3 Equilibrium

This model assumes subgame perfection.

3.1 Period 2

Working backwards, consider A's optimal behavior conditional on not invoking D to go to war.[1] If State A chooses to hassle, A will select a level of hassling h_2^* such that

$$h^*(x) \in \underset{h}{argmax}\{x + \alpha h(1-x) - K_A(h)\} \tag{1}$$
$$s.t. h \in [0,1],$$
$$\&(1-x)(1-\alpha h) - K_D(h) \geq 1 - p - c_D.$$

The bottom constraint implies in the third move D will not go to war.

It is possible to identify h^* explicitly. When the constraints do not bind, the optimal level of hassling is

$$\tilde{h} s.t.\, \alpha(1-x) = K_A'(\tilde{h}).$$

When the bottom constraint binds but the $h \in [0,1]$ constraint does not bind, then the optimal level of hassling is

$$\hat{h} s.t.\, (1-x)(1-\alpha\hat{h}) - K_D(\hat{h}) \geq 1 - p - c_D.$$

It is then possible to define h^* as

$$h^*(x) = \begin{cases} \tilde{h}(x) & \text{if } \tilde{h}(x) \leq \hat{h}(x) \,\&\, \tilde{h}(x) \in [0,1] \\ \hat{h}(x) & \text{if } \tilde{h}(x) > \hat{h}(x) \,\&\, \hat{h}(x) \in [0,1], \\ 0 & \text{if} \tilde{h}(x) > \hat{h}(x) \,\&\, \hat{h}(x) < 0, \\ 1 & \text{if} \tilde{h}(x) > \hat{h}(x) \,\&\, \hat{h}(x) > 1. \end{cases}$$

Next consider D's actions in the first move of period 2. By definition, D receives a weakly

[1] This section will later consider A's decision to go to war rather than use hassling. However, because A can always choose to go to war instead of hassling, it is unnecessary to consider A selecting a hassling level that would compel D to go to war.

better payoff when $\tilde{h}(x)$ is implemented relative to $\hat{h}(x)$. Let $\tilde{X}$ define the set of offers x such that $\tilde{h}(x) \leq \hat{h}(x)$. Additionally, $\bar{X}$ define the set of offers x where A does weakly better hassling rather than going to war, or $x + \alpha h^*(x)(1-x) - K_A(h^*(x)) \geq p - c_A$.

When the intersection of $\tilde{X}$ and $\bar{X}$ is non-empty, then D will select the x from this set that maximizes D's expected utility utility. When $\tilde{X} \cap \bar{X} \neq \emptyset$, D will receive a payoff that is weakly greater than D's war payoff. When the intersection of $\tilde{X}$ and $\bar{X}$ is empty, then so long that D makes an offer of $x \in [0,\, p + c_D]$, D will receive the war payoff either (a) because A hassles up to that point, (b) because A goes to war, or (c) because an offer of $p + c_D$ is made to A, and A cannot do better than this (because $p + c_D > p - c_A$). When $\tilde{X} \cap \bar{X} = \emptyset$, any offer where $x > p + c_D$ will result in a lower payoff for D.

It is therefore possible to define D's optimal offer x^* as:

$$x^* = \begin{cases} \underset{x \in \tilde{X} \cap \bar{X}}{argmax} \{(1-x)(1-\alpha h^*(x)) - K_D(h^*(x))\} & \text{if } \tilde{X}_2 \cap \bar{X}_2 \neq \emptyset, \\ \\ \text{any } x \in [0,\, p + c_D] & \text{otherwise.} \end{cases}$$

Finally, it is possible to define equilibria behavior.

Lemma 1: The following behavior constitutes a subgame perfect equilibria.

If $\tilde{X} \cap \bar{X} \neq \emptyset$, then D offers $x = x^*$. A then selects $h = \tilde{h}(x)$. D accepts.

If $\tilde{X} \cap \bar{X} = \emptyset$, then D offers some $x \in [0,\, p + c_D]$. If $x + \alpha h^*(x)(1-x) - K_A(h^*(x)) \geq p - c_A$, then A selects $h^*(x)$. D accepts.

If $\tilde{X} \cap \bar{X} = \emptyset$, then D offers some $x \in [0,\, p + c_D]$. If $x + \alpha h^*(x)(1-x) - K_A(h^*(x)) < p - c_A$, then A goes to war.

Proof: Follows from construction.

4 Discussion

This model represents the first time that diplomacy before possible gray zone conflict was possible. Allowing for an endogenous offer provides several insights.

First, it suggests one reason why gray zone conflict exists at all. Hassling can arise in equilibrium due to State A's inability to commit to not hassle. Consider the equilibrium stemming from the following parameter values:$c_A = 0.9$, $c_D = 0.9$, $K_A(h) = 0.5h + h^2$, $K_D(h) = 0.5h$, $p = 0.5$, $\delta = 0.8$, $\alpha = 1$. This results in an equilibrium with hassling. Under these parameter values, D offers A $x = 0$, A selects $h = 0.25$, and D accepts this hassling. There are two interesting aspects to this equilibrium. First, while both A and D could do weakly better by agreeing to a peaceful settlement of $x = 0.25$ and $h = 0$, this is not sustained as a peaceful equilibrium because A could deviate from this, select $h = 0.125$, and do better. Thus, because A cannot commit to not hassle, D takes this into consideration when making A an offer, and may choose offers knowing that A will hassle. Second, while it would be possible to identify an offer that would keep A from hassling (here $x = 0.5$), D prefers to simply allow the hassling to occur as accepting hassling gives D a greater utility– under the hassling equilibrium, $U_D = 0.625$.

Second, this section formalizes the conditions where hassling is State A's "optimal strategy" and when it is "the result of deterrence success." In determining which is the case, the existence of offers in the set $\tilde{X} \cap \bar{X}$ is important. The equilibrium in Lemma 1 was such that any time D can make an offer within $\tilde{X} \cap \bar{X}$, D will make an offer from that set. Verbally, this means that any time D can make an offer that results in A willingly shading their level of hassling away from D's wartime constraint due to the costs or limited efficacy of hassling, D will do so. This implies that D is making an offer with the intent of pushing hassling into the state where it resembles A's "optimal strategy." In contrast, when $\tilde{X} \cap \bar{X}$ is empty, then no matter what offer D makes, A will select an action (hassling or war) that results in D receiving D's war payoff. Thus, when hassling arrives in this case, it is "the result of deterrence success" because the level of hassling stops at D's wartime constraint. Perhaps counter-intuitively, keeping D's costs of war fixed, D does worse when A's hassling is the result of deterrence success and relative to when D is choosing optimal strategy.