Models of Grey Zone Conflict

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Abstract

Russian support for rebels in Eastern Ukraine, cyberattacks like the Stuxnet virus against the Iranian nuclear program, China's expansion in the South China Sea, and many other examples demonstrate that grey zone conflict, or political conflict falling short of war, is a pervasive feature of the international order. However, this phenomena has received little formal attention. This paper identifies a specific type of grey zone conflict, hassling, where one side commits costly violence against the other to extract a larger share of a political settlement. This paper then develops a series of models describing this phenomena. Model 1 shows that a state's inability to commit to not hassle can lead to hassling occurring in equilibrium, despite the existence of a mutually beneficial peaceful offer. Model 2 demonstrates that uncertainty over how states value hassling can lead to war. Model 3 provides a game-free analysis of hassling models with information asymmetry and speaks to the conditions under which hassling is more likely to occur. Model 4 shows that because hassling can undermine a state's future capacity in wartime, despite the mutual costs states face in an equilibrium with hassling, a Pareto-improving peaceful equilibrium may not exist.

Part I

Introduction and Baseline Model

Contents

1 Introduction

When state and non-state actors interact, they may engage in destructive political conflict that falls outside of peace or conventional war. Here are six contemporary examples of this phenomena.

1. Russia in Ukraine: After Russia annexed Crimea in 2015, it has committed violence and supported rebel groups in the Donbass region of Ukraine.
2. Iran in Iraq: Following the 2003 US invasion of Iraq, Iran supported Abu Zarqawi, the Sunni militant who formed al Qaeda in Iraq (the precursor for the Islamic State). Later in the war, Iran also supported "Special Groups," Shia Paramilitary whose actions at times undermined Baghdad.
3. Pakistan in Afghanistan and India: Pakistan has provided explicit support and safe havens to militant and terrorist groups operating in Eastern Afghanistan and India.
4. Cyberwarfare against Iran: The designers of the Stuxnet computer virus (most likely the United States, Israel, or both) undermined Iranian uranium enrichment facilities to slow Iran's nuclear weapons program.
5. Palestinian groups in Israel: Groups like Hamas periodically engages in low-level conflict, launch rockets, or use suicide attacks against targets in Israel.
6. China's territorial disputes in the South China Sea: Recently China has claimed several islands, reefs, and passages within the South China Sea, effectively uprooting existing maritime boundaries. China has built islands, established military facilities, and has moved aircraft carriers into these disputed areas.

These examples are commonly identified in the policy community as "grey zone" conflict. This paper will define grey zone conflict as a political interaction where one or both sides use costly directed military technology to alter the status quo, and where both sides willingly do not escalate to a conventional or nuclear war.

The examples above demonstrate that grey zone conflict is an important feature of the international system today. And, grey zone conflict is not new. State sponsored piracy during the age of exploration falls under the above definition of grey zone conflict. Likewise for France's support of colonial rebels against Great Britain during the United States' Revolutionary War, and white Southerners committing violence to suppress black's political rights during the Reconstruction Era in America.

The existence of grey zone conflict, an inefficient and costly outcome distinct from war and peace, has not been embraced in the formal literature on conflict.[1] This paper addresses this shortcoming and demonstrates that the strategic option to resort to grey zone conflict has important implications for how scholars should think about war and peace.

This paper will explore a specific type of grey zone conflict, which will be referred to as "hassling." Hassling has one side committing grey zone violence against the other to extract a larger share of a political settlement. In other words, hassling is an inefficient means for one party to seize a larger share of the pie. In this regard, hassling differs from an activity like economic sanctions because sanctions are commonly viewed as mutually detrimental, while hassling, even if costly, is productive for the hassler. And, when one actor hassles another actor, and the actor being hassled does not escalate to war, hassling can be an equilibrium outcome. Each of the five contemporary examples of grey zone conflict above could qualify as hassling.

This paper is split into four parts. First, the paper discusses grey zone conflict and hassling in more depth, discusses similar literature, and introduces Model 1, the baseline model of hassling. Model 1 shows how a state's inability to commit to not hassle can lead to hassling occurring in equilibrium, despite the existence of a mutually beneficial peaceful equilibrium. In the next part, this paper considers one-period models with asymmetric information. Model 2 demonstrates that uncertainty over how states value hassling can lead to war. Model 3 provides a game-free analysis of a type of hassling models and speaks to the conditions under which hassling is more likely to occur. The next part discusses a dynamic model of hassling, Model 4. Model 4 shows that because hassling can undermine a state's future capacity in wartime, despite the mutual costs states face in an equilibrium with hassling, a Pareto-improving peaceful equilibrium may not exist. In the final part, the paper concludes by suggesting directions for future research.

[1] Powell (2015) and formal work on international sanctions are notable exceptions, as discussed below.

2 On Hassling

2.1 Defining Hassling

This paper defines grey zone conflict as a political interaction where one or both sides use costly directed military technology to alter the status quo, and where both sides willingly do not escalate to a conventional or nuclear war. This definition embraces the Mazarr (2015) definition of grey zone conflict being a tool of "measured revisionism" and place a particular emphasis on what grey zone conflict is not (peace or conventional war). This definition also rules out several activities, including weapons development (this is not directed), limited wars (where one or both actors limit their use of force, but where one or both sides are engaged in a conventional war), or economic sanctions (this does not use military technology). While this definition misses some activities that could qualify as grey zone conflict– for example, Russia's potential financial support for anti-EU candidates in France's election– it also encompasses all activities listed in the previous section.

The term "hassling" refers to a type of grey zone conflict. This paper will explore models that allow states to hassle, which is where one party commits grey zone violence against the other to extract a larger share of a political settlement. If hassling occurs, the state doing the hassling is better off and the target state is worse off, both relative to hassling not occurring. That being stated, because hassling uses destructive military technology, hassling does come with costs to both sides, making it inefficient relative to a peaceful settlement. This payoff structure can arise if the side doing the hassling is conducting all the violence, or if the target state does not aggressively respond.[2]

Most important to the analysis below, hassling is modeled as a unilateral action. For hassling to be observed in equilibrium, the state doing the hassling must gain from hassling, and the target state must be unwilling to offer the concessions necessary to prevent the hassling and unwilling to go to war.

It is useful to ground the discussion above into empirical examples. While each of the five examples given at the outset of this paper could qualify as a hassling, Pakistan's support of militants in Afghanistan is a particularly good fit. Given Pakistan's precarious geopolitical position of bordering India, China, and Afghanistan, several scholars have conjectured that Pakistan benefits from an allied and supportive or a destabilized Afghanistan (Rashid, 2010; Hussain *et al.*, 2005). Pakistan's activities in Afghanistan support this conjecture, as since

[2]Short of escalation to war, this paper will not analyze the strategy behind different degrees of responses to hassling. This is admittedly a shortcoming to this paper and should be the topic of future analysis.

the 1990s Pakistan's military has supported militants that were either (1) supportive of Pakistan, (2) able to destabilize the Afghan government, or (3) both. This hassling has clearly been detrimental to Afghanistan's development, but it has also weighed Pakistan with financial, military, and diplomatic costs. However, in the absence of an existing durable political solution, Pakistan seems willing to absorb these costs for the security benefits it provides. Additionally, this hassling has been occurring for some time; this duration suggests that hassling in this case is fairly stable and can be thought of as an equilibrium outcome.

2.2 Most Similar Research

To the best of the author's knowledge, this is the only paper to model the type of grey zone conflict (hassling) as described above. However, there are several similar formal papers that are substantively similar or model similar phenomena.

Substantively, this paper is similar to Schultz (2010), which models, among other political phenomena, one state supporting rebel groups within another state.[3] There are two key differences. First, Schultz models support for rebels as a policy outcome being negotiated over. This paper instead treats supporting rebels as an inefficient means that is used in lieu of a political solution. Referring back to the Pakistan example, Schultz assumes there is some agreed upon level of support that Pakistan can give to militants, and that deviating from this level can lead to war. This paper assumes there is a political means to appease Pakistan to keep Pakistan from supporting rebels in Afghanistan, but also that Afghanistan could go to war to attempt to resolve this issue.[4] Second, Schultz is primarily concerned with how war may arise due to uncertainty regarding the extent of rebel support (the implemented policy decision). Model 2 differs by demonstrating that uncertainty over a state's tolerance for grey zone conflict can also lead to war.

This paper is also similar to Powell (2015), which models limited war. Powell introduces a model with two types of conflict (conventional war and nuclear war) where states decide to (1) commit varying degrees or types of conventional forces to a conflict and (2) publicly commit to brinkmanship practices, which Powell models as a probabilistic mapping from the opposing state's commitment of conventional forces to the likelihood of a nuclear strike. This paper is different in the type of conflict it attempts to describe. The probabilistic brinkmanship decision embraced in Powell seems appropriate for capturing the risk of a nuclear exchange in the face of a conventional or limited war. However, the idea that a state's

[3]Schultz aims to model phenomena where one state is in full control of a policy position that is being bargained over, and where there is uncertainty regarding what the other state is doing. As another example, this model could also describe a state's investment in a covert nuclear program.

[4]Also, as it is modeled, Schultz treats varying degrees of support for rebels as efficient policy outcomes, given there are no additional costs for either state for greater levels of support- this paper assumes supporting rebels is inefficient.

decision makers commit to a probabilistic chance of escalation seems inappropriate when considering escalation from low-level conflict to conventional war. For example, if Pakistan engages in violence level p in Kashmir, it is not as if India has publicly committed with probability $r(p)$ to send troops to Kashmir, and with probability $1 - r(p)$ to not escalate.[5] This paper instead adopts the approach that a state responds to a low-level attack not through a predetermined probabilistic function but rather responds based on their preferences, over which there may be uncertainty. In other words, when Pakistan engages in low-level violence in Kashmir, India may find this unacceptable and escalate, but Pakistan is uncertain if India will behave in such a manner. This difference is important; in the model below, states may possess incentives to misrepresent in order to achieve more favorable outcomes, whereas this never happens in Powell.

This paper is also similar to work on the role of sanctions in international crisis bargaining. This paper is similar to that on sanctions because sanctions fall outside of war or peace and are an inefficient outcome. However, the literature on sanctions has generally embraced the empirical observation that sanctions are costly to both the target and sender of sanctions.[6] Because sanctions are bad for both actors, sanctions can be viewed as potential signals of resolve (as modeled in Gartzke *et al.* (2001) as a modification of Fearon (1997)) or as actions in a war of attrition game (like that modeled in Smith (1974)). As is modeled here, the inefficient action (hassling) can arise because one player can achieve a better outcome by doing so (in other words, hassling is cost effective). As shown in Coe (2014), this assumption can also apply to certain situations with sanctions; Coe shows that sanctions may destabilize a regime and lead to internal conflict in a way that benefits the state doing the sanctioning.

Model 4 below resembles work on arming and mobilization (Powell, 1993; Fearon *et al.*, 2007; Fearon, 2011; Kydd, 2000; Debs and Monteiro, 2014; Bas *et al.*, 2016). In these models, states select a level of arming or mobilization, and this alters the payoffs from war. In these models, arming and mobilization is costly, but equilibria with mobilization and arming can be more effective at preventing war than equilibria where mobilization or arming does not occur.[7] This is similar to hassling as examined here because hassling is costly, hassling alters the payoffs from war, and, as shown below, an equilibrium with hassling may exist in settings where a peaceful equilibrium does not. The key difference between between a hassling equilibrium and a peaceful equilibrium with arming or mobilization is that hassling can

[5]Of course the argument could be made that India may commit to a certain level of troops in Kashmir, that these troops may be fired upon, and this could escalate into war with some probability. This paper will assume that state's leaders do have some control over their military.

[6]Lektzian and Sprecher (2007) provides an excellent review of why this is the case.

[7]Peace can be fostered partly through mechanisms like tying hands or signaling, but also because mutual arming can have a deterrent effect.

be beneficial to the state doing the hassling, while arming or mobilization is not immediately beneficial in peace time. This difference alters the constraints that must hold to prevent war.

3 Model 1: Hassling can be sustained through commitment problems.

3.1 Model 1 Intuition

Hassling is an inefficient and destructive outcome relative to a peaceful, bargained solution. Why then do states fail to arrive at a bargained outcome instead of hassling?

To answer this question, two points on hassling should be introduced. First, hassling can occur after a deal or concession is made. The inability for a state to commit to not hassle can create perverse incentives at the bargaining stage. Second, hassling can only be so effective. The use of low-level violence, cyberwarfare, or support for militants is only be so effective in altering the political status quo. Additionally, hassling may be constrained if the hassling state fears the other side escalating to war.

Hassling is inefficient, which means a bargained political outcome exists where both states could be better off. Assume a setting where there is some political settlement and one state is hassling the other. While a different, peaceful political settlement does exist that is mutually beneficial, if states were to renegotiate to this point, there is nothing to stop the formerly hassling state from continuing to hassle. While there can exist a bargained offer that would prevent the potentially hassling state from hassling, this concession may be more than the target state is willing to make. Because hassling has its limitations, the target state may be unwilling to make concessions and instead accept some hassling rather than make the concession necessary to prevent any hassling.

To illustrate this point, consider Russia's involvement in Ukraine since 2014. Assume that Russia benefits from a fractured Ukraine both due to local security concerns and from any destabilizing effects a fractured Ukraine has on NATO. Presumably, there exists some bargained political settlement where Russia and Ukraine could mutually be made better off relative to the existing state of hassling. However, unless Ukraine or NATO were willing to make Russia significant political concessions, Russia could still likely benefit from some degree of hassling. Therefore, Ukraine or the EU may prefer to accept some Russian hassling rather than commit to a significant political concession that would fully accommodate Russia.

This model also makes a point on deterrence with respect to grey zone conflict. Specifically, the presence of grey zone conflict does not indicate a deterrence failure, even if the target state is attempting to deter grey zone conflict. The hassling state may be restricting the degree to which they are hassling due to not wanting to go to war.

A comment should be placed up-front. This is a baseline model that is later modified to have asymmetric information over preferences (in Models 2 and 3) or to be an infinite horizon game where hassling today affects states' wartime capacities tomorrow (Model 4). While the results here are straightforward, they should not be viewed as the final word on hassling.

3.2 Model 1 Game Form

Consider a modification of the bargaining model in Fearon (1995) without uncertainty. In this model, after a settlement is reached, one state can hassle the other as a way to achieve a greater portion of the settlement.

3.2.1 Actors

There are two actors in the game, State A and State D. At the onset of the game, State D possesses the entirety of an asset valued at 1. At times, these states will be referred to as A and D.

3.2.2 Game Order

First, D makes an offer of $x \in [0, 1]$ to A. Whatever offer is made, D keeps the remainder $(1 - x)$.

Second, A can accept the offer, go to war, or accept the offer then hassle. The degree of hassling is denoted by $h \in (0, 1]$, with higher values indicating stronger degrees of harassment. If A selects "war" or "accept," the game terminates and utilities are realized. If A selects some $h \in (0, 1]$, then the game goes into the third stage.

Third, D can respond to by either going to war (denoted "war") or accepting the hassling (denoted "accept"). After this action, utilities are realized.

3.2.3 Utility Functions

U_A and U_D will denote A's and D's utility from set of actions.

The utilities from peace are

$$
\begin{aligned}
U_A =& x, \\
U_D =& 1 - x.
\end{aligned}
$$

The utilities from war are

$$
\begin{aligned}
U_A =& p - c_A, \\
U_D =& 1 - p - c_D.
\end{aligned}
$$

When a state wins the war, they capture the entirety of the asset. p is the probability A wins in war, and $c_A \geq 0$ and $c_D \geq 0$ denote each state's costs of war.

The utilities from hassling are

$$
\begin{aligned}
U_A =& x + (1 - x)h - hk_{A1} - h^2 k_{A2} \\
U_D =& (1 - x)(1 - h) - hk_D
\end{aligned}
$$

Hassling is a costly technique that captures a portion of the D's share of the asset. A derives positive utility from the offer x, as well as any portion of the D's share captured through hassling $h(1 - x)$. Hassling leaves D with share $(1 - x)(1 - h)$. Hassling is also costly, which is formalized in the $-hk_A - h^2$ and hk_D terms. A's marginal costs are increasing in h because hassling becomes less efficient at share capture as it increases. To ground this assumption in reality, sponsoring rebels, engaging in low level violence, and cyberwarfare are only so effective at share capture; after a certain point, these activities begin costing more and delivering less. D's marginal costs of hassling are assumed to be linear.

3.3 Model 1 Equilibrium

This model assumes subgame perfection.

Consider D's and A's optimal behavior conditional on hassling occurring and D not going to war. Working backwards, the first order conditions imply that A will select optimal hassling level $\tilde{h} \in argmax_h \left\{x + (1 - x)h - hk_{A1} - h^2 k_{A2}\right\}$ (constrained by $\tilde{h} \in [0, 1]$) when not facing the risk of war, defined as

$$
\tilde{h}(x) = min \left\{1,\ max\left\{\frac{1 - x - k_{A1}}{2k_{A2}},\ 0\right\}\right\}.
$$

Alternatively, A may select a level of hassling that keeps D indifferent between accepting hassling and going to war. This is the value of h such that $(1-x)(1-h)-hk_D = 1-p-c_D$. This is defined as

$$\hat{h}(x) = min\left\{1,\ max\left\{\frac{c_D - x + p}{1 - x + k_D},\ 0\right\}\right\}.$$

It is then possible to, for a fixed x, define the following function

$$h^*(x) = \begin{cases} \tilde{h}(x) & \text{if } \tilde{h}(x) \leq \hat{h}(x) \\ \hat{h}(x) & \text{if } \tilde{h}(x) > \hat{h}(x). \end{cases}$$

$h^*(x)$ is the optimal level of hassling for A that will not trigger D to go to war.

Finally, it is possible to define D's utility as a function of x

$$U_D(x) = \begin{cases} 1-x & if \begin{cases} x > p - c_A, \ \& \\ x > x + (1-x)h^*(x) - h^*(x)k_A - h^*(x)^2 \end{cases} \\ \\ p - c_D & if \begin{cases} p - c_A > x, \ \& \\ p - c_A > x + (1-x)h^*(x) - h^*(x)k_A - h^*(x)^2 \end{cases} \\ \\ (1-x)(1-h) - hk_D & if \begin{cases} x + (1-x)h^*(x) - h^*(x)k_A - h^*(x)^2 > x, \ \& \\ x + (1-x)h^*(x) - h^*(x)k_A - h^*(x)^2 > p - c_A \end{cases} \end{cases}$$

The game's equilibrium is as follows:

Proposition 1.1. *In Model 1, the Subgame Perfect Nash Equilibrium has D start the game by selecting*

$$x^* \in argmax\,\{U_D(x)\}$$

Then A selects

$$acceptif \begin{cases} x > p - c_A, \,\& \\ x > x + (1-x)h^*(x) - h^*(x)k_A - h^*(x)^2 \end{cases}$$

$$warif \begin{cases} p - c_A > x, \,\& \\ p - c_A > x + (1-x)h^*(x) - h^*(x)k_A - h^*(x)^2 \end{cases}$$

$$h^*(x)if \begin{cases} x + (1-x)h^*(x) - h^*(x)k_A - h^*(x)^2 > x \\ x + (1-x)h^*(x) - h^*(x)k_A - h^*(x)^2 > p - c_A \end{cases}$$

Then, if A selected $h^*(x)$, D always accepts.

3.4 Model 1 Analysis

The equilibrium above contains several moving parts. For that reason, to analyze behavior, it is helpful discuss equilibrium behavior for a fixed set of parameter values.

Hassling can arise in equilibrium due to State A's inability to commit to not hassle. Consider the equilibrium stemming from the following parameter values: $c_A = 1$, $c_D = 1$, $k_{A1} = 0.5$, $k_{A2} = 1$, $k_D = 0.5$, $p = 0.5$. This results in a hassling equilibrium. Under these parameter values, it is optimal for D to offer A $x = 0$, A selects $h = 0.25$, and D accepts this hassling. There are two interesting aspects to this equilibrium. First, while both A and D could do weakly better by agreeing to a peaceful settlement of $x = 0.25$, this is not sustained as a peaceful equilibrium because A could deviate, select $h = 0.125$ and still do better. In this regard, the inability of A to commit to not hassle implies that hassling is always a reality that D must take into consideration. Second, while it would be possible to identify an offer that would keep A from hassling (here $x = 0.5$), D prefers to simply allow the hassling to occur as accepting hassling gives D a greater utility– under the hassling equilibrium, $U_D = 0.625$.

This model also demonstrates that the existence of hassling should not be viewed as a failure of deterrence. There are two reasons for this observation. First, an equilibrium with hassling implies that A was successfully deterred from war. This means that State A determined that they had too low a probability of winning war or that war was too costly to directly engage. Under the example in the previous paragraph, were the costs of war for state A much lower, then D would have needed to make a larger concession in order to avoid a costly war. Second, the threat of war can alter a state's hassling behavior. Under certain parameter values where D faces a low cost of war and a high probability of victory,[8] if State A is attempting to avoid war, then A will not select the hassling level defined by $\tilde{h}$ (State A's optimal level) and will instead choose $\hat{h}$, a lower level of hassling that would keep D from escalating to war. For both cases, hassling should not be viewed as an outright failure of deterrence.

Naturally questions may arise regarding the existence of peaceful equilibria within a dynamic setting. Analysis of prisoners-dilema-like games in economics and political science suggests that through repeated interactions and sufficient patience, Pareto-improving equilibria, like a peaceful equilibrium, can also be supported. Thus, a dynamic model will be discussed in Model 4.

Part II

Information Asymmetry and Hassling

4 Model 2. Uncertainty over a state's tolerance for hassling can lead to war.

4.1 Model 2 Intuition

Uncertainty over a state's tolerance for hassling can lead to war. This can still occur in a situation where any peaceful solution is preferred to war.

How does war occur in this situation? Referencing the model above, suppose there can be two types of state D. One type of state D (call this type "type 1"), if hassled, is willing to escalate and go to war. The other type of state D (call this type "type 2"), if hassled, would not be willing to go to war. Also suppose that State A prefers hassling to a peaceful settlement, but does not want to go to war. With these preferences, an equilibrium exists where sometimes type 2 Ds imitate type 1 Ds, sometimes State A hassles a type 1 D, and this leads to war. Essentially, sometimes State A believes a type 1 D is actually a type 2 D, A hassles, and type 1 Ds escalate.

The model below demonstrates that incentives to misrepresent preferences over hassling

[8] For example, $c_A = 1$, $c_D = .1$, $k_{A1} = 0.2$, $k_{A2} = 1$, $k_D = 0.8$, $p = 0.1$. Under these values, it is an equilibrium for $x = 0$, $h = 0.1667$, and for D to accept the level of hassling.

can lead to conflict. This is very similar to a well-established result that private information about the costs of war, the likelihood of winning war, or the value of the pie being fought over can all lead to war. The result here is different because it is shown in the setting where both states' payoff to war is strictly less than *any* peaceful settlement. In a typical one-period crisis bargaining game without hassling, these preferences imply that war would never occur because in the choice between war or any peaceful settlement, both sides prefer any peaceful settlement.[9] The introduction of hassling introduces a new way that war can arise between states.

This result is important in that it identifies a new way for war to emerge. Specifically, one state may hassle another while weighing the trade-offs between claiming a larger share of an asset and war occurring.

4.2 Model 2 Game Form

This model follows a similar structure as Model 1. There are two states (A and D) in a crisis over an asset. The game order is as follows.

First, this game begins with nature selecting D to be one of two types. Nature designates D to be Type 1 with probability θ and Type 2 with probability $1-\theta$ with $\theta \in [0,1]$. Type designates parameters in D's utility function and will be discussed below. While state D knows their own type, A does not observe this designation.

Second, D makes an offer of $x \in [0,1]$ to A. D keeps the remainder $(1-x)$.

Third, A can accept the offer, go to war, or accept the offer then hassle. Unlike Model 1, the degree of hassling is assumed to be a fixed $h \in (0,1]$. If A selects "war" or "accept," the game terminates and utilities are realized. If A selects some hasslng level h, then the game goes into the third stage.[10]

Forth, D can respond to by either going to war (denoted "war") or accepting the hassling (denoted "accept"). After this action, utilities are realized.

The payoffs for different outcomes are summarized below.

[9] This holds true for any game that ends with one state making an offer and the other state accepting the offer or going to war

[10] The decision to fix h is done as a technical simplification. If h is allowed to take all possible values between 0 and 1, the tension still exists that A would want to select different levels of hassling for different types of defenders and that hassling-tollerant type Ds want to pool with hassling-intollerant type Ds to achieve a greater peaceful settlement.

Eqm\Actor	**D Type 1**	**D Type 2**	**A**
Peace	$1-x$	$1-x$	x
Hassling	$(1-x)(1-h)-hk_{D1}$	$(1-x)(1-h)-hk_{D2}$	x+$h-xh-hk_A-h^2$
War	$(1-p)-c_{D1}$	$(1-p)-c_{D2}$	$p-c_A$

This Model will limit itself a specific preference ordering. This is done for the sake of more cleanly describing a specific type of setting. These preferences are discussed immediately below.

Assume for either state, any peaceful settlement exceeds their payoff from war. If this was a setting without hassling, D could offer A $x=0$ and A would always accept. Formally, this implies that $0>p-c_A$, $0>1-p-c_{D1}$, and $0>1-p-c_{D2}$.

Regarding State A's preferences, assume that so long as it does not result in war, A does strictly better hassling after some low offers. Formally, this implies $h-hk_A-h^2>0$, which is A's utility from hassling after an offer of $x=0$ is made. This assumption also implies that there exists an $x\in[0,1]$ where A is indifferent between hassling and accepting an offer; this offer will be denoted $\tilde{x}$, where $\tilde{x}=1-k_A-h$.

Regarding type 1 State D's preferences, assume that if an offer of $x=0$ is made and A hassles, type 1's are willing to go to war. Formally, this is $1-p-c^W_{D1}>1-h-hk_{D1}$.

Regarding type 2 State D's preferences, assume that D prefers the outcome where A accepts offer $\tilde{x}$ to the outcome where A hassles after offer $x=0$. Formally, this is $1-\tilde{x}>1-h-hk_{D2}$. Additionally, it is assumed that if, after an offer of $x\in[0,\tilde{x}]$, state A hassles, type 2 D's prefer to accept the hassling rather than go to war. Formally, this is $(1-\tilde{x})(1-h)-hk_{D2}\geq 1-p-c_{D2}$.

Taken together, it is possible to formalize and define the preference orderings.

Definition: *Under* ***Preference Ordering 2.1****,*

$$1 > h - hk_A - h^2 > 0 > p - c_A,$$

$$0 > 1 - p - c_{D1} > 1 - h - hk_{D1},$$

$$1 - \tilde{x} > 1 - h - hk_{D2} > (1 - \tilde{x})(1 - h) - hk_{D2} > 1 - p - c_{D2} \;\&\; 0 > 1 - p - c_{D2},$$

hold.

4.3 Model 2 Equilibrium

This section will discuss a semi-pooling equilibrium. In it, type 1 Ds make offer $x = 0$. Type 2 D's either make offer $x = \tilde{x}$, or will imitate type 1 Ds and make offer $x = 0$. State A will with some probability hassle after receiving an offer of $x = 0$; when A hassles a type 1 D, war occurs.[11] This section formally describes the equilibrium, then describes the additional conditions necessary to sustain it.

This semi-pooling equilibrium is sustained through indifference conditions. First, let α denote the probability that A hassles after an offer of $x = 0$. For a type 2 D to be indifferent between making an offer of $x = \tilde{x}$ and offering $x = 0$ (imitating a type 1 D), it must be that

$$1 - \alpha + \alpha(1 - h - hk_{D2}) = 1 - \tilde{x}.$$

Which can be re-written

$$\alpha = \frac{\tilde{x}}{h + hk_{D2}}. \tag{1}$$

Next, let β denote the probability that a type 2 D imitates a type 1 D by making an offer of $x = 0$. For State A to be indifferent between accepting this offer and hassling, it must be that

$$\frac{\beta(1-\theta)}{\beta(1-\theta)+\theta}\left(h - hk_A - h^2\right) + \frac{\theta}{\beta(1-\theta)+\theta}\left(p - c_A\right) = 0.$$

[11] Note that a fully separating equilibrium cannot exist. Consider a (hypothetical) fully separating equilibrium where type 1 D's offer A $x = 0$, and type 2 D's offer A $x = \tilde{x} = 1 - k_A - h$. If A was made an offer of $x = 0$, A would know hassling in this case would result in war because only type 1 D's offer $x = 0$. Thus, A would never hassle if made offer $x = 0$ and would always accept if made offer $x = 1 - k_A - h$. However, in this setting, type 2 D's could imitate type 1 D's, make an offer of $x = 0$, never go to war, and remain strictly better off.

This can be re-written

$$\beta = \frac{\theta(c_A - p)}{(1-\theta)(h - hk_A - h^2)}. \tag{2}$$

Due to Preference Ordering 2.1, α and β as defined above fall within $[0, 1]$.

Before introducing the equilibrium, an assumption must be in place.

Assumption 2.1*: Assume*

$$\alpha(p - c_{D1}) + (1 - \alpha) \geq 1 - \tilde{x}. \tag{3}$$

holds.

Assumption 2.1 implies that type 1 Ds prefer making offer $x = 0$ and occasionally going to war over a peaceful equilibrium where they offer $\tilde{x}$.

With these in place, it is possible to introduce Proposition 2.1.

Proposition 2.1. *For model 2, supposes states possess Preference Ordering 2.1 and Assumption 2.1 holds. A semi-pooling equilibrium exists with the following sequence of play and beliefs.*

1. *Type 1 Ds offer $x = 0$. Type 2 Ds offer $x = 0$ with probability β and $x = \tilde{x}$ with probability $1 - \beta$.*
2. *State A accepts an offer of $\tilde{x}$. If State A receives an offer of $x = 0$ they hassle with probability α and accept with probability $1 - \alpha$.*
3. *If type 1 Ds are hassled, they go to war. If type 2 Ds are hassled, they accept.*

A believes that if $x = 0$ is offered, then State D is type 1 with probability $\theta/(\beta(1-\theta)+\theta)$ and type 2 with probability $\beta(1-\theta)/(\beta(1-\theta)+\theta)$. If $x \in (0, 1]$, then A believes State D is type 2 with probability 1.

Proof: Follows from above.

4.4 Model 2 Analysis

This subsection provides a brief discussion of the equilibrium.[12] In this regard, focusing on the feasibility constraints is most informative. The conditions on α implies that type 2 State Ds are indifferent between offering $x = 0$ and $x = \tilde{x}$ because State A does not always hassle when faced with an offer of $x = 0$. The conditions on β implies A is indifferent between hassling or not hassling when seeing an offer of $x = 0$ because only a share of type 2 D's offer $x = 0$. The final feasibility condition implies that as bad as war may be, type 1 D's stand to gain by offering $x = 0$ despite knowing that it sometimes can lead to war rather than conceding and offering $x = \tilde{x}$ knowing it will always lead to peace.

Importantly, this model demonstrates a simple point. In a situation where either side is willing to take any peaceful settlement, the existence of hassling implies that war can still break out. This model provides some evidence that the existence of hassling can lead to a more war-prone international order.

5 Model 3: Game-Free Analysis

5.1 Model 3 Intuition

The result from Model 2, that the availability of hassling can lead to war in settings where either side prefers any peaceful settlement to war, is shown for a specific model and set of parameter values. Given that international international conflict, war or grey-zone, does not take place within a given institution, the game form of conflict models should play a minimal role in influencing equilibrium behavior. Therefore, it would be beneficial to have a result that speaks to a large set of games where grey zone conflict (not just hassling) can arise regardless of the game form. This is possible through Bayesian Mechanism Design.

Banks (1990) and Fey and Ramsay (2011) also conduct a game-free analysis of conflict models. Given the results put forward in these papers, the technical aspects of the results below are not groundbreaking. What is novel here is that game free analysis has not been used to analyze models with grey zone conflict, and that the results here are applied to better explain how grey zone conflict occurs on both a case-by-case basis and in aggregate.

The results below hold for a set of models where there is uncertainty over each state's respective costs of grey zone conflict. The first result is that the realized costs of grey zone conflict

[12] This is done in lieu of a rigorous discussion on comparative statics. Because this section is only concerned with a specific equilibrium stemming from a specific preference ordering with specific restrictions in place, any discussion of comparative statics would not speak to what occurs under different feasible preference orderings and different equilibria.

play a role in the likelihood of grey zone conflict occurring within a crisis; specifically, when states have a lower realized cost of grey zone conflict, then grey zone conflict is more likely to occur. The intuition here is that states bargain with each other expecting some distribution of costs. When one state has a lower cost value within that distribution, then the other state may not expect this, fail to make adequate peaceful concessions, and grey zone conflict may result. The second result leverages this first result to comment on the perceived prevalence of grey zone conflict in the Cold War and Post-Cold War periods. Specifically, the rise in grey zone conflict may be the result of more interactions and more interstate crises occurring.

5.2 Model 3 Form

Before beginning, it is helpful to state the Revelation Principle.

Revelation Principle: *If s^* is a Bayesian-Nash equilibrium of game G, then there exists an incentive-compatible direct mechanism yielding the same outcome.*

Proof: See Myerson, 1979.

Consider a game G with two players, State Y and State Z. Each state selects some set of actions from the possible set of actions, denoted $a_z \in A_z$, $a_y \in A_y$, with $a \in A_z \times A_y$ denoting the full action profile. Note that action spaces a_z and a_y can be multidimensional.

States select actions that lead probabilistically into outcomes of a settlement, a war, or a grey zone conflict. Let $\alpha(a) \in [0, 1]$ denote the probability of settlement, $\beta(a) \in [0, 1]$ denote the probability of war, and $\gamma(a) \in [0, 1]$ denote the probability of grey zone conflict, with $\alpha(a) + \beta(a) + \gamma(a) = 1$.

The selected actions also influence the payoff each actor receives from the settlement, war, or grey zone conflict outcomes. This follows logically for the settlement outcome, because the actions could include an offer one state makes to another. A similar logic holds for grey zone conflict, as demonstrated in the hassling example above; when states are given generous offers, then hassling has diminished returns. This also applies to the war outcome, because the actions undertaken before the conflict breaks out could influence the final war outcome or the costs of war (one such action could be moving troops in preparation for a war). For $i \in \{y, z\}$, this is modeled as the utility from a peaceful settlement being $p_i : A_z \times A_y \to \mathbb{R}$, the utility from war being $w_i : A_z \times A_y \to \mathbb{R}$, and the partial utility from grey zone conflict being $g_i : A_z \times A_y \to \mathbb{R}$.

Expanding on the utility of grey zone conflict, this paper will assume that states know their own utility from grey zone conflict, but that the other state internalizes the utility of grey zone conflict in an idiosyncratic and unobserved manner. For example, while the United States or members of Europe are able to calculate the costs of Russian action in Eastern Ukraine, because they do not know how these actions are a part of Russia's long-range foreign policy plans, they would struggle to precisely calculate Russia's benefit to these actions. The unobserved qualities of each state are modeled as a uni-dimensional type denoted $\theta_i \in \Theta_i$ with $i \in \{y, z\}$. Each state's type is part of their utility function by generating value $h : \Theta_i \to \mathbb{R}$, with h_i increasing in θ_i. Each player knows their own type, but not the type of their opponent. The distribution of types F_i is common knowledge.

Letting $i \in \{y, z\}$ and a denote the actions selected by agents, each player's utility is a function of their actions and type, or

$$u_i(a, \theta_i) = \alpha(a)p_i(a) + \beta(a)w_i(a) + \gamma(a)\left(g_i(a) + h_i(\theta_i)\right).$$

Let s^* define a perfect Bayesian equilibrium to game G with the utility function above. The strategy will be a mapping from each actor's type onto a set of actions. As is typical, s^* is a Bayesian-Nash equilibrium if each type of player is selecting a best response to the strategies used by the other player. This is what would be called an "indirect game" as it is the state's types that determine their payoffs, but these types are transmitted indirectly through the strategies s^*.

For equilibrium s^* of game G, $U_i(\theta_i)$ denotes player i of type θ_i's expected utility, or

$$U_i(\theta_i) = \int_{\Theta_j} u_i(s^*(\theta_i, \theta_j), \theta_i) dF(\theta_j)$$

with $i \neq j$.

An assumption must be in place:

Assumption 1: *For all $\Theta_y \times \Theta_z$, there exists some equilibrium $s^*(\theta_i, \theta_j)$.*

At this point the revelation principle can be used. The outcome of game G is the realization of α, β, γ, and each player's utility. This implies that the equilibria outcomes stemming from s^* can also arise as part of a direct mechanism, or a game where states report their type their action in a game that results in identical outcomes and utilities. This also implies that instead of studying the optimal strategies and comparative statics of a specified game form G and players of any given type pair $\theta' \in \Theta_y \times \Theta_z$, this same result could arise through an incentive compatible direct mechanism, where players of type pair θ' report their types and these types result in the same outcomes that would arise through playing game G. Thus, the study of this incentive comparable direct mechanism can speak to a range of possible game forms, as stated in Result 3.

Proposition 3.1. Let G be a game and let s^* be an equilibrium of that game. For $i \in \{y, z\}$, $\gamma(\theta_i, \theta_j)$ is weakly increasing in θ_i.

Proof. Referencing the revelation principle, incentive compatibility implies that states will accurately report their type, or

$$\theta_i \in argmax_{\hat{\theta}_i \in \Theta} \left\{ U_i(\hat{\theta}_i | \theta_i) \right\}.$$

This constraint must hold for both states across all possible state pairs.

What does this mean for outcomes? Let $\bar{\theta}$ and $\underline{\theta}$ denote values such that $h_i(\bar{\theta}) > h_i(\underline{\theta}_i)$. The revelation principal and incentive compatibility imply $U_i(\bar{\theta}_i|\bar{\theta}_i) \geq U_i(\underline{\theta}_i|\bar{\theta}_i)$ and $U_i(\underline{\theta}_i|\underline{\theta}_i) \geq U_i(\bar{\theta}_i|\underline{\theta}_i)$, which written out is:

$$\int_{\theta_j} \left[\alpha(\bar{\theta}_i, \theta_j) p_i(\bar{\theta}_i, \theta_j) + \beta(\bar{\theta}_i, \theta_j) w_i(\bar{\theta}_i, \theta_j) + \gamma(\bar{\theta}_i, \theta_j) \left(g_i(\bar{\theta}_i, \theta_j) + h_i(\bar{\theta}_i) \right) \right] dF_j(\theta_j) \geq \quad (4)$$

$$\int_{\theta_j} \left[\alpha(\underline{\theta}_i, \theta_j) p_i(\underline{\theta}_i, \theta_j) + \beta(\underline{\theta}_i, \theta_j) w_i(\underline{\theta}_i, \theta_j) + \gamma(\underline{\theta}_i, \theta_j) \left(g_i(\underline{\theta}_i, \theta_j) + h_i(\bar{\theta}_i) \right) \right] dF_j(\theta_j) \quad (5)$$

and

$$\int_{\theta_j} \left[\alpha(\underline{\theta}_i, \theta_j) p_i(\underline{\theta}_i, \theta_j) + \beta(\underline{\theta}_i, \theta_j) w_i(\underline{\theta}_i, \theta_j) + \gamma(\underline{\theta}_i, \theta_j) \left(g_i(\underline{\theta}_i, \theta_j) + h_i(\underline{\theta}_i) \right) \right] dF_j(\theta_j) \geq \quad (6)$$

$$\int_{\theta_j} \left[\alpha(\bar{\theta}_i, \theta_j) p_i(\bar{\theta}_i, \theta_j) + \beta(\bar{\theta}_i, \theta_j) w_i(\bar{\theta}_i, \theta_j) + \gamma(\bar{\theta}_i, \theta_j) \left(g_i(\bar{\theta}_i, \theta_j) + h_i(\underline{\theta}_i) \right) \right] dF_j(\theta_j) \quad (7)$$

Terms (9) and (11) and terms (10) and (12) can be summed (which preserves the inequality) and re-written as

$$\left(h_i(\bar{\theta}_i) - h_i(\underline{\theta}_i) \right) * \int \left[\left(\gamma(\bar{\theta}_i, \theta_j) - \gamma(\underline{\theta}_i, \theta_j) \right) \right] dF_j(\theta_j) \geq 0.$$

Because $h_i(\bar{\theta}_i) - h_i(\underline{\theta}_i) \geq 0$, the inequality must hold. ■

5.3 Model 3 Analysis

The final inequality suggests that all else held equal, as a player's realized utility from grey zone conflict increases (the change from $h_i(\underline{\theta}_i)$ to $h_i(\bar{\theta}_i)$), the likelihood of grey zone conflict occurring is weakly increasing. Thus, in settings where a state's realized type leads to lower benefits or higher costs from grey zone conflict, then there will be a weakly reduced likelihood of grey zone conflict occurring.

It is useful split this section into two parts.

5.3.1 Interpreting Proposition 3.1: Case-by-Case Basis

Consider an example with two states (State Y and State Z) in a crisis over some asset. In this crisis, the costs of war are high enough that no state would be willing to go to war. In the crisis State Y is atypical in that its unobserved characteristics has it it highly benefiting from grey zone conflict. This would imply that its θ_y value is in the right-tail of the distribution of possible θ_y values.

Information asymmetry implies Z does not know this. Therefore, when bargaining with Y, Z may not assume that Y's θ_y is in the tail of the distribution. This could mean that Z would treat Y different in the bargaining model if Z knew what Y's true θ_y was; in fact, Z may make offers to Y that Y finds unacceptable, and Y would escalate to grey zone conflict. It is straightforward to imagine how this game could play out different if θ_y been lower; with a lower θ_y, State Y may be more willing to accept an offer rather than escalate to grey zone conflict.

Result 3 suggests that in these games with asymmetric information over type, each state enters the crisis with beliefs over the type of state they are in crisis with, and this will affect how they interact with one another. In two crises with all else being equal, (Crisis A and Crisis B) if in Crisis A one state benefits more from grey zone conflict than in Crisis B, then Crisis A is more likely to end in grey zone conflict.

Proposition 3.1's applicability to the real world is that when looking at a crisis that ended in grey zone conflict, the the result could have arisen from one or both sides possessing unexpectedly high benefits to the grey zone conflict.

5.3.2 Interpreting Proposition 3.1: In Aggregate

Some papers have claimed that the Cold War and Post-Cold War periods have experienced an increase in low-level conflict.[13] One explanation for this has been the advent and proliferation of nuclear weapons, as now states are unwilling to settle conflicts using conventional war out of fears that it could lead to a nuclear exchange.

If there has been an increase in instances of low-level conflict, then Proposition 3.1 suggests a different explanation: there may simply be more crises occurring that could lead to grey zone conflict. Proposition 3.1 pertains to realizations of states' costs and benefits of grey zone conflict within a distribution. For there to be more realizations of the costs/benefits of grey zone conflict within a distribution that lead to grey zone conflict, there may simply be more crises occurring.

Why would this be the case? It is beyond the scope of the paper to explain, but two simple explanations exist. First, the number of states has increased, which suggests that there are more dyads that could experience these crises. Second, the capability of states has increased. Today, there are more states that possess a greater capacity to project military force than in the past; this suggests that there may be more opportunities for crises to arise.

Part III
Dynamic Models of Hassling

6 Model 4. The Future Impact of Hassling Can Undermine Peaceful Equilibria

6.1 Model 4 Intuition

Model 2 demonstrated that when one state is able to hassle and there is asymmetric information, it can lead to war. Essentially, one state may be willing to run the risk of war by hassling if hassling provides a great enough payoff. However, Model 4 demonstrates it is premature to suggest that hassling only leads to war; in dynamic models, peaceful equilibria may not always exist, and an equilibrium with hassling may be all that stands between two states and war.

[13] Other sources dispute this point- for example de Mesquita (1990).

This section introduces a feature of hassling that has not been discussed before. This section assumes that hassling can provide the hassling state a greater likelihood of winning in a future war. This is assumed because in many cases, the actions undertaken during hassling could prove useful during a war. If State A has been supporting insurgent movements inside State D, then State A is more likely to be successful in a war with State D as it provides A with natural allies. For example, two days before the September 11, 2001 attacks, al Qaeda killed Ahmad Shah Massoud, the military and political leader of the Northern Alliance, the primary resistance to the Taliban in Afghanistan; al Qaeda likely carried out this assassination at least in part to undermine what would be the United States' primary ally in Afghanistan. Additionally, if State A has been engaging in low-level cyberwarfare with State D, then presumably State A has some means to create significant damage to State D should a war be declared. For example, the Stuxnet virus was only used to disrupt Iranian centrifuges, but could have been used to disrupt any sort of industrial processes, including energy facilities or weapons manufacturing.

This feature of hassling can lead to two interesting outcomes. First, if State A is hassling, there may not exist a peaceful equilibrium that gives A as great a utility as A receives from hassling. If A stops hassling, D's wartime capabilities will grow. This implies that in the future, A may need to make political concessions to accommodate D's rise in power. This feature of hassling then may motivate A to continue hassling rather than accept peace. This result can help explain why hassling can occur, even in a dynamic setting. Second, there may be cases where a peaceful equilibrium does not exist, but a hassling equilibrium does. Following models like Schultz (2010), it will be assumed that the owner of the asset in dispute may benefit by deviating and going to war (because the owner would not make concessions for the time before war occurs and during the war). In this case, a peaceful equilibrium may not always exist. However, if hassling can reduce the owner's benefit from deviating, a hassling equilibria may still exist. Thus, while a hassling equilibrium may be inefficient , it may also be the only type of equilibrium that prevents the outbreak of war.

6.2 Model 4 Game Structure

Consider two states, A and D. Assume D begins the game in control of an asset worth 1 in each period. So long that D is in control of the asset, D decides the share that State A receives.

This is a simultaneous move, multi-period game, with $t = 0, 1, 2, ...$ denoting periods. This is not a true infinite horizon game. If states go to war in a given period, then the outcome of war is determined probailistically. From that point on, states will continue to receive payoffs, but states will no longer act.

So long that the game has not terminated in period $t-1$, in each period t, D has two types of actions. D can make A an offer of $x_t \in [0,1]$ or D can declare war.

So long that the game has not terminated in period $t-1$, in each period, A has two types of actions. A can select some hassling level $h_t \in [0,1]$, or A can declare war.

Importantly, the actions in the previous period have an effect on the payoffs from actions in the current period. Specifically, hassling in period $t-1$ gives A a greater likelihood of winning war in period t. This assumption is made because hassling can take the form of supporting rebel groups within D's state or degrading some of D's capabilities via cyberwarfare, both of which would be beneficial to A if a war broke out. This dynamic effect is formalized below in the utilities.

6.3 Model 4 Actions and Utilities

The per-period game payoffs are summarized below for all $t \in \{1,2,3,...\}$. A discussion on $t=0$ will be included below.

Regarding actions in the repeated game, if in any period State A ever selects "War," then the states are at war and the actors stop selecting actions in future periods. If State D selects "War" and A does not also select "War," then D has escalated in that period to counter some of the hassling (if it is occurring) and to take full control over the asset in dispute. The difference here in what happens with the "War" option lies in the asset ownership. Because D holds the asset, if A declares war on D, then the states are at war. However, if D declares war and A does not, then D is not at war with A over the asset; rather, here D claims full ownership of the asset and starts countering A's hassling (if $h_t > 0$).

State A\State D	Offer $x_t \in [0,1]$	War
$h_t \in [0,1]$	$x_t + h_t(1-x_t) - h_t k_A - h_t^2$, $(1-x_t)(1-h_t) - h_t k_D$	$r_A(h_t)$, $r_D(h_t)$
War	$p(h_{t-1}) - c_A$, $1 - p(h_{t-1}) - c_D$	$p(h_{t-1}) - c_A$, $1 - p(h_{t-1}) - c_D$

Per-period payoffs in Model 3.

6.3.1 (x_t, h_t) Payoffs

The payoff structure here captures hassling as an inefficient means to transfer part of an agreed upon outcome. x_t is the offer that D makes to A. h_t is A's selected level of hassling. The terms $h_t k_A$, h_t^2, and $h_t k_D$ capture the inefficiency costs of hassling, where $k_A > 0$ and

$k_D > 0$.

The following notation will be used: $H_A(h_t, x_t) = x_t + h_t(1-x_t) - h_t k_A - h_t^2$ and $H_D(h_t, x_t) = (1-x_t)(1-h_t) - h_t k_D$.

6.3.2 (h_t, War) Payoffs

Under this action pairing, A has selected some $h_t \in [0,1]$, and D has unilaterally escalated and taken control of the asset. The functions $r_A(h_t)$ and $r_A(h_t)$ capture this outcome. For example, when A is not hassling, then D has full control of the asset, and $r_A(0) = 0$ and $r_D(0) = 1$. Additionally, to capture the inefficiencies associated with hassling, it is assumed $r_A(h_t)$ is increasing and strictly concave in h_t and $r_D(h_t)$ is decreasing and strictly convex in h_t. Finally, it is assumed for all h_t, $r_A(h_t) \leq x_t + h_t(1-x_t) - h_t k_A - h_t^2$ and $r_D(h_t) \geq (1-x_t)(1-h_t) - h_t k_D$; this final condition implies that by escalating to war, D captures a greater share of the settlement that arises from hassling, and A captures a diminished share.

6.3.3 (War, x_t) or (War, War) Payoffs

Under these action pairs, the states are at war. When war occurs in period t, A wins control of the asset for all remaining periods with probability $p(h_{t-1})$, and D wins with probability $1-p(h_{t-1})$; thus, while payoffs continue, the states stop conducting actions after these action pairs are reached. $p(h_{t-1})$ is weakly increasing in its argument, with $p : [0,1] \rightarrow [\underline{p}, \bar{p}]$, where $[\underline{p}, \bar{p}] \subset [0,1]$. The terms c_A and c_D capture the costs of war, where $c_A > 0$ and $c_D > 0$.

State A's present value in period t of going to war is $(p(h_{t-1}) - c_A)/(1-\delta)$, and State D's present value of going to war in period t is $(1 - p(h_{t-1}) - c_D)/(1-\delta)$, where δ are the states' discount rate.

6.3.4 Assumptions

Assumption 1: *(War, War) is a stage-game Nash equilibrium. Formally, $p(h_{t-1}) - c_A \geq r_A(h_t)$.*

Assumption 1 formalizes that if either state selects "war", the other state does weakly better also selecting "war." Thus, in the supergame, it is a subgame perfect Nash equilibrium in period $t \in \{01, 2, ...\}$ for both states to declare war.[14]

[14] Admittedly, this is a strong assumption. A different model could be written where both states declaring war is a subgame perfect equilibrium for a non-infinite number of periods–for example, if A has lost ten rounds of war, the game changes and A

Assumption 2: *The stage game payoffs in $t = 0$ take the form defined above, with modifications to $p(h_{t-1})$. The probability of winning war in period $t = 0$ will be assigned and will depend on the equilibrium being analyzed.*[15]

The function $p(h_{t-1})$ has the actions in period $t - 1$ affect A' probability of winning a war in period t. This leaves the probability of A winning in war in $t = 0$ undefined. This paper will address this by assigning the probability of A's victory in period $t = 0$; it will do so in a way that, based on the equilibrium being studied, so the $t = 0$ expected payoffs match the expected payoffs for all other $t \in \{1, 2, ...\}$ if the equilibrium play is sustained.

Assumption 3: *Equilibria must be subgame perfect.*

6.4 Game 4 Equilibrium

Consider two possible non-war equilibria: a Peaceful Equilibrium and a Hassling Equilibrium.

Definition*: In the* ***Peace Equilibrium****, for each $t \in \{0, 1, 2, 3, ...)$, D offers a fixed $x_t \in [0, 1]$ and A selects $h_t = 0$. If either player deviates in period t, then both states select "war" in period $t + 1$. For period $t = 0$, the payoffs take the form as above with $p(h_{t-1}) = \underline{p}$.*

Definition*: In the* ***Hassling Equilibrium****, for each $t \in \{0, 1, 2, 3, ...)$, D offers a fixed $x_t \in [0, 1]$ and A selects $h_t = h$ for some $h \in (0, 1]$. If either player deviates in period t, then both states select "war" in period $t + 1$. In period 0, the payoffs take the form as above with $p(h_{t-1}) = p(h)$.*

For the Hassling Equilibrium or Peace Equilibrium to be sustained as a subgame perfect Nash equilibrium, they must be robust to both State A's and State D's best unilateral deviations. To state the necessary conditions formally, one additional definition is needed.

Definition*: Let $\hat{h}$ be defined as A's best non-war unilateral deviation in period t, where*

does strictly better hassling or accepting when D declares war. This would require adding additional costs or other complications to the model, and the model would not substantively change (other than D being more incentivized to defect from a hassling or peaceful equilibrium, and A less incentivized to defect).

[15] This assumption is not critical, and could be relaxed, but at the expense of adding more technical constraints.

deviating in t leads to war in $t+1$. Formally, for a fixed $x_t = \hat{x}$, this is

$$\hat{h} \in argmax_h \left\{ H_A(h, \hat{x}) + \frac{\delta(p(h) - c_A)}{1-\delta} \right\}.$$

With this in place, the necessary constraints to support the Peace and Hassling Equilibria are in Lemmas 4.1 and 4.2.

Lemma 4.1: *For a fixed $x_t = x$, the Peace Equilibrium is sustained if*

$$\frac{1-x}{1-\delta} \geq 1 + \frac{\delta(1 - \underline{p} - c_D)}{1-\delta}, \tag{8}$$

$$\frac{x}{1-\delta} \geq H_A(\hat{h}, x) + \frac{\delta(p(\hat{h}) - c_A)}{1-\delta}, \tag{9}$$

and

$$\frac{x}{1-\delta} \geq \frac{\underline{p} - c_A}{1-\delta} \tag{10}$$

hold.

In condition (8), D deviates by escalating to "War." In (9), A deviates by selecting their optimal level of hassling knowing that there will be war in $t+1$. In (10), A deviates by selecting "War."

Lemma 4.2: *For a fixed $x_t = x$ and $h_t = h$, the Hassling Equilibrium is sustained if*

$$\frac{H_D(h, x)}{1-\delta} \geq r_D(h) + \frac{\delta(1 - p(h) - c_D)}{1-\delta}, \tag{11}$$

$$\frac{H_A(h, x)}{1-\delta} \geq H_A(\hat{h}, x) + \frac{\delta(p(\hat{h}) - c_A)}{1-\delta}, \tag{12}$$

and

$$\frac{H_A(h, x)}{1-\delta} \geq \frac{p(h) - c_A}{1-\delta} \tag{13}$$

hold.

In condition (11), D deviates by escalating to "War." In (12), A deviates by selecting their optimal level of hassling knowing that hassling in t will lead to war in $t+1$. In (13), A deviates by selecting "War."

6.5 Game 4 Equilibrium Analysis

6.5.1 Comparing A's Utility from Hassling or Peace Equilibria

The proposition below demonstrates there may not exist a Peace Equilibrium that gives State A at least as great a stage-game payoff as an existing Hassling Equilibrium.

This result can help explain why hassling can occur, even in a dynamic setting. Hassling is destructive and burdens states with costs, whereas a peaceful settlement does not. Therefore, instead of one state hassling the other, it seems plausible that states could find a way to reach a mutually beneficial Peace Equilibrium. This proposition shows that a Peace Equilibrium that weakly benefits both A and D may not exist.

Before showing this, a condition must be introduced.

Definition: *When* ***Condition 4.1*** *holds,* $H_A(h,x) > \delta \underline{p} + \delta c_D$.

This is used in Proposition 4.1:

Proposition 4.1: *Consider a Hassling Equilibrium satisfying the conditions in Lemma 2 where* $x_t = x$ *and* $h_t = h$ *for all* t. *If Condition 4.1 holds, then there does not exist a Peace Equilibrium where State A receives a stage-game payoff greater than or equal to their Hassling Equilibrium stage-game payoff.*

Proof: Consider a Peace Equilibrium with some per-period offer x. From Lemma 1, (4) implies that $\frac{1-x}{1-\delta} \geq 1 + \frac{\delta(1-\underline{p}-c_D)}{1-\delta}$,or re-written, $\delta \underline{p} + \delta c_D \geq x$. Therefore, the greatest offer A can receive in the Peace Equilibrium is $\delta \underline{p} + \delta c_D$.

This Proposition stems from D's inability to commit to future settlements and D's advantage when going to war in the Peace Equilibrium relative to a Hassling Equilibrium. Consider a Hassling Equilibrium. For the Hassling Equilibrium to be sustained, D must not be able to benefit by deviating and going to war (as captured in line (7) in Lemma 2). Because some positive level of hassling h disadvantages A relative to a hassling level of 0 both in the one-shot defection payoff ($r_D(h) < 1$) and in the war payoff ($1 - \underline{p} > 1 - p(h)$), the Hassling Equilibrium can be sustained with D receiving a lower stage-game payoff than the Peace

Equilibrium. Therefore, so long that hassling is not too destructive, it is possible that A will obtain a larger payoff from a Hassling Equilibrium (given D's low defection constraint and therefore offer) than from a Peace Equilibrium (given D's high defection constraint and therefore offer).

Proposition 4.1 has implications for stage-game payoffs.[16] How are these stage-game payoffs relevant to a situation where one state is hassling the other but a transition to a Peace Equilibrium is feasible? One way to analyze this transition is to consider the possibility of a multilateral deviation, so long as it leaves both states weakly better off. In other words, consider a modified Hassling Equilibrium where, in period $\tilde{t} \in \{1, 2, 3, ...\}$, both States can continue playing the Hassling Equilibrium or change to some possible Peace Equilibrium. By comparing the present value of continuing hassling from $\tilde{t}$ onward to the present value of transitioning to some Peace Equilibrium in $\tilde{t}$ onward, it is possible to see if there is Pareto improvement that can be made through a multilateral deviation to Peace.

When Condition 4.1 holds, it is much more difficult for this transition to occur. If Condition 4.1 holds, then for all periods $\tilde{t}+1$, A receives a lower per-period payoff relative to the hassling payoff. While it may be possible that in period $\tilde{t}$ some offer can be made that makes A better off than hassling, if A is sufficiency patient (possesses a high enough δ), then this one-period benefit will not be enough to motivate the change from hassling to peace. Instead, at period $\tilde{t}$, A may prefer to continue hassling as stopping hassling leads to D becoming more powerful and A needing to make future concessions.

6.5.2 On the Existence of Non-War Equilibria

Lemmas 4.1 and 4.2 also speak to the conditions under which Peace or Hassling Equilibria exist. These are Propositions 4.2 and 4.3.

Proposition 4.2. *If*

$$c_A + \delta c_D \geq \underline{p} \tag{14}$$

and, for some $x \in [0, 1]$,

$$\delta \left(c_A + c_D - p(\hat{h}) + \underline{p} \right) \geq (1-\delta) H_A(\hat{h}, x), \tag{15}$$

then there exists a Peace Equilibrium.

[16] One example of parameter values supporting Condition 4.1 and satisfying the conditions in Lemmas 1 and 2 are $x = 0.6$, $h = 0.3$, $\delta = 0.6$, $\underline{p} = 0.2$, $\bar{p} = 0.5$, $k_D = 0.3$, $r_D(h) = 0.55$, $c_D = 0.8$, $k_A = 0.05$, and $c_A = 0.5$, with $p(h) = \underline{p}(1-h) + h\bar{p}$.

Proof: First for (14). For a Peace Equilibrium to be supported, there must be some offer $x \in [0,1]$ that satisfies (8) and (10) in Lemma 4.1. By summing (8) and (10) and maintaining the inequality, one has

$$\frac{1}{1-\delta} \geq 1 + \underline{p} - c_A + \frac{\delta(1-\underline{p}-c_D)}{1-\delta} + \frac{\delta(\underline{p}-c_A)}{1-\delta}.$$

Through algebra, condition (14) can be derived. This can similarly be done for (15) by summing (8) and (9). □

Because there are benefits to breaking away from the Peace Equilibrium, it may be impossible to support a Peace Equilibrium. As (14) shows, if the costs of war are too low, agents do not sufficiently value the future (δ is too small), or $\underline{p}$ is too high, then there does not exist a set of offers that can satisfy both states.

A similar statement for the Hassling Equilibrium is made in Proposition 2.

Proposition 4.3. *If, for a fixed h*

$$\frac{1-hk_A-h^2-hk_D}{1-\delta} \geq r_D(h) + p(h) - c_A + \frac{\delta(1-c_A-c_D)}{1-\delta} \tag{16}$$

and, some $x \in [0,1]$,

$$\frac{1-hk_A-h^2-hk_D}{1-\delta} \geq r_D(h) + \frac{\delta(1-p(h)-c_D)}{1-\delta} + H_A(\hat{h},x) + \frac{\delta(p(\hat{h})-c_A)}{1-\delta}, \tag{17}$$

then there exists a Hassling Equilibrium where $h_t = h$ and $x_t = x$ for all t.

Proof: (16) follows from summing (11) and (13), and (17) follows from summing (11) and (12).

While Proposition 4.3 is dense, several straightforward comparative statics follow. First, the right hand side of the inequalities in (16) and (17) are decreasing in the costs of war (c_A and c_D), implying that when one or both sides face high costs from the (war, war) equilibrium, hassling is easier to be sustained. Second, the more hassling reduces State D's utility from unilaterally defecting (reduces $r_D(h)$), the easier hassling is to sustain in equilibrium. Third, the more A can gain by unilaterally deviating to $\hat{h}$ (captured in $H_A(\hat{h},x)$), hassling becomes more difficult to sustain.

Proposition 4.2 and Proposition 4.3 together illustrates an important point; there are parameter values under which the Hassling Equilibrium is sustained and the Peace Equilibrium is not,[17] and there are parameter values where the reverse holds. This point demonstrates what was shown in Model 2, that the availability of hassling as an action can lead to war, is not the final word on what hassling means for the outbreak of war. Instead, because actions like supporting rebel groups and cyberwarfare have the ability to make D worse off when deviating from the equilibrium path, these actions can allow for a non-war equilibrium in settings where a peaceful equilibrium does not exist.

6.6 Model 4 Analysis

Model 4 demonstrates two points. First, due to the power shifts that occur when a state stops hassling, there may not exist a peaceful equilibrium that compensates the hassling state enough to motivate it to stop hassling. Second, there exist parameterizations where an equilibrium with hassling can be sustained and not dissolve to war whereas a peaceful equilibrium would end in war.

The first point, that a hassling state may do better hassling than accepting any peaceful equilibrium, has relevance to the examples motivating this paper. Consider the example of Russia's hassling in Eastern Ukraine. At this point, there could exist a peaceful political settlement that, in the short term, gives Russia an equivalent or greater utility to what the existing hassling does. However, by stopping hassling today, Russia would be strengthening Ukraine and NATO in the future. Therefore, by stopping hassling today, in the future Russia may need to make political concessions. If these concessions are significant, they could leave Russia worse off, making the decision today to stop hassling less desirable.

The second point, on the possible existence of hassling equilibrium when a peaceful equilibrium does not exist, has relevance for how grey zone conflict should be perceived. This is a different view on grey zone conflict than has been formalized in the past. Schultz (2010) suggests that because support for rebels is difficult to monitor, it creates perverse incentives to over-support rebels, and this can lead to war. While this is an important consideration, because hassling can leave the target state at a disadvantage should war break out, this model showed that support for rebels and other forms of hassling can at times prevent war.

This point on the durability of hassling equilibrium raises two issues. First, if the pre-

[17] One example of parameter values violating (14) is $\delta = 0.6$, $\underline{p} = 0.6$, $\bar{p} = 0.7$, $k_D = 0.1$, $r_D(h) = 0.55$, $c_D = 0.4$, $k_A = 0.05$, and $c_A = 0.3$. Letting $p(h) = \underline{p}(1-h) + h\bar{p}$, a hassling equilibrium exists at $x = 0.6$, $h = 0.3$.

vention of war is viewed as a normative good, then hassling can, at times, be desirable. Second, even in settings where hassling is less robust to war than a peaceful equilibrium, hassling as an action can provide a possible, non-war equilibrium in settings where a peaceful equilibrium does not exist. Why would peaceful equilibrium not exist? As Fearon (1995) suggests, sometimes issues are not divisible. Alternatively, perhaps a peaceful equilibrium exists, but perhaps identifying the correct bargain is difficult or prone to failure, perhaps how players interpret the space of possible bargains is not well defined or understood, or perhaps forming an agreement simply takes time.[18] In all of these cases, in the absence of a bargaining peaceful equilibrium, the hassling equilibrium may be adequate in preventing war.

Part IV

Conclusion

7 Future Research

Because each model included a discussion of implications, the conclusion here will be limited to suggestions for future research.

First, this paper's had a limited discussion on models of asymmetric information. Model 2 was concerned with a single preference ordering within a specific model; more can be done exploring that model under different parameter values or with modified models. Additionally, the game-free analysis in Model 3 was limited to settings where there was only uncertainty regarding how states value grey zone conflict. Ideally, future game-free analysis could account for uncertainty over the costs and benefits of both hassling and war outcomes for states. That being stated, to the extent that this analysis requires modeling actors as possessing multidimensional types, this research will face significant challenges given the generally agreed upon difficulty of mechanism design where actors have multidimensional types (see Carroll (2015) for a discussion on this).

Second, the formal political science literature has begun examining how the difficulties of policy-making interacts with political institutions (for example, see Callander (2011)). To the best of my knowledge, this has not been explored in the context of making peace agreements for international conflict; given the relative simplicity of hassling as a means of share transfer, it might be expected that hassling could serve a purpose to stall war as negotiations occur. This type of research could help explain violent political behavior during negotiations to resolve international crises.

[18] One paper exploring these types of issues in the legislative context is Callander (2011).

Third, by providing the first set of models of hassling, this paper aims to motivate further empirical analysis. Given the list of incidents motivating this paper, one could conclude that there are certain empirical regularities regarding grey zone conflict, like how the hassling methods used commonly give the state conducting the hassling some opportunity for plausible deniability. However, without a more rigorous analysis of incidents of grey zone conflict, it is difficult to say if this observation holds true for the full sample of grey zone conflict incidents. More work on compiling and cataloging incidents of grey zone conflict is needed.

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