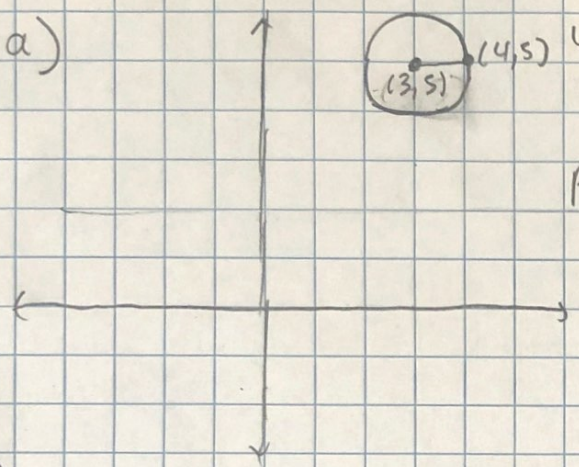
Quiz #2
Riya Tyagi

1. 1)

$t=0$, by parametric equation

$P_t = (x,y) + (3+\cos(t), 5+\sin(t))$

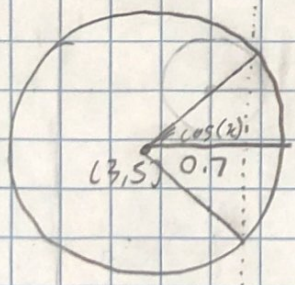
a)



Unit circle centered at (3,5)
Starting point: (4,5)

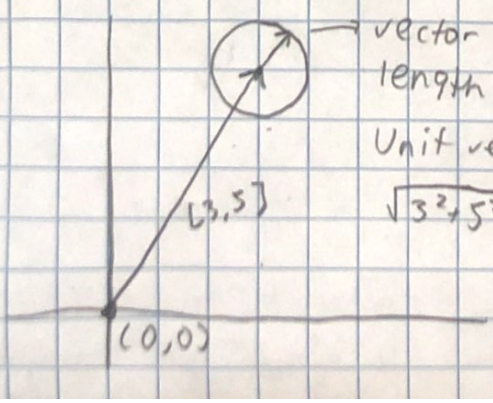
Particle's Motion:
$(x-3)^2+(y-5)^2=1$

b) All t, t in degrees $0 < t < 500°$
crosses line $x = (3,7)$



$x = 3.7$ line
$\cos(t) = 0.7$
$t = \boxed{45.572996°}$
$-45.572996, = \boxed{314.427004°}$
$\boxed{405.572996°}$

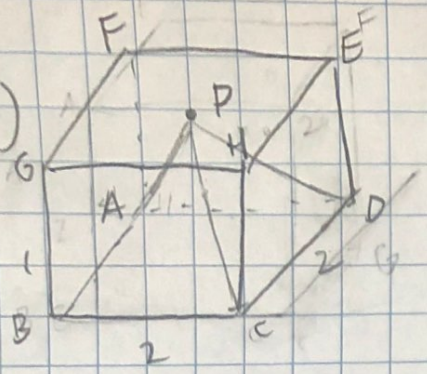
c) How far from origin, so farthest & closest points are along diameter of particle path (d along (0,0) line & (3,5) btwn)



vector w/ direction of (3,5) & length of $r_1 = 1$
Unit vector of [3,5]
$\sqrt{3^2+5^2} = \sqrt{34}$

$\frac{1}{\sqrt{34}}[3,5] + [3,5] + (0,0)$
$\approx (3.514, 5.857)$

1, 2)

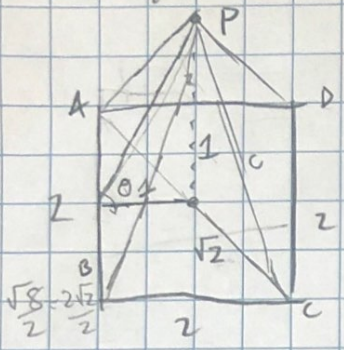


$\frac{1}{3}$ volume of box

Box volume = $4\ u^2$

a) $\boxed{\frac{4}{3}\ u^2}$

b) equilateral pyramid (all edges congruent)



height = 1

$\sqrt{2}^2 + 1^2 = c^2$

$c^2 = 3$

$c = \sqrt{3}, \pm 2$

* Not to scale

$\boxed{\text{NO}}$

c) Dihedral $\angle$, btwn face PAB & square face ABCD

$\tan\theta =$ ⊥ in upper diagram

$\frac{1}{1} = 1$

$\tan^{-1}(1) = \boxed{45°}$

d) Candies in pyramid shape, how many fit into 6×6×6 candy box

volume-based

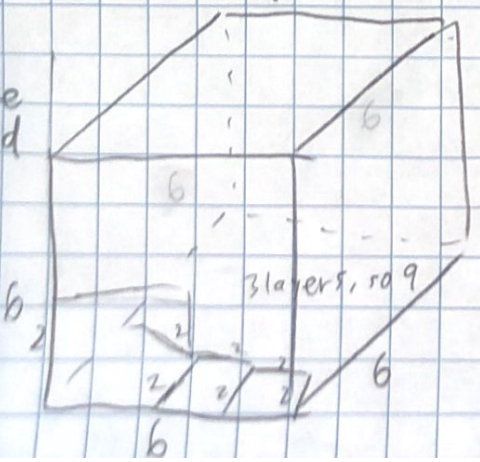
Volume of pyramid = $\frac{4}{3}u^2$

Box-volume: 6×6×6 = $216\ u^2$

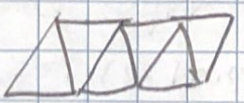
$\frac{216}{4/3} = \frac{216 \cdot 3}{4} = \boxed{162 \text{ candies}}$

e) How many we can actually fit

* No space wasted

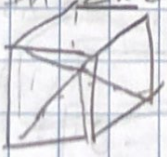


We can pack like such:



square base of 2×2

In 2×2×2 prism, we can pack 6



base of 2×2 h=1

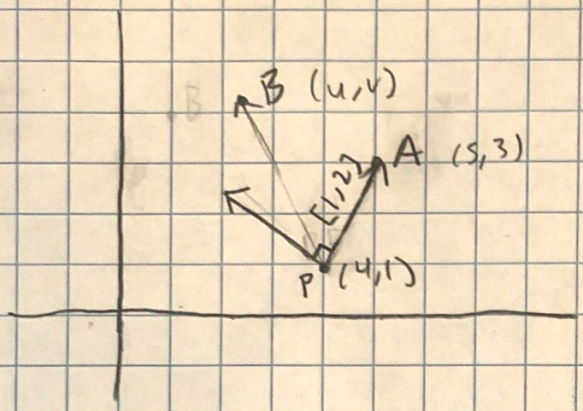
3 on base 1 layer, 9 on base total, h=2

9×3=27, 27×6= 162 candies

1, 3)

P = (4, 1)
A = (5, 3)
B = (u, v)

} points in coordinate plane



Rotating around circle with radius = PA / PB

a) 90° vector for rotation

[-2, 1]

A' = (4, 1) + [-2, 1] = (2, 2)

b) vector $\overrightarrow{AB}$

= [u-5, v-3]

vector $\overrightarrow{A'B'}$

= [5-v-2, u-3-2]

= [3-v, u-5]

The two vectors are perpendicular

Now for B

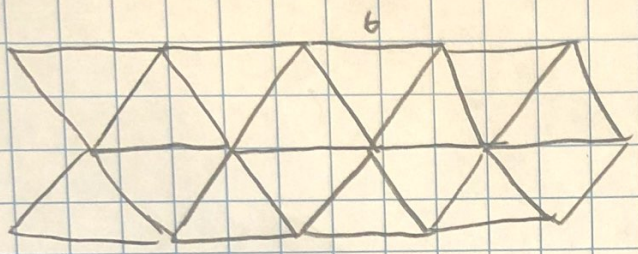
vector $\overrightarrow{PB}$ = [u-4, v-1]

Perp vector = [1-v, u-4]

B' = (4+1-v, 1+u-4)

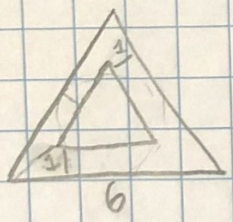
= (5-v, u-3)

4)

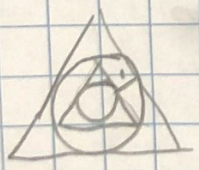


Coin w/ $r = 1$ ft
equilateral Δs w/ sidee length 6 ft

a) Prob that coin will not cover sides, but will touch them



Inner triangle 1 ft away from sides
Similarity ratios^2 give area ratios

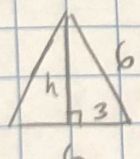


Inradius of larger Δ

$r = \frac{2A}{P}$

$A = \frac{6 \cdot 3\sqrt{3}}{2} = 9\sqrt{3}$

$r = \frac{18\sqrt{3}}{18} = \sqrt{3}$



$\sin(60) = \frac{h}{6}$

$\frac{\sqrt{3}}{2} = \frac{h}{6}$

$2h = 6\sqrt{3}$, $h = 3\sqrt{3}$

Inradius of smaller
$r = \sqrt{3} - 1$

Similarity ratio
$\frac{\sqrt{3}-1}{\sqrt{3}}$

Area ratio

$\frac{(\sqrt{3}-1)^2}{3} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{3}$

$\Rightarrow \frac{3 - \sqrt{3} - \sqrt{3} + 1}{3} = 4 - 2\sqrt{3} : 3$

Percent $= \frac{4-2\sqrt{3}}{3} \times 100 \approx 17.86\%$

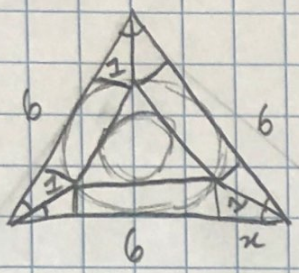
$\approx 18\%$

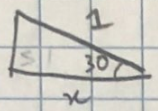
b) E.C

b) E. C

won't cover any vertices

so must be 1 away from vertices





$\cos(30) = \frac{x}{1}$

$x = \frac{\sqrt{3}}{2}$

Thus, length of one of sides of inner triangle $= 6 - 2 \cdot \frac{\sqrt{3}}{2} = 6 - \sqrt{3}$

Similarity ratio $= \frac{6-\sqrt{3}}{6}$

Area ratio $= \frac{(6-\sqrt{3})(6-\sqrt{3})}{36} = \frac{39-12\sqrt{3}}{36}$

Percent $= \frac{(39-12\sqrt{3}) \cdot 100}{36}$

$\approx \boxed{50.598\%}$

$\approx \boxed{51\%}$