Development and testing of a new model of the variable connected fractions of Prairie basins

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Key Points:

- List up to three key points (at least one is required)
- Key Points summarize the main points and conclusions of the article
- Each must be 100 characters or less with no special characters or punctuation

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Abstract

TBD

1 Introduction

It has been known for more than 60 years (Stichling & Blackwell, 1957) that the regions within Canadian Prairie basins which can contribute flow to their outlet are temporally and spatially variable. The cause of the variability is the unusual hydrography of the Prairies, which was formed by glacial and post-glacial processes. As the region is semi-arid, and flat, and deglaciation ocurred ˜10,000-12,000 years B.P. (Christiansen, 1979), there has not been sufficient water, energy, or time for conventional drainage networks to form. Consequently, there are few streams within the region. The largest rivers in the region (the Saskatchewan and its tributaries) are sourced from the Rocky Mountains and their foothills.

In the absence of streams, much of the region is dominated by the presence of millions of depressions, known locally as "sloughs" or "potholes". The depressions trap water from direct precipitation, the trapping of blowing snow, and can recieve surface runoff. Because much of the region is underlain by deep deposits of glacial till, which is largely impermeable, there is generally little interaction between the depressions and groundwater, although very shalllow interchanges can occur. In the absence of artificial drainage, most of the losses of water from the depressions are due to evaportation and evapotranspiration from the surrounding vegetation.

Many attempts have been made to model the variability using a variety of models. All of the models developed to date have had limitations which have restricted their general application within the region. The methodology of (Shaw, Pietroniro, & Martz, 2012) was restricted to a single basin. As the relationships controlling the connected fraction were not quantified, it could not be applied to other locations. K. R. Shook and Pomeroy (2011) and K. Shook, Pomeroy, Spence, and Boychuk (2013) developed 2 models for simulating the connected fractions of Prairie basins. The Wetland DEM Ponding Model (WDPM) applies simulated fluxes of water (precipitation, runoff, evaporation, infiltration) to a digitial elevation model (DEM). Using the algorithm of Shapiro and Westervelt (1992), the applied water is redistributed over the DEM, ponding in depressionsm, and draining from the basin's outlet. Although the spatial distributions of water produced by the program appeared to agree well with remotely- sensed data (Armstrong, Kayter, Shook, & Hill, 2013; K. Shook et al., 2013), the WDPM was too slow to be used as a component of a hydrological model.

K. R. Shook and Pomeroy (2011) and K. Shook et al. (2013) developed the Pothole Cascade Model (PCM) which used discrete ponds, similar to the model of Shaw (2009), to simulate the variable connected fractions of Prairie basins. When parameterised with pond sizes and distribtutions for a given basin, PCM produced very similar connected-fractions to those estimated by the WDPM for the same location (K. Shook et al., 2013). Both models demonstrated very strongly hysteretic relationships between the connected fraction and the depressional storage of a basin. The hysteresis was demonstrated to be caused by the disconnection of all depressions as soon as subtractive fluxes were applied, and to the very differing effects of additive and subtractive fluxes on the frequency distributions of pond areas (K. Shook et al., 2013). Hysteresis was demonstrated to be a feature of the response of depression-dominated hydrological basins, occurring between the upland runoff and consequent inflows to a given depression (K. Shook, Pomeroy, & van der Kamp, 2015).

The PCM algorithm was added to the Cold Regions Hydrological Modelling (CRHM) platform, which is a physically-based model capable of simulating Prairie hydrological processes (Pomeroy et al., 2007). The addition of the PCM was shown to dramatically improve the modelled hydrological responses of a Prairie basin (Pomeroy et al., 2014). However, as was demonstrated by K. R. Shook and Pomeroy (2011), the simulated connected fraction produced by the PCM depends on the number of simulated depressions used. When few depressions are used in a simulations, they do not well represent the shape of the connected-fraction curve of a basin with many depressions. Unfortunately, it is difficult to incorporate the very large numbers of depressions found in a Prairie basin in a hydrological model. Each depression has several parameters (area, depth, volume, shape) which add to the complexity of the model. Worse, the connectivity of each depression must also be determined, which is an onerous task. In the case of Pomeroy et al. (2014), 46 depressions were used for each sub-basin modelled, which was too few to give a good representation, but was about all that could be managed by the model. The outflows from the small number of simulated depressions were upscaled to represent the outflows from the sub-basin.

Mekonnen et al. (2014) developed PDMROF, which is a very simple representation of the connected fraction of a Prairie basin, and which can be easily incorporated in any hydrological model. PDMROF is based on the PDM model of Moore (2007). PDMROF is based on the assumptions:

1. that the volumes of depressions can be represented as a power-law distribution
2. that the depressions are filled in order from smallest to largest, and that the contributing fraction can therefore be estimated by integrating the power-law volume distribution, based on the volume of storage within the basin.
3. that the contributing fraction decreases due to water removal in the same manner as which it increases.

There are several problems with this approach.

Researchers have indeed demonstrated that the areas of Prairie ponds, and therefore of the depressions which contain them *approximate* power-law distributions, although it has been demonstrated that they are better approximated by Pareto II distributions []. The relationship between the volume and area of a single depression was shown by (Hayashi & van der Kamp, 2000) to approximate that of a paraboloid. Therefore the frequency distrbution of the volumes of depressions is not the same as that of the areas.

The assumption that depressions are filled in the order of their sizes has several flaws. The first is that it ignores the role of gatekeeping in controlling flows. The second flaw lies with the implementation of the filling. As explained by Mekonnen et al. (2014), runoff generated within a landscape unit is assigned to the integrated storage volume. The fraction of the storage volume which is filled then generates runoff, the rest of the runoff filling up the remaining storage. The problem with this methodology is that it ignores the actual mechanisms by which depressions fill. As described above, Prairie depressions are filled through a) direct precipitation, b) trapping of blowing snow and c) through runoff. All three of these mechanisms have very different contributing areas. As was shown by K. Shook et al. (2013), the relationship between the basins contributing runoff directly to a Prairie depression, and the areas of depressions, can be described by power-laws, with exponents smaller than 1.0. Therefore the relative area draining to a depression decreases with increasing depressional area. Not only does the PDMROF filling use the incorrect runoff area, it cannot properly calculate the contributing fraction of a basin.

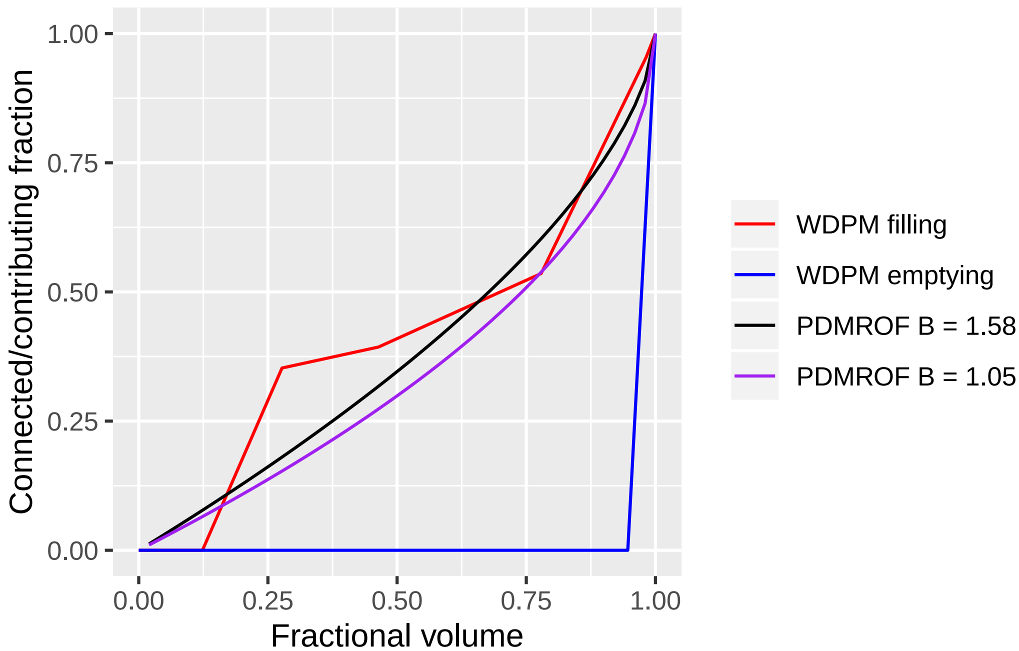
The third assumption is evidenly incorrect. When a set of filled depressions is subjected to subtractive fluxes (evaporation, infiltration) they all become unfilled. Thus their individual, and collective, connected fractions are instantly zero. As water is progressively removed, ponds will shrink, disconnect and disappear. Therefore, as was found by K. R. Shook and Pomeroy (2011) and K. Shook et al. (2013), the size distributions of pond areas change very differently when adding and removing water. The processes are not reversible, so there is no way that the connected fraction curves can be of the same shape.

Nevertheless, PDMROF has been shown to do a good job in simulating the contributing fraction of a small Prairie basin in a hydrological model (Mengistu & Spence, 2016). It is noticable that it was necessary to calibrate the model parameters, unlike those of PCM. The requirement for calibration is related to the deficiencies of PDMROF noted above - were the model based on better assumptions, its parameters might be set from remote sensing/GIS.

Figure 2 plots the filling and emptying curves computed using WDPM for the St. Denis sub-basin above pond 90. The curves computed by PDMROF, using parameters determined by Mengistu and Spence (2016) are also plotted. It is noticeable that the PDMROF curves are quite similar the WDPM filling curve. Mengistu and Spence (2016) reported that in all of their simulations the contributing fraction of the basin was smaller than 0.35. In this region, PDMROF splits the difference between the rising and falling limbs of the WDPM curves. Had the volumetric storage been greater, there would likely have been greater discrepencies between the two models.

The need to calibrate PDMROF is also its great weakness. In the Canadian Prairies, there are few stream gauges, and consequently stream basins are very large. Of the 217 currently active stream gauging stations within the Canadian Prairies, the median gross basin area is 573 km^2 (mean = 1238 km^2). Therefore, any stream basin modelled in the Prairies will have very large regions which are combined together for calibrating the PDMROF parameters. No matter how well the model is distributed for its forcings, and state variables, the hydraulic parameters will always be lumped at the scale of the basin.

The implications of the lumping are demonstrated by the behaviour of the full St. Denis basin. The full basin is dominated by the presence of a large pond near the outlet. In fact, the basin does not have a traditional outlet, as it has no permanent stream. The



126 **Figure 1.** Connected/contributing fractions vs. fractional volume for WDPM and PDMROF,
127 for St. Denis Basin above Pond 90.

outlet is simply the lowest point on the basin divide - much of the basin lies below the outlet. Therefore St. Denis basin is endorheic, and has never been known to spill in the period of monitoring. As demonstrated in , the connected-fraction curve for St. Denis basin is completely different from that of the sub-basin above pond 90. Note that as there have never been any flows recorded from the outlet, it would be impossible to calibrate PDMROF for the basin. Therefore, it is not a good practice to use a calibrated PDM-ROF relationship for any other basin, even when it lies entirely within the same land-form and land use type. As Prairie basins are very large, the use of calibrated models like PDMROF will invariably lump together many different regions, with very different contributing fractions, resulting in poor simulations.

1.1 Research objectives

The overall objective of this research is to develop and test a new method for simulating the variable conencted/contributing fractions of Prairie basins. The method is to have the following characteristics

1. It must be generic, able to estimate the connected/contributing fraction for any Prairie basin.
2. It must be able to simulate the hysteretic relationship between the connected fraction and the storage of water in a basin.
3. It must be able to be parameterized from GIS/remote sensing, without use of any form of calibration.
4. It must have a reasonably small number of parameters and state variables.
5. It must be able to execute quickly.

The ideal model will therefore have the generic capacity, hystertic relationships and simple parameterization of WDPM and PCM, as well as the small numbers of parameters and states and simplicity of PDMROF.

2 Materials and Methods

2.1 Model background

The development of the new model is based upon the work of ??, which found the following:

1. When the a basin is small, the sizes, arrangements and locations of all of the depressions are important in controlling the conencted fraction.
2. As the number of depressions is increased (i.e. when the basin is large), the effect of any single depression is reduced.
3. When there are large numbers of depressions, whose size distributions approximate a Pareto II distribution (with many more small depressions than large ones),

then the spatial arrangement of the depressions is unimportant, as gatekeeping is very small.

4. When there are large numbers of depressions, and there is a single large depression, or a few large depressions, then the location of the large depression(s) within the basin is important.

In the case of a large number of depressions without a dominant depression, the connected fraction curve is the integrated area of the depressions and basins, vs. the integrated volumes. Although this may seem to be related to PDMROF, the shape of the curve is very different, because a) it specifically includes the depression basin areas and b) the depressions are filled by runoff. The shapes of the rising limbs were found to approximate 1:1 lines.

2.2 Model concept

The concept behind the new model is to combine the effects of the large number of small depressions, and the few large depressions, in a flexible manner. The guiding assumption is that the model will be used in basins large enough that the spatial arrangement of the small depressions is not terribly important. From [?] this would appear to be the case when the number of depressions, with areas greater than 25 m^2, is at least 1,000.

2.2.1 Small depressions

The state of the small depressions is defined by their fractional depressional storage and connected areas. Figure 3 plots the connected fraction computed from 10,000 realizations of 1,000 depressions simulated from the relationships described in []. In each realization, the depressions were filled from their initial state, by repeated additions of water. In each addition, the water was applied evenly to the depression, and its basin, assuming a runoff coefficient of 1. With each addition of water, the connected fraction was computed based on the assumption of no gatekeeping - each depression is assumed to be connected directly to a stream channel. The curves in Figure 3a represent filling the simulated depressions from their being empty. The curves in Figure 3b were generated by filling all of the depressions, and remving 100 mm from each. This results in all of the ponds having surface elevations being below theier basins' outlets, which causes the connected fraction of the basin to be zero. Of course, all depressions with maximum depths smaller than 100 mm will have their water depth reduced to zero. Following the water removal, the simulated depressions were then filled iterattively as before. The process was repeated for removals of 200 mm and 300 mm, as plotted in Figure 3c and d.

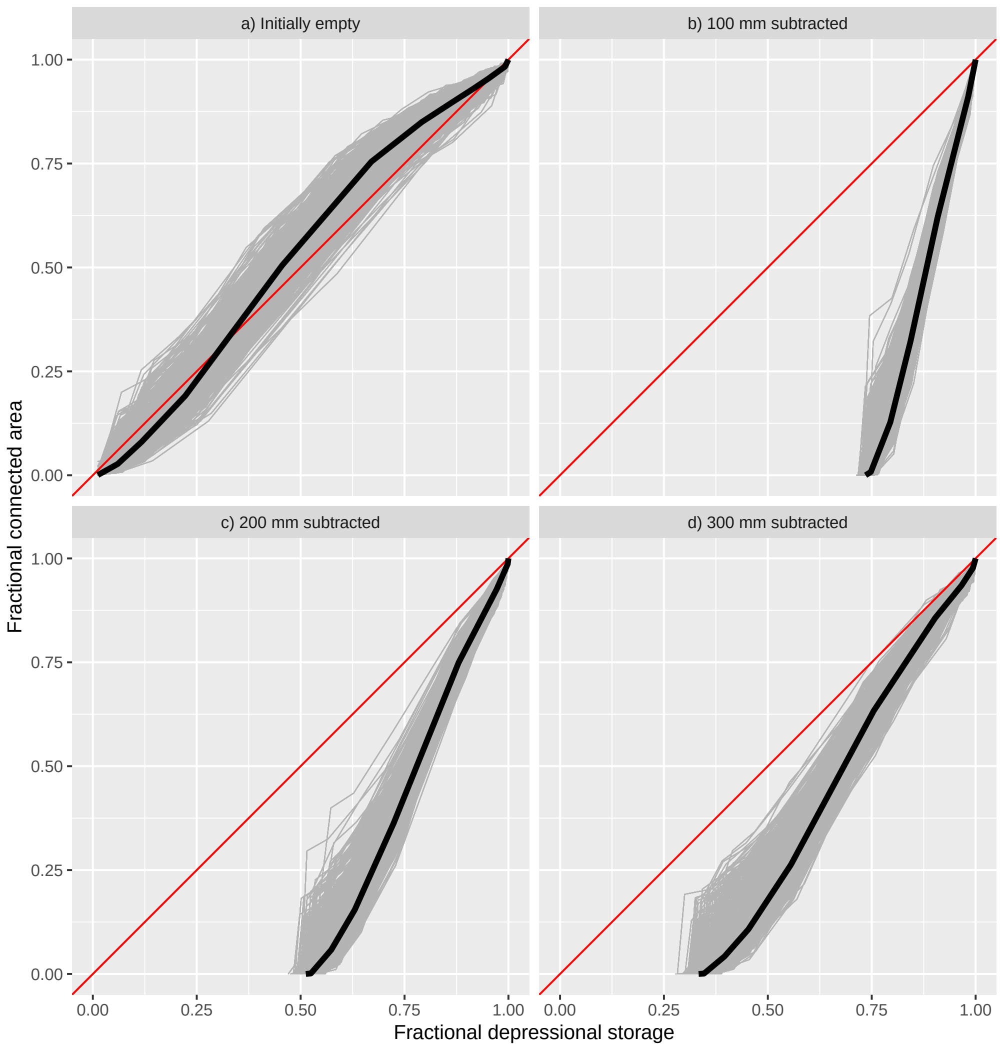


Figure 2. Connected/contributing fractions vs. fractional volume for 10,000 realizations of 1,000 depressions. The realizations are plotted in grey, the median of all realizations is plottted as the heavy black line. A 1:1 line is plotted in red.

Although there is some considerable scatter among the curves, the median curves are slightly sigmoidal, approximating lines. Very similar results were found by K. R. Shook and Pomeroy (2011) for applications and removals of water at Smith Creek Research basin using the WDPM.

It is important to note that [] demosntrated that the degree of scatter among realizations of this type is a function of the number of depressions being simulated. Only 1,000 depressions are used here; [] found over 2000 depressions greater than 25 m^2 in a Prairie basin of only 11 km^2. Therefore, in a large basin, the degree of scatter, and therefore the degree of uncertainty in the filling trajectories, is reduced.

The operation of the proposed parametric model of the hystertic connected fraction is illustrated by the schematic diagram in Figure 4. The initial state of the modelled basin (its fractional water volume and connected fraction) is plotted as point a. When water is removed from depressional storage, either by evaporation or by infiltration, the connected fraction becomes zero, and the volume is reduced, as shown by point b. As water is further removed, the connected fraction remains at zero, and the fractional water is reduced, as is shown by point c. When water is added, the connected fraction and fractional volume are increased, along the trajectory from the current state (point c) to the filled state, i.e. (1,1), resulting in the new state at point d.

Such a model is very easily implemented. All that is required is to store the basin's state at each interval. Whenever water is removed, the connected fraction is set to zero, and the volumetric storage is reduced as usual. The computation of the connected fraction from the trajectory to (1,1) only takes a few lines of code. Because the connected fraction and the volumetric storage are connected, it would be simplest to solve them iteratively, by dividing the additions of water into very small depths. Since the Prairies are semi-arid, and hydrological models of the region are generally run on sub-daily time intervals this is unlikely to be problematic.

2.2.1.1 Water areas The existing physically-based hydrological models used for the Canadian Prairies, MESH and CRHM, are based on GRUs and HRUs, respectively. Incorporation of the parametic hystersis model would result in a single HRU/GRU being used to represent the water storage. Unfortunately, neither model currently simulates the relationship between the volume of water stored in a GRU/HRU and its area, which is important for calculating the actual evaporation. K. R. Shook and Pomeroy (2011) showed that the relationship between the fractional water-covered area and the fractional water storage is slightly hysteretic.

Figure 5 plots the fractional water cover (fraction of the maximum possible water-covered area) vs. the fractional depressional storage for simulated filling and emptying

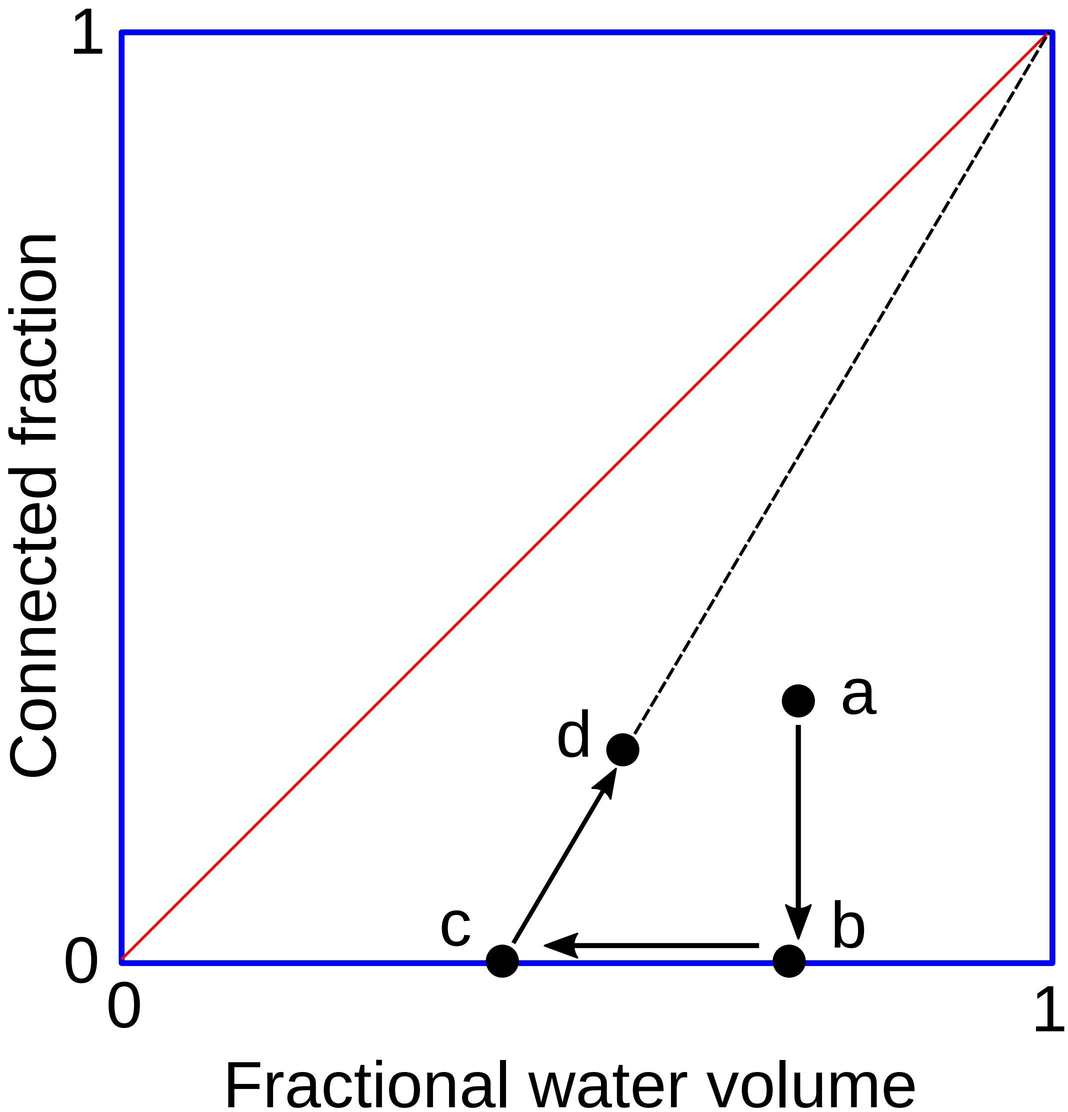


Figure 3. Operation of the parametric hysteresis model of the states of Prairie basins. The original state is shown at point a. Points b through d show the effects of removing, then adding water. All states exist below the 1:1 line plotted in red.

of a Prairie basin. The blue line plots the median of the filling curves from an initially-empty basin, the red line plots the median of the emptying curves, using the 10,000 realizations of 1,000 depressions discussed above. The green line plots the result that would be expected for a single depression using the relationship of Hayashi and van der Kamp (2000), with an exponent of 3.33. The purple line plots the same relationship using an exponent of 1.72. The two values were used by Pomeroy et al. (2014) for depressions greater than or equal to and smaller than 10,000 m^2, respectively. The same values were used for all of the realizations.

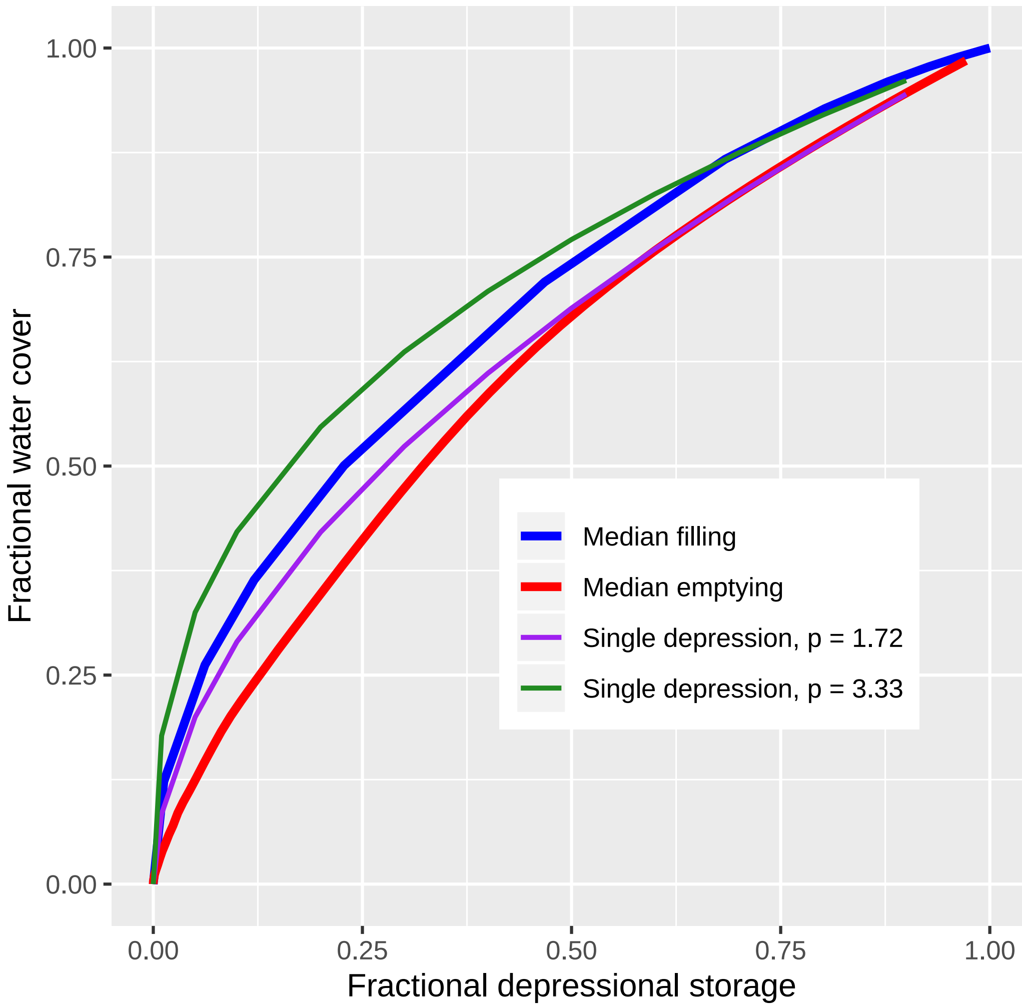


Figure 4. Fraction water cover vs. fractional volume for 10,000 realizations of 1,000 depressions. The median rising limb is plotted in blue, the median falling limb is plotted in red. Curves for a single depression, based on the Hayashi-van der Kamp equations are plotted in green, for p = 3.33, and purple for p = 1.72

The median filling curve lies midaway between the single depression curves, for values of the fractional depressional storage smaller than about 0.5, becoming tangent to the p = 3.33 curve. As the depressions fill roughly in their size order (also affected by their basin areas), the small ponds are filled first, so the mdeian water area filling curve will come to resemble that of the large depressions. When the depressions are emptied, the ponds in small depressions are progressively elimiated. Thus the emptying curve lies below even the curve of the single depressions with exponent values of 1.72.

The original graphs of K. R. Shook and Pomeroy (2011) were derived from WDPM simulations at SCRB, and showed a greater degree of hysteresis than that of Figure 5, probably because of the actual values of the scaling exponents being more variable than the values used here. Nevertheless, as the degree of hysteresis is small, it is probably simplest to use the relation for a single depression, with a relatively small value of the scaling exponent, to represent the water area within a basin. The alternative of developing a set of hysteretic water area curves is left as an exercise for the reader.

2.3 Large depressions

While the small depressions are hysteretic as a group, they do not exhibit strong gatekeeping effects. By contrast, the large depressions can cause gatekeeping, the effects depending on the sizes of the depressions and their locations [].

As was stated above, models such as MESH and CRHM can simulate the hydrological processes affecting large ponds by modelling them as GRUs/HRUs. Therefore, it is feasible to model at least a few large depressions, if they exist, in a given sub-basin without requiring changes to the program structure.

Figure 6 is a schematic diagram of how a basin containing a single large depression, and many small ones, might be represented. The large depression pond (shown in blue) would be subject to direct precipitation, evaporation and evapotransipration from the riparian vegetation, all of which are affected by the area of the water, which would be modelled. The large depression above the water surface (in red), and the basin of the large depression (in yellow) trap snow and contribute surface runoffa nd subsurface flows without any fill-and-spill ocurring. The grey region represents the fraction of the basin controlled by the depressional storage of the small ponds, which can be modelled using the parametric hysteresis model. A single HRU/GRU could be used to model the runoff from this region. However, the differences in the sizes and locations of the fraction above and below the large depression will result in variations of the timings of streamflows from these regions, so it would probably be advisable to model them separately.

3 Data

4 Results

Or section title might be a descriptive heading about the results

5 Conclusions

Acknowledgments

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