Neural Operators: Advancing Real-Time Structural Response

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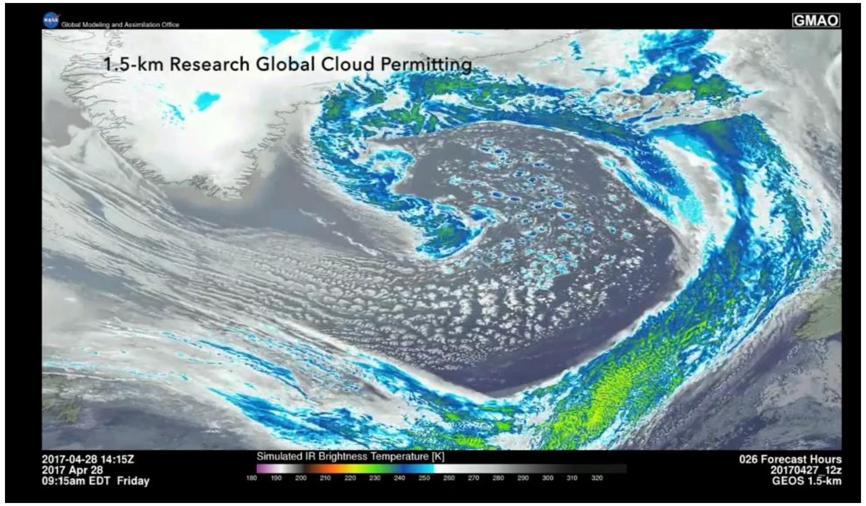
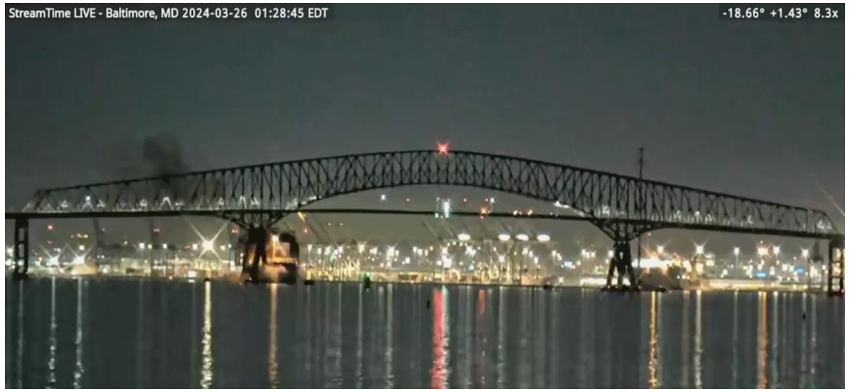
DesignSafe Training Webinar

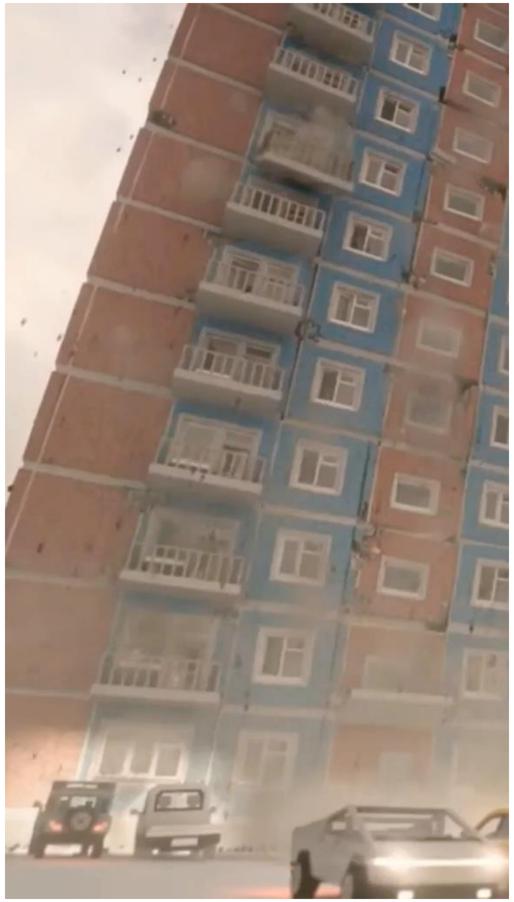
Processes of Nature



Evolution of an Icelandic Low in the North Atlantic Ocean over three days
Source: https://www.nas.nasa.gov/SC17/ (NASA)



Collapse of the Francis Scott Key Bridge in Baltimore on March 26, 2024
Source: https://www.youtube.com/watch?v=2kw1hH5XCYU



Bhuj Earthquake in India in 2001
https://www.youtube.com/shorts/SkNaxcxVz98

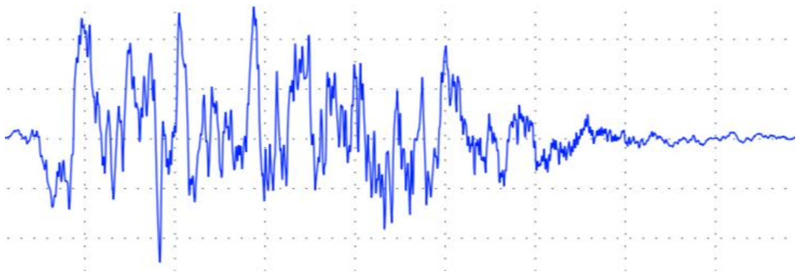
Physics-based Models

Can represent the **Processes of Nature**

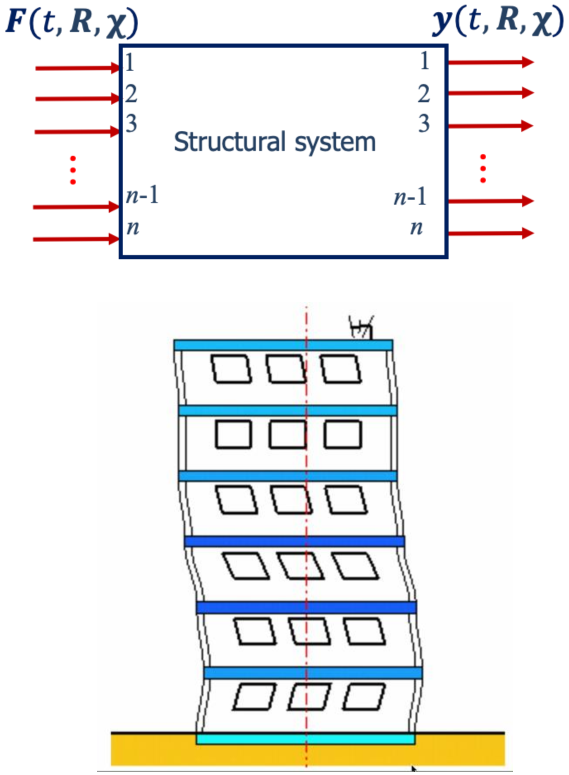
- Physics-based models are approximated via **ODEs/PDEs**

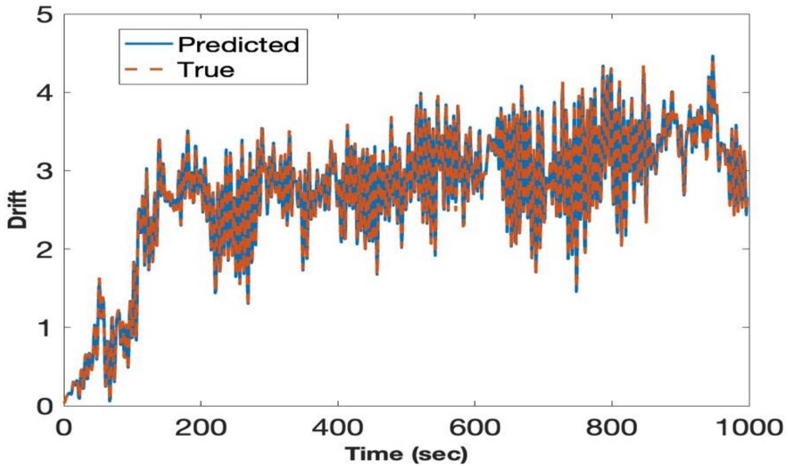
  To model earthquake: $m\frac{d^2u}{dt^2} + \mathrm{k}\frac{du}{dt} + F_0 = 0$

- Computational Mechanics helps us simulate these equations.



Input excitation





System response

Performance-based design

Extreme events

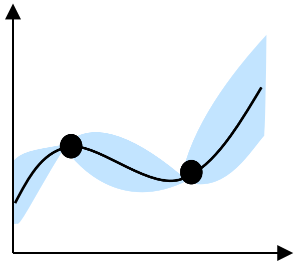
Non-linear Structural systems

Dynamic responses

Performance

State-of-the-art Performance assessment centered on **probabilistic** metrics

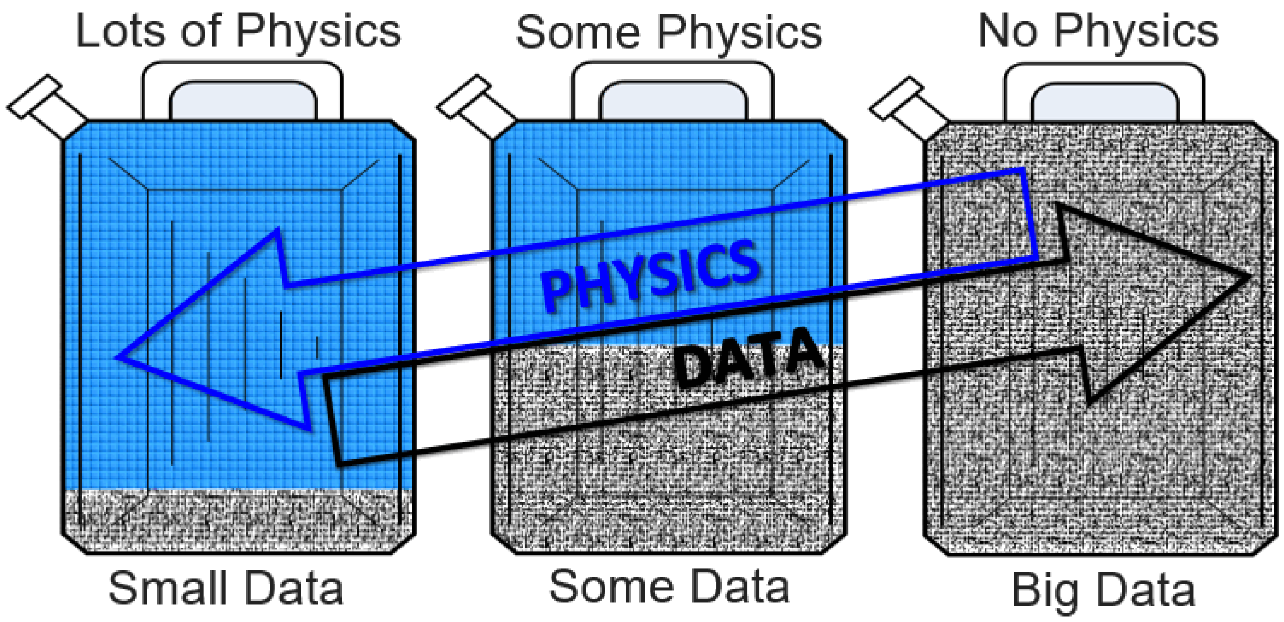
Surrogate Models



Computationally Demanding

Data + Law of physics

Three scenarios of
Physics-Informed Learning Machines



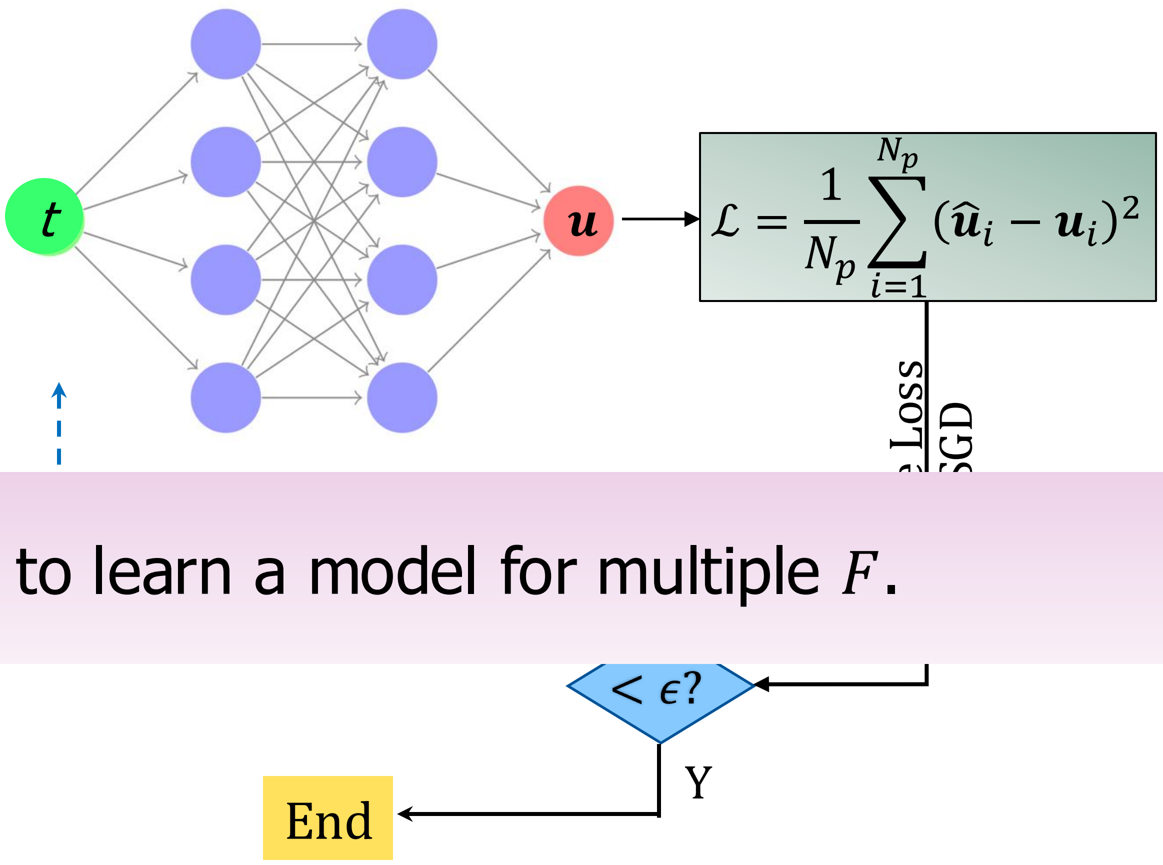
The 5-D Law: Dinky, Dirty, Dynamic, Deceptive Data

Training data-driven NNs

$$m\frac{d^2u(t)}{dt^2} + \mathrm{k}\frac{du(t)}{dt} + F = 0$$

$\mathcal{N}: t \rightarrow u(t)$ For a single F

$\hat{\boldsymbol{u}}$: Predicted solution
$\boldsymbol{\theta}$: Weights and biases
N_p: # of collocation points



Let's say we want to learn a model for multiple F.

Training data-driven NNs

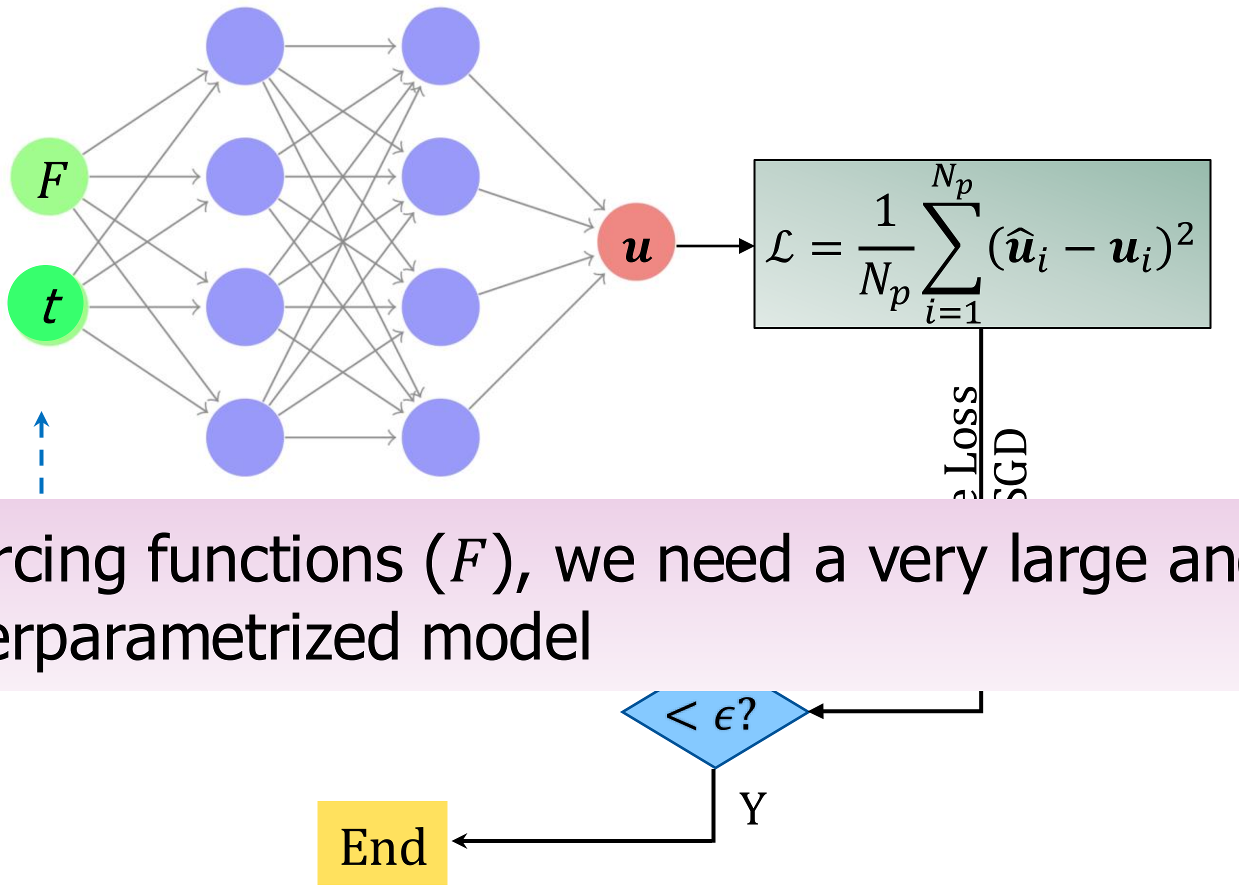
$$m\frac{d^2u}{dt^2} + \mathrm{k}\frac{du}{dt} + F = 0$$

$$\mathcal{N}: [t, F] \rightarrow u(t)$$

$\hat{\boldsymbol{u}}$: Predicted solution

$\boldsymbol{\theta}$: Weights and biases

N_p: # of collocation points



To learn a model for 1M Forcing functions (F), we need a very large and overparametrized model

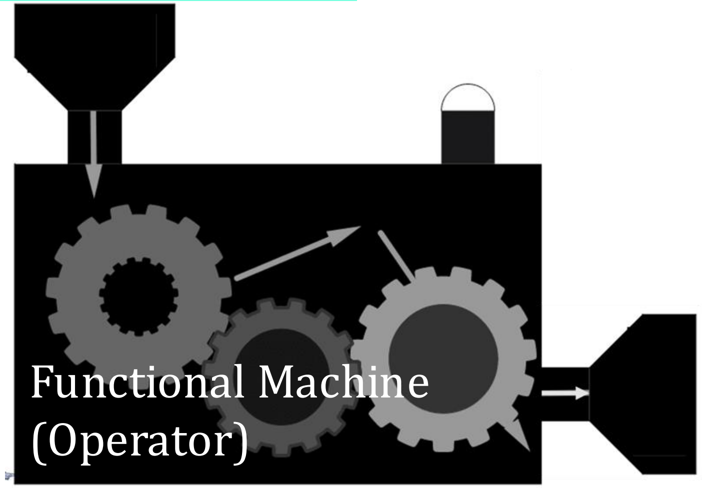
Learning a Neural Operator

Inputs

$\mathcal{F} = \{F_1, F_2, \ldots, F_n\}$

$\Phi : \mathcal{F} \to \mathcal{U}$

$\mathcal{F}, \mathcal{S}$ are infinite dimensional function space



Outputs

$\mathcal{U} = \{u_1(\boldsymbol{F_1}), u_2(\boldsymbol{F_2}), \ldots, u_n(\boldsymbol{F_n})\}$

What we know

$\{\mathcal{F}_n, \mathcal{U}_n\}_{n=1}^{N}$

$\mathcal{U}_n = \Phi(\mathcal{F}_n)\,, \mathcal{F}_n \sim \mu \; i.i.d$

Task

Operator, $\Psi : \mathcal{F} \times \Theta \to \mathcal{U}$ such that $\Psi(., \theta^*) \approx \Phi$

Training $\theta^* = \mathrm{argmin}_\theta l(\{\mathcal{F}_n, \Psi(\mathcal{U}_n, \theta)\})$

Surrogate Modeling Techniques

- Discretized Data
- Discretization dependent
- Queries on mesh
- Learning functions between vector spaces

- Functional Data
- Discretization Invariant
- Continuous quantities
- Learning operators between function spaces

Data-driven

PCA **Auto-encoders**

K-PCA **Diffusion maps**

f-PCA **DeepONet** **LNO**

FNO **WNO**

F-RKHS

Finite Dimensional

Infinite Dimensional

Physics Informed

PINNs*

PI-DeepONet

PINO

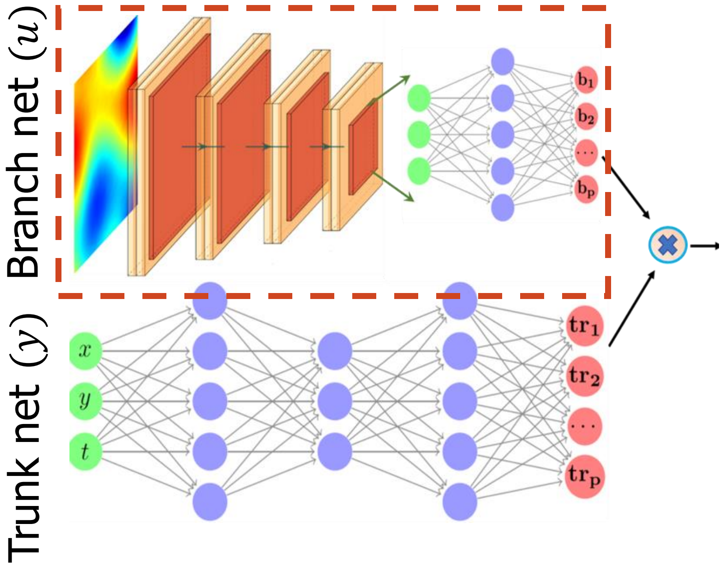
*continuous output

Deep Operator Networks (DeepONet)

- Generalized Universal Approximation Theorem for Operator [Chen '95, Lu et al. '19]
- **Branch net**: Input $\{u(x_i)\}_{i=1}^{m}$, output: $\left[b_1, b_2, .., b_p\right]^T \in \mathbb{R}^p$
- **Trunk net**: Input y, output: $\left[t_1, t_2, .., t_p\right]^T \in \mathbb{R}^p$
- Input u is evaluated at the fixed locations $\{y_i\}_{i=1}^{m}$

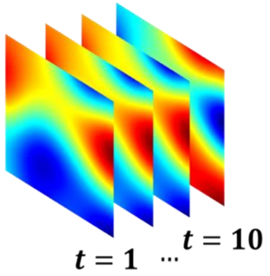
$$G_\theta(u)(y) = \sum_{i=1}^{p} \underbrace{b_i(u(x_1), u\,(x_2), \dots, u\,(x_m))}_{\text{branch net}} \cdot \underbrace{tr_i(y)}_{\text{trunk net}}$$

Can be FNN/CNN/U-Net/LSTM etc.



Minimize loss

$$G_\theta(u)(y) \downarrow \mathcal{L}_r(\theta) + \mathcal{L}_i(\theta) \downarrow \theta^*$$



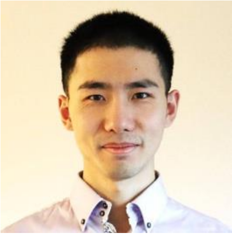
Other neural operators

- Fourier neural operator
- Wavelet neural operator
- Laplace neural operator

[1] Kovachki, N. et al. (2021), [2] Li, Z. et al. (2020), [3] Tripura, T. and Chakraborty, S. (2022), [4] Cao, Q. and Goswami S. (2023).

Manuscript in preparation:

Neural operators for stochastic modeling of system response to natural hazards

A collaboration between Johns Hopkins University, University of Texas Tech University and University of Michigan.

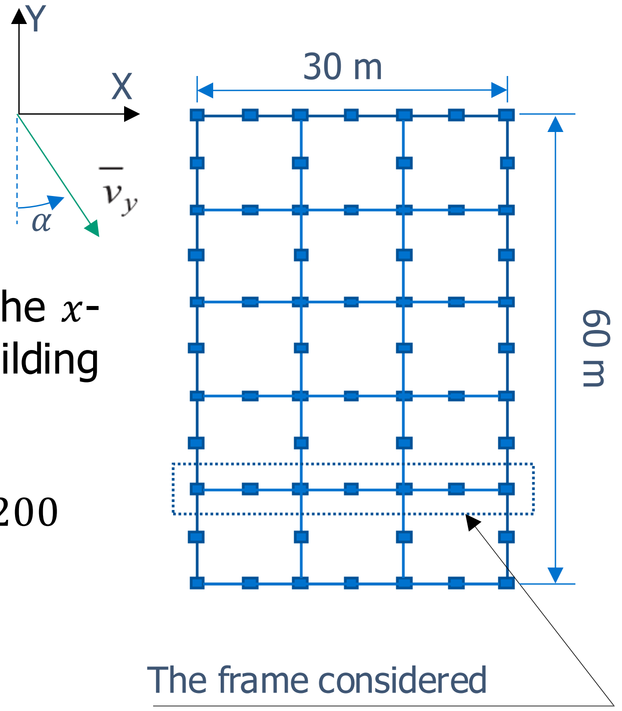
NO for wind response approximation

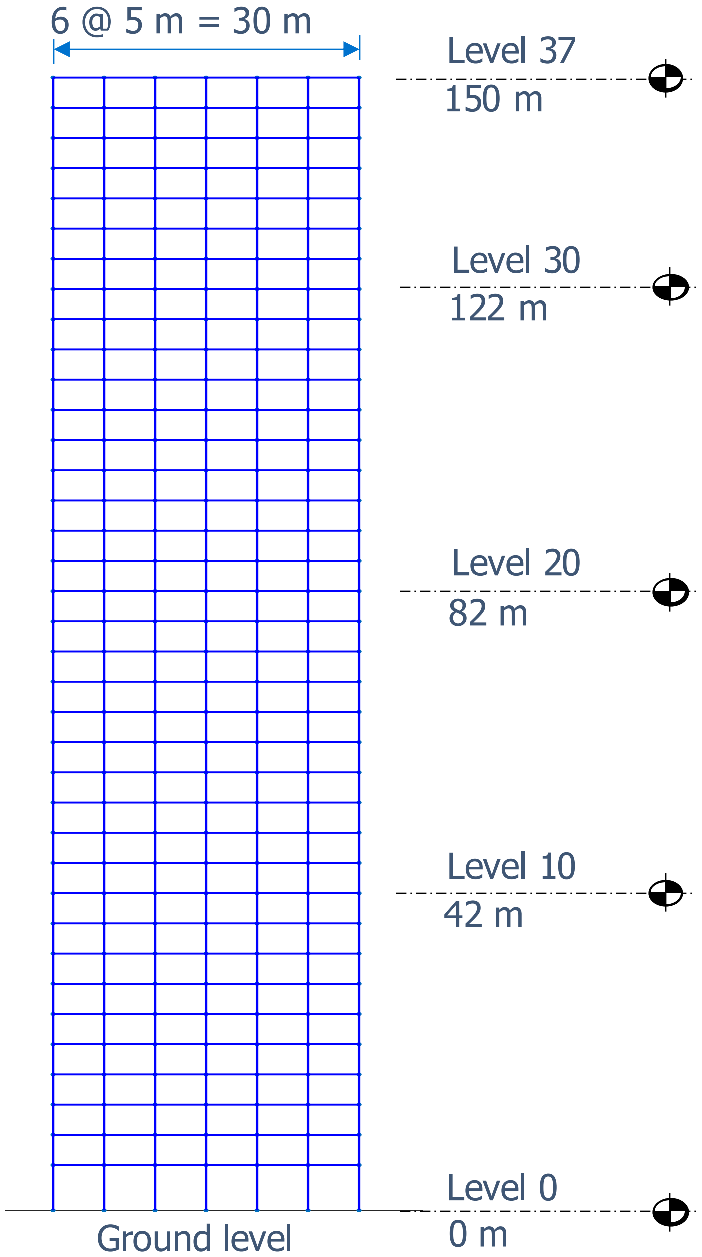
Structural system:
- 37-story 3D steel moment resisting frame;
- Elastic-perfectly plastic material model;
- Wind direction: α = 90°;
- Wind speed: $\bar{v}_y = 65$ m/s at the building top;
- Wind load motion: Wind tunnel data informed stochastic wind load model;

OBJECTIVE

Use DeepONet to map the wind excitation to the x-displacement of the right-most node of the building

Labelled dataset: 1000: $N_{train} = 800, N_{test} = 200$





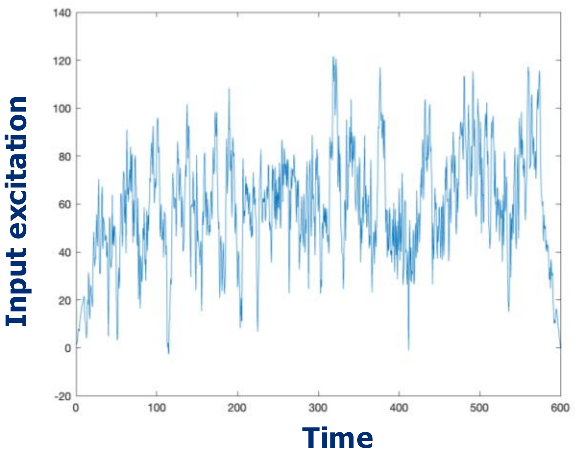
NO for wind response approximation

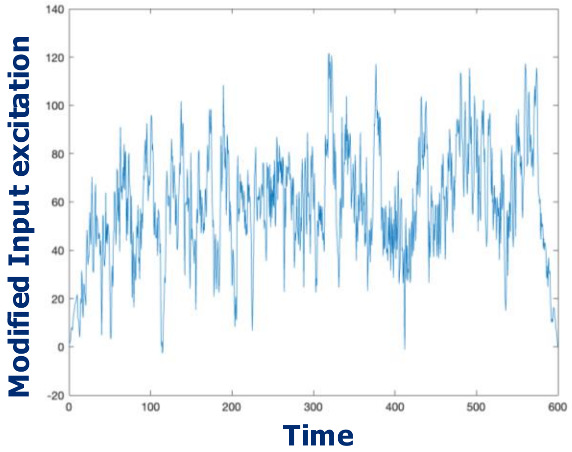
Data

- H_i : Time history of excitation at i^{th} floor
- Δt^H : 0.5s
- T : 600 s
- R_i : Displacement at i^{th} floor (R_i^x, R_i^y, R_i^z)
- Δt^R : 0.02s
- $\mathcal{G}: [H_1, H_2, \ldots, H_{37}] \rightarrow R_{37}$

Preprocessing

- Input excitation frequency $(\mathrm{H_{freq}})$ $\neq$ Response frequency $(\mathrm{R_{freq}})$
- **Option 1**: Filtering the response and reducing its frequency to $\mathrm{H_{freq}}$
  - Loss of information
- **Option 2:** Interpolation of $\mathrm{H_i}$ on the time discretization of $\mathrm{R_i}$
  - Taking time delay of 20s to remove the initial zeros





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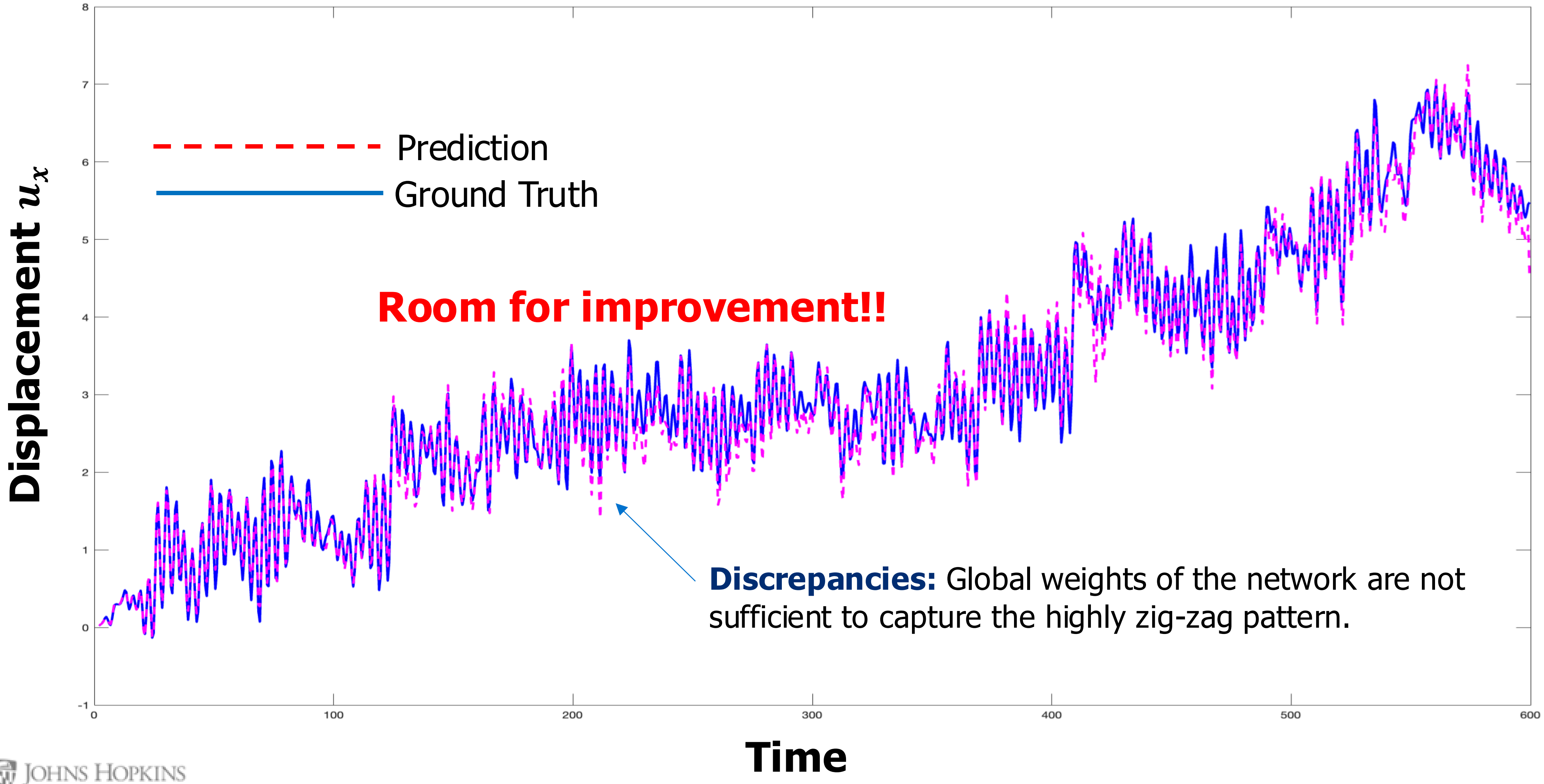
NO for wind response approximation



NO for wind response approximation

Solution 1: DeepOnet with Self-adaptive Weights

- Introduce **a weight** at every time stamp.
- Such weights can be manually modulated. For the given problem size – impractical.
- Hence, adaptive weights to automatically update along with the network parameters can be used.

Original loss function:

$$\mathcal{L}(\theta) = \frac{1}{N}\sum_{i=1}^{N}\left(u_i(\theta) - \mathcal{G}_\theta(\mathbf{v}_i)(\theta)\right)^2$$

Weighted loss function:

$$\mathcal{L}(\theta) = \frac{1}{N}\sum_{i=1}^{N} w_j\left(u_i(\theta) - \mathcal{G}_\theta(\mathbf{v}_i)(\theta)\right)^2 \Rightarrow$$

$$\mathcal{L}(\theta, \boldsymbol{\lambda}) = \sum_{i=1}^{N} g(\boldsymbol{\lambda})\left(u_i(\theta) - \mathcal{G}_\theta(\mathbf{v}_i)(\theta)\right)^2$$

Masking function

NO for wind response approximation

DeepOnet with Self-adaptive Weights

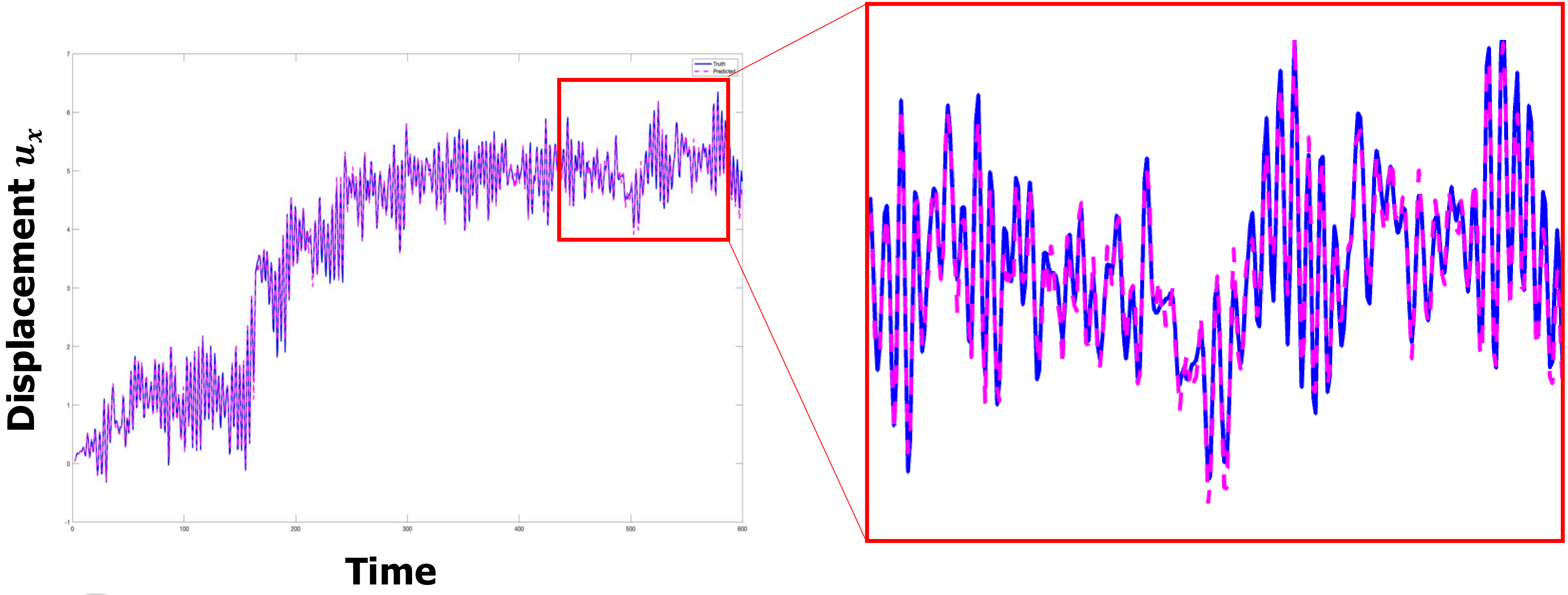
Optimization problem: $$\min_{\theta}\max_{\boldsymbol{\lambda}}\mathcal{L}(\theta,\boldsymbol{\lambda})$$

Updating the masking function: $$\boldsymbol{\lambda}^{\mathrm{k}+1}=\boldsymbol{\lambda}^{\mathrm{k}}+\eta_{\boldsymbol{\lambda}}\nabla_{\boldsymbol{\lambda}}\mathcal{L}(\theta,\boldsymbol{\lambda})$$ **Gradient ascent**

Constraints on the masking function: $$g(\boldsymbol{\lambda}_i)>\mathbf{0}$$

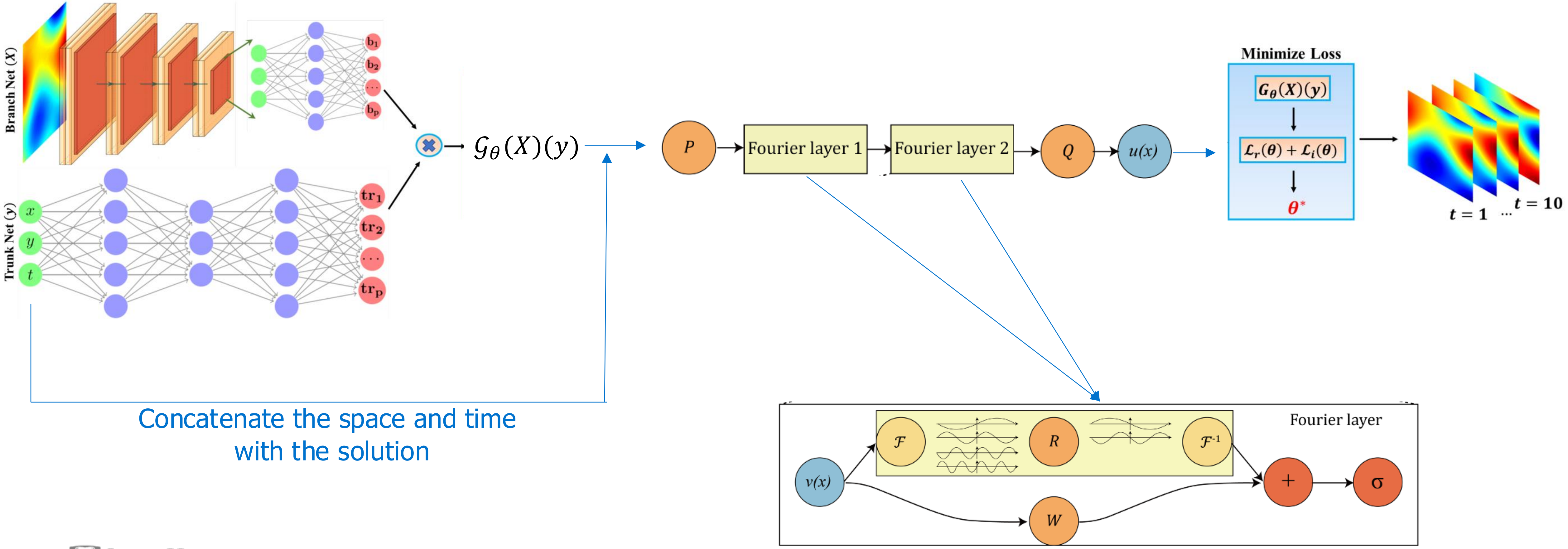
NO for wind response approximation

DeepOnet with Self-adaptive Weights



NO for wind response approximation

Solution 2: DeepOnet integrated with FNO



NO for wind response approximation

Relative L_2 error (%)

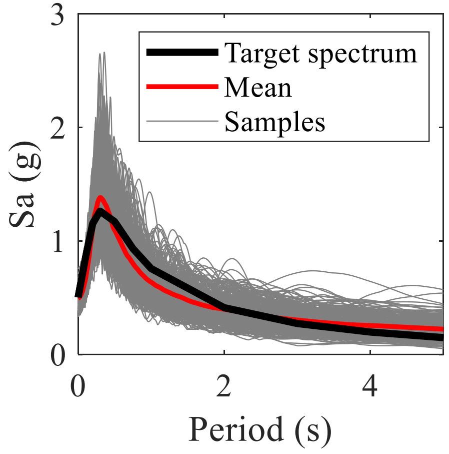
Method		u_x	u_y	u_z
DeepONet in time	Vanilla DeepONet	13.12	14.89	13.11
	Self-adaptive DeepONet	7.87	7.89	7.89
DeepONet in space and time	Vanilla DeepONet	15.92	15.66	15.12
	Self-adaptive DeepONet	7.91	7.56	7.62
	Integrated framework	1.12	1.34	1.76

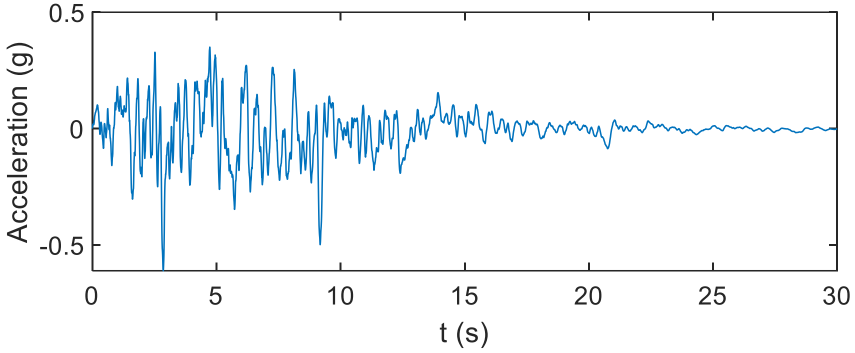
DeepONet in time: Mapping response to the location of the highest interstorey drift
DeepONet in space and time: Mapping response to the nodes on the right surface of the building for all 37 floors.

NO for earthquake response approximation

Structural system:

- 6-story nonlinear shear building
- Ground motion model: Filtered white noise;
- Target spectrum: Generated using USGS tool at Loma Prieta with 10% exceedance in 50 years.

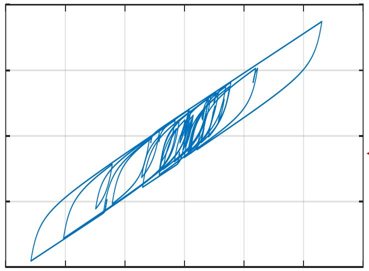




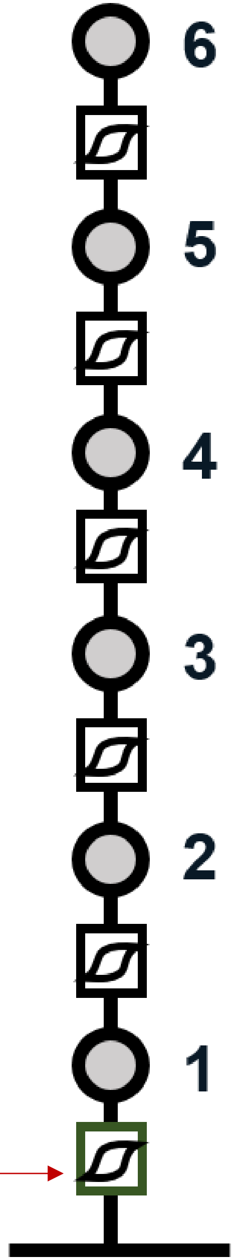
OBJECTIVE

Use FNO to map the ground motion to the acceleration at the 6 floors

Labelled dataset: 1000: $N_{train} = 800, N_{test} = 200$



Steel 02 Material



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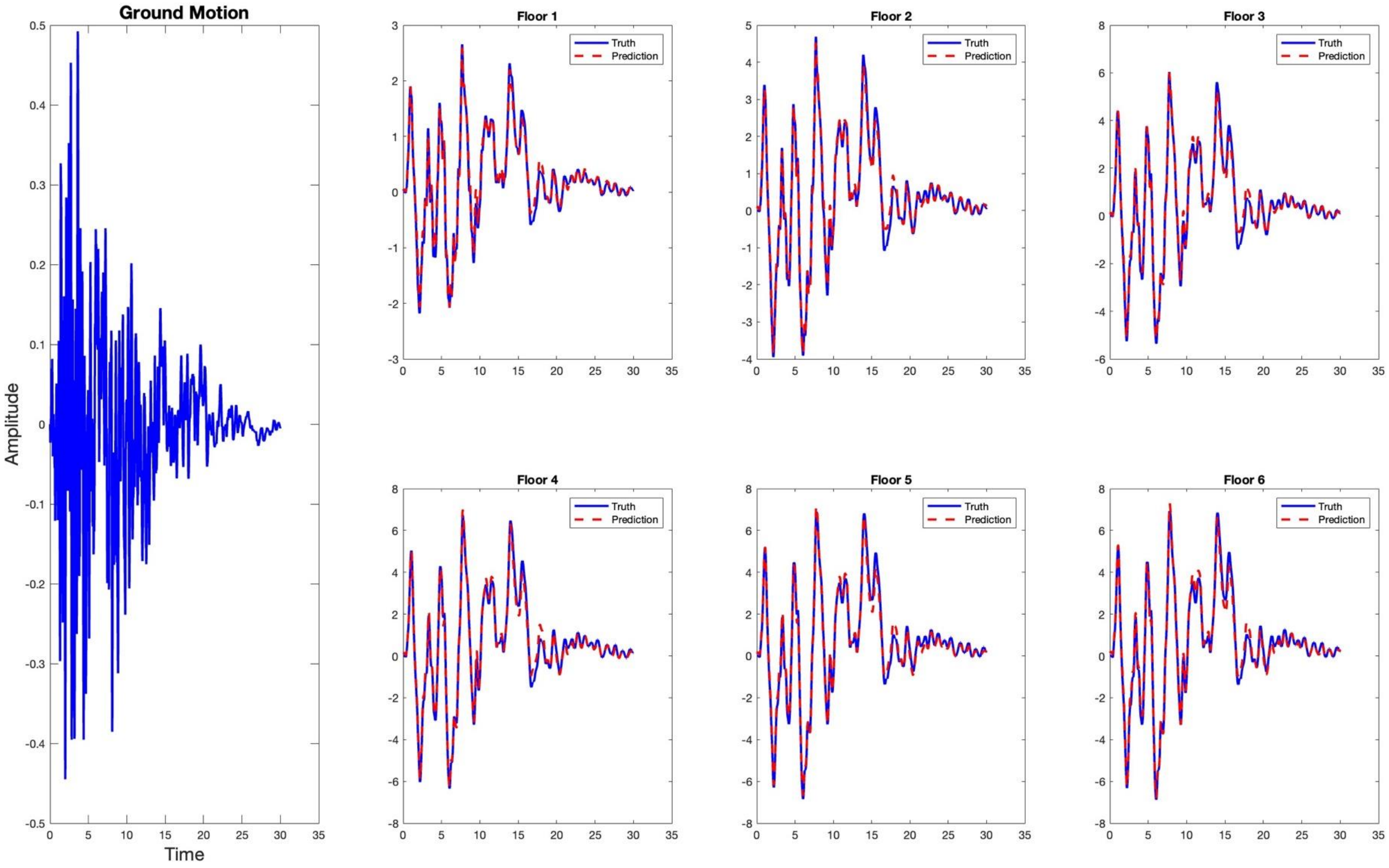
NO for earthquake response approximation



Relative L_2 error (%)

Method		Response
FNO 1D	Vanilla FNO	8.1
	SA - FNO	4.23

Hands-on training

Let us learn to construct a DeepONet for modeling the response of a cantilever beam under applied displacement

Training of DeepONet

$$m\frac{d^2u}{dt^2} + \mathrm{k}\frac{du}{dt} + F = 0$$

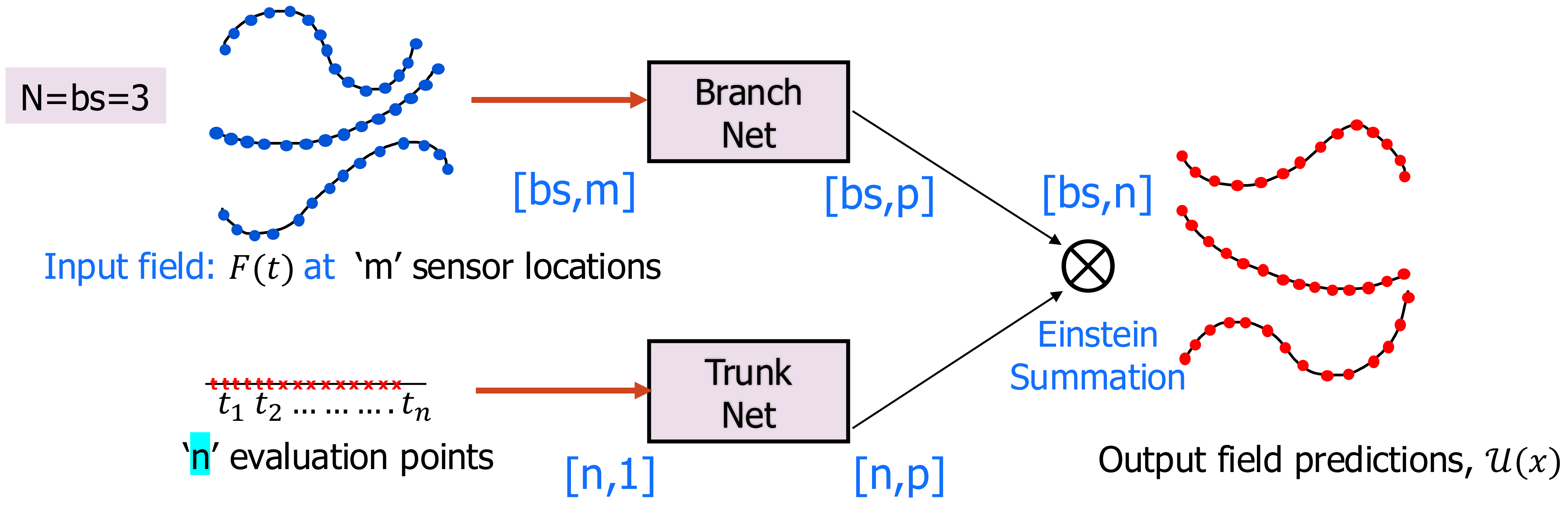
Nonlinear operator $\mathcal{G} : \mathcal{F} \to \mathcal{U}$

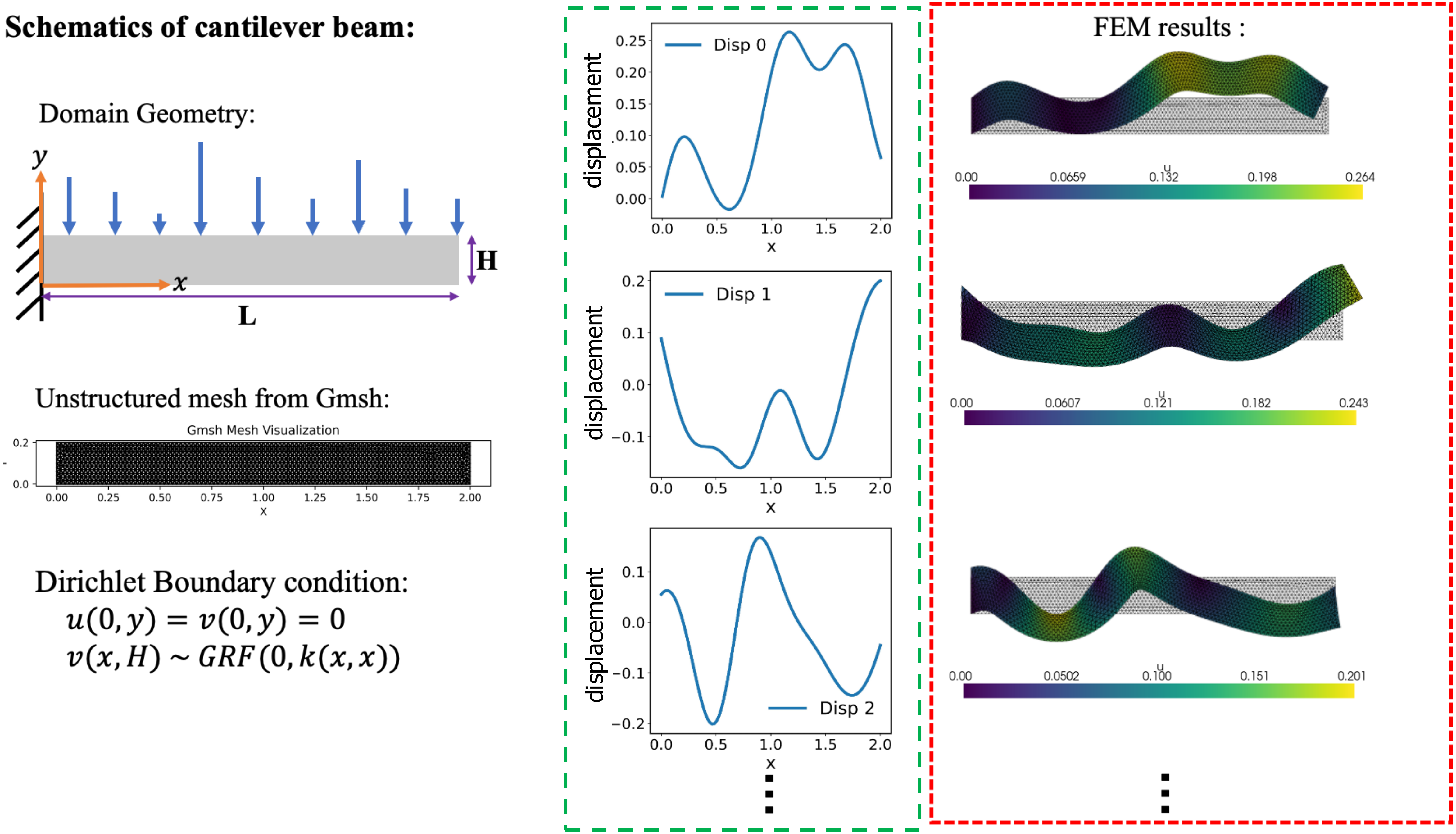
Neural operator $\mathcal{G}_\theta : \mathcal{F} \to \mathcal{U}, \quad \theta \in \Theta$

Training data $\{F^i(t_k), u^i(t_j)\}_{i=1,j=1,k=1}^{N,n,m}$

Training Dataset

S.No	Input field data	Output field data
1	$F^1(t_1), F^1(t_2), \cdots\cdots\cdots, F^1(t_m)$	$u^1(t_1), u^1(t_2), \cdots\cdots\cdots, u^1(t_n)$
2	$F^2(t_1), F^2(t_2), \cdots\cdots\cdots, F^2(t_m)$	$u^2(t_1), u^2(t_2), \cdots\cdots\cdots, u^2(t_n)$
.	.	.
N	$F^N(t_1), F^N(t_2), \cdots\cdots\cdots, F^N(t_m)$	$u^N(t_1), u^N(t_2), \cdots\cdots\cdots, u^N(t_n)$





Schematics of cantilever beam:
Domain Geometry:
y
x
H
L
Unstructured mesh from Gmsh:
Gmsh Mesh Visualization
Dirichlet Boundary condition:
$u(0, y) = v(0, y) = 0$
$v(x, H) \sim GRF(0, k(x, x))$
Disp 0
Disp 1
Disp 2
displacement
x
FEM results :
u
0.00 0.0659 0.132 0.198 0.264
0.00 0.0607 0.121 0.182 0.243
0.00 0.0502 0.100 0.151 0.201

The codes and the associated details are available on:
https://designsafe-training.github.io/deeponet/deeponet.html
and
https://github.com/DesignSafe-Training/deeponet

Centrum IntelliPhysics






Thank you!