Real-Time Inference of Defects and Impedance Using Deep Operator Networks

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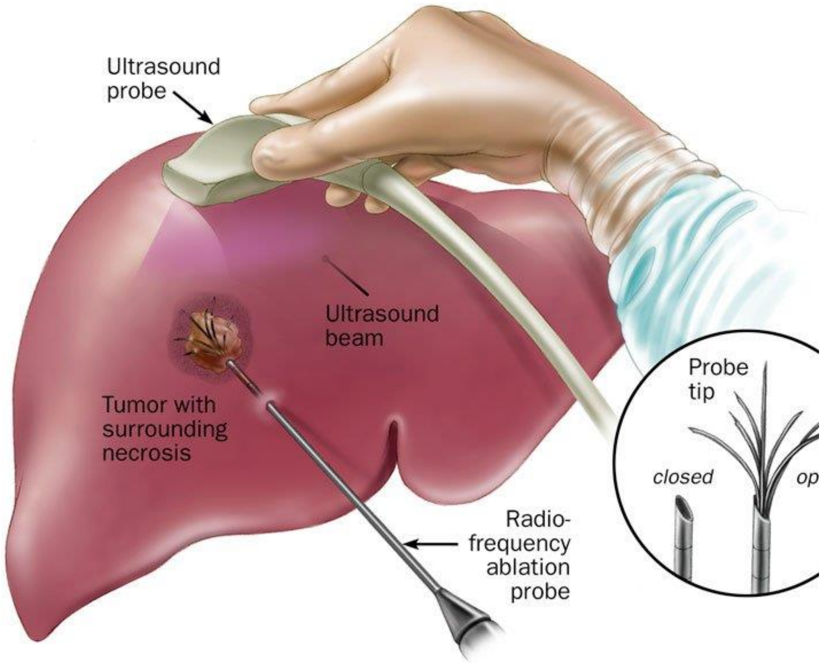
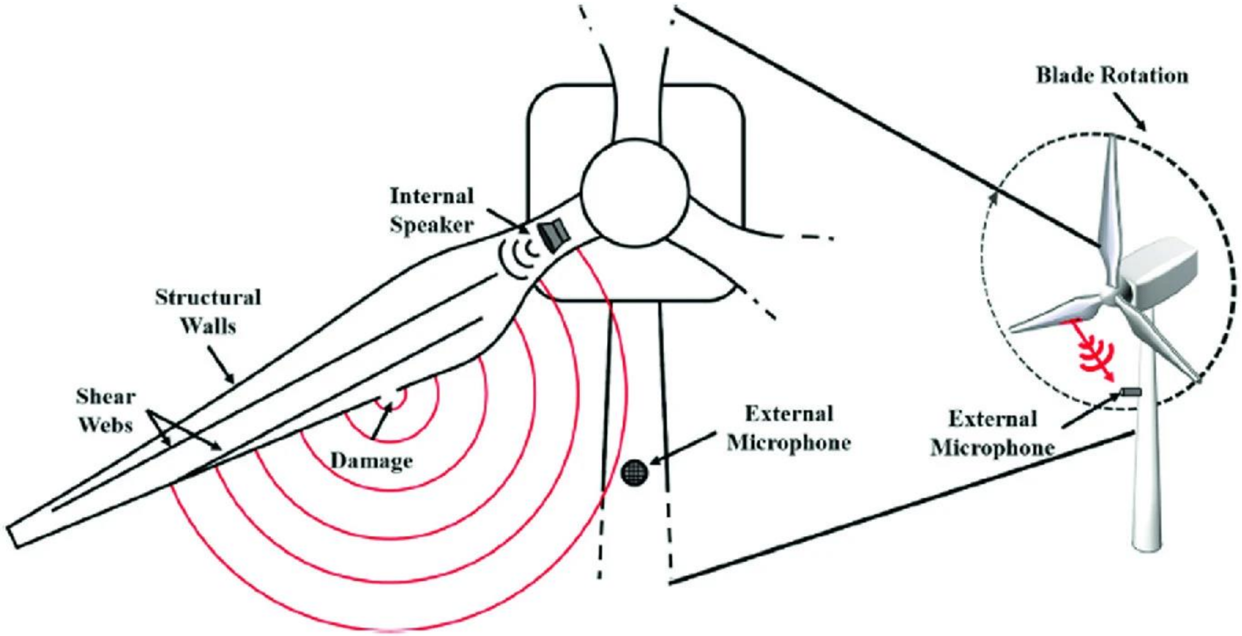
Outline

- Motivation
- Forward wave scattering
- Numerical implementation
- Neural Operators
- Results
- Future work

Motivation



Source: South Florida Surgical Oncology. (n.d.). Liver Tumor Ablation. Retrieved from https://southfloridasurgicaloncology.com/liver-tumor-ablation/



Source: Malekimoghadam, R., Krause, S., & Czichon, S. (2020). A Critical Review on the Structural Health Monitoring Methods of the Composite Wind Turbine Blades. (pp. 409-438).

Liver tumor radiofrequency ablation:

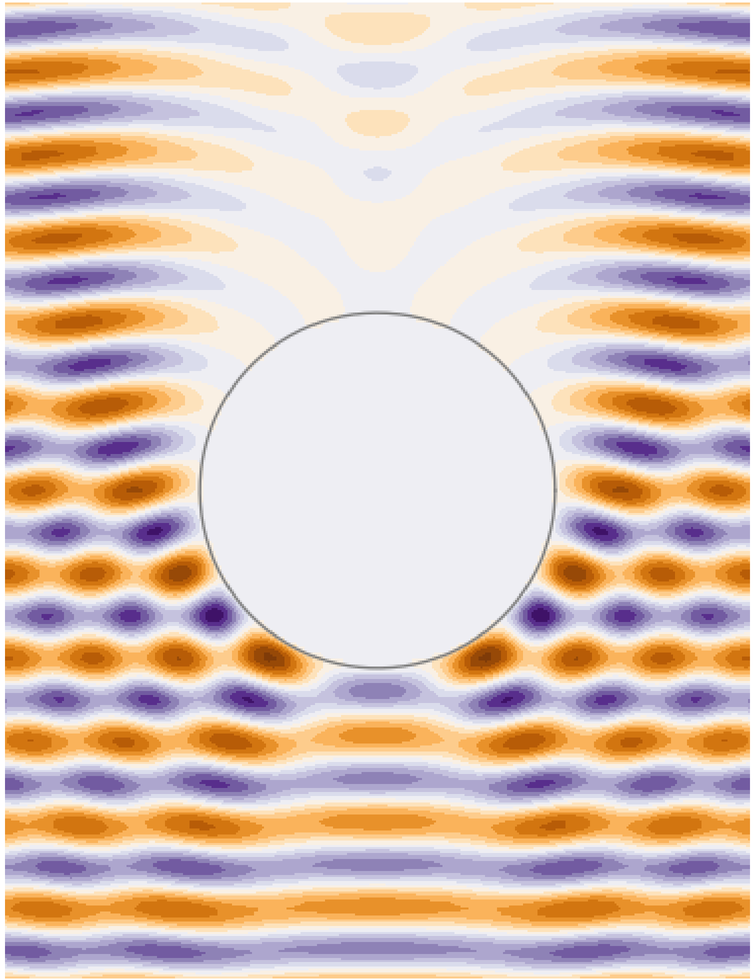
- The tumor boundary changes during treatment as tissue is destroyed.

Wind Turbine Blade Monitoring:

- Dynamic shape changes in turbine blades under operational loads can indicate potential failures.

Wave scattering

Wave scattering is a physical phenomenon that occurs when waves encounter an obstacle or a medium with
different properties, causing the waves to change direction, amplitude, or phase.



Source: https://arturgower.github.io/publication/effectivewaves-3d/

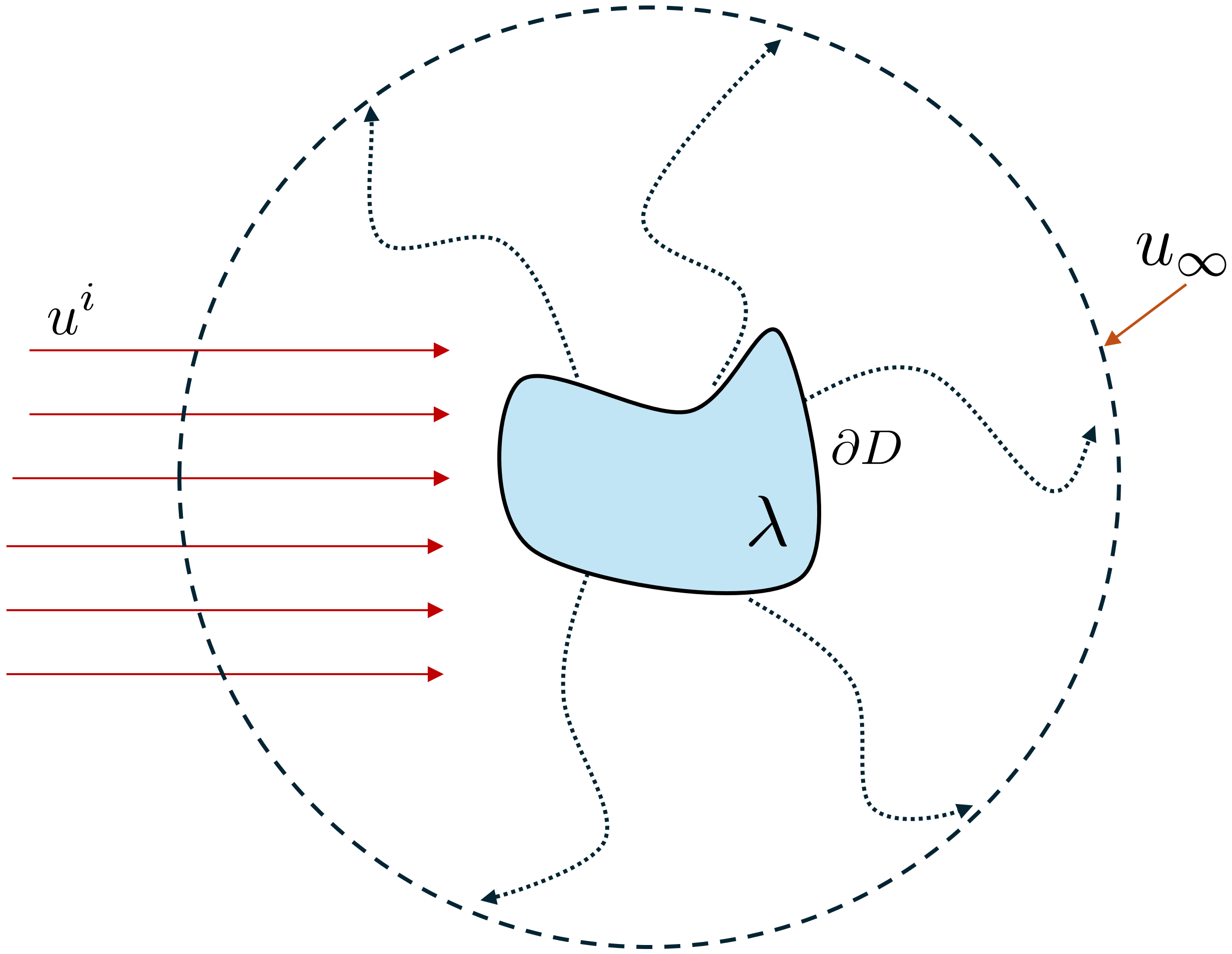
Forward scattering problem

Find the total field $u = u^i + u^s$ such that:

$$\begin{aligned} \Delta u + k^2 u = 0 \quad & \text{in } \mathbb{R}^2 \setminus \overline{D} \quad \text{(Helmholtz equation)} \\ \frac{\partial u}{\partial \nu} + ik\lambda u = 0 \quad & \text{on } \partial D \quad \text{(Impedance boundary condition)} \\ \lim_{r\to\infty} \sqrt{r}\left(\frac{\partial u^s}{\partial r} - iku^s\right) = 0 \quad & \text{(Sommerfeld radiation condition)} \end{aligned}$$

The incident field u^i is typically a plane wave:

$$u^i(x) = e^{ikx\cdot d}$$



u^i: Incident wave
u^s: Scattered wave
u: Total field
∂D: Obstacle boundary
λ: Impedance

k: Wave number
ν: Normal vector
r: Radius
d: direction unit vector
u_∞: Far field solution

Boundary Integral Equation approach (Nyström Method)

General solution: Green's function, $\Phi(x,y) = \frac{i}{4} H_0^{(1)}(k|x-y|)$

$H_0^{(1)}$: Hankel function of the first kind of order zero

Single-Layer Potential Representation, $u^s(x) = \int_{\partial D} \Phi(x,y)\phi(y)\,ds(y), \quad x \in \mathbb{R}^2 \setminus \overline{D}$

Apply boundary condition:

$$\phi(x) - (K'\phi)(x) - ik\lambda(x)(S\phi)(x) = 2\frac{\partial u^i}{\partial \nu}(x) + 2ik\lambda(x)u^i(x), \quad x \in \partial D$$

where,

$$(S\phi)(x) = 2\int_{\partial D} \Phi(x,y)\phi(y)\,ds(y), \quad x \in \partial D$$

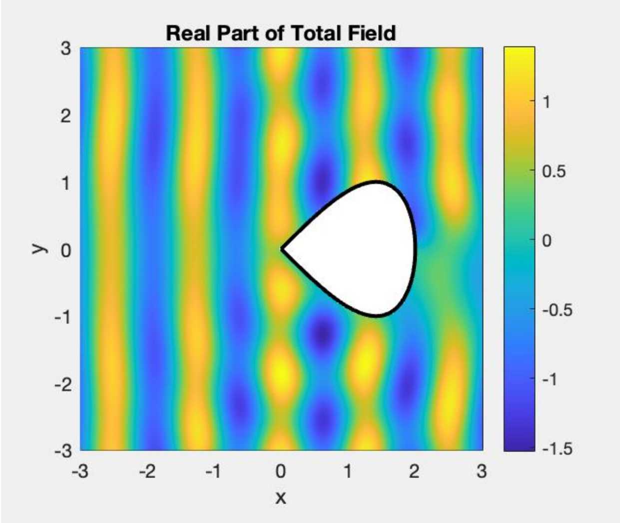
$$(K'\phi)(x) = 2\int_{\partial D} \frac{\partial \Phi(x,y)}{\partial \nu(x)}\phi(y)\,ds(y), \quad x \in \partial D$$

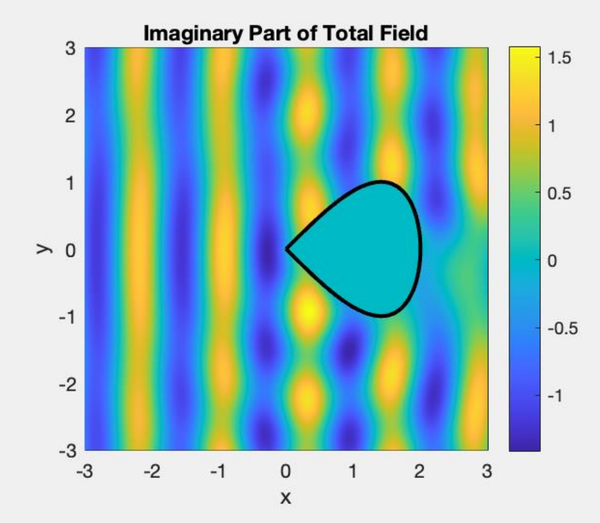
$$u_\infty(\hat{x}) = \frac{e^{i\pi/4}}{\sqrt{8\pi k}} \int_{\partial D} e^{-ik\hat{x}\cdot y}\phi(y)\,ds(y)$$

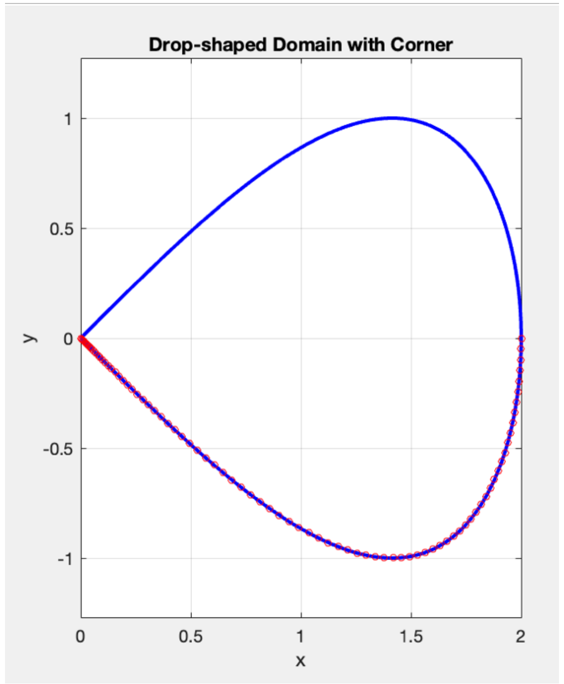
$$u^s(x) = \frac{e^{ik|x|}}{\sqrt{|x|}} \left\{ u_\infty(\hat{x}) + O\left(\frac{1}{|x|}\right) \right\},$$

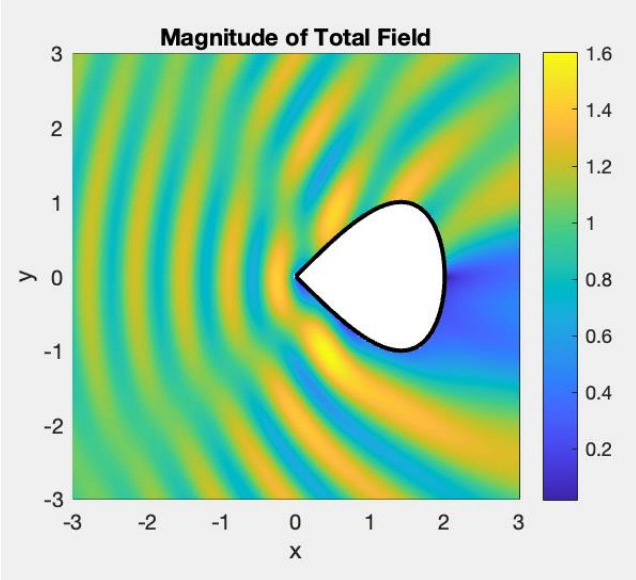
Numerical example

$k = 5$ (wave number)
$\lambda = 5$ (impedance)
$n = 64$ (quadrature points)
$d = [1, 0]$ (direction normal vector)









Kress, Rainer, and William Rundell. "Inverse scattering for shape and impedance revisited." *The Journal of Integral Equations and Applications* 30.2 (2018): 293-311.

Boundary integral equations for BVPs, and their high-order Nystr¨om quadratures: a tutorial Alex Barnett

Data generation

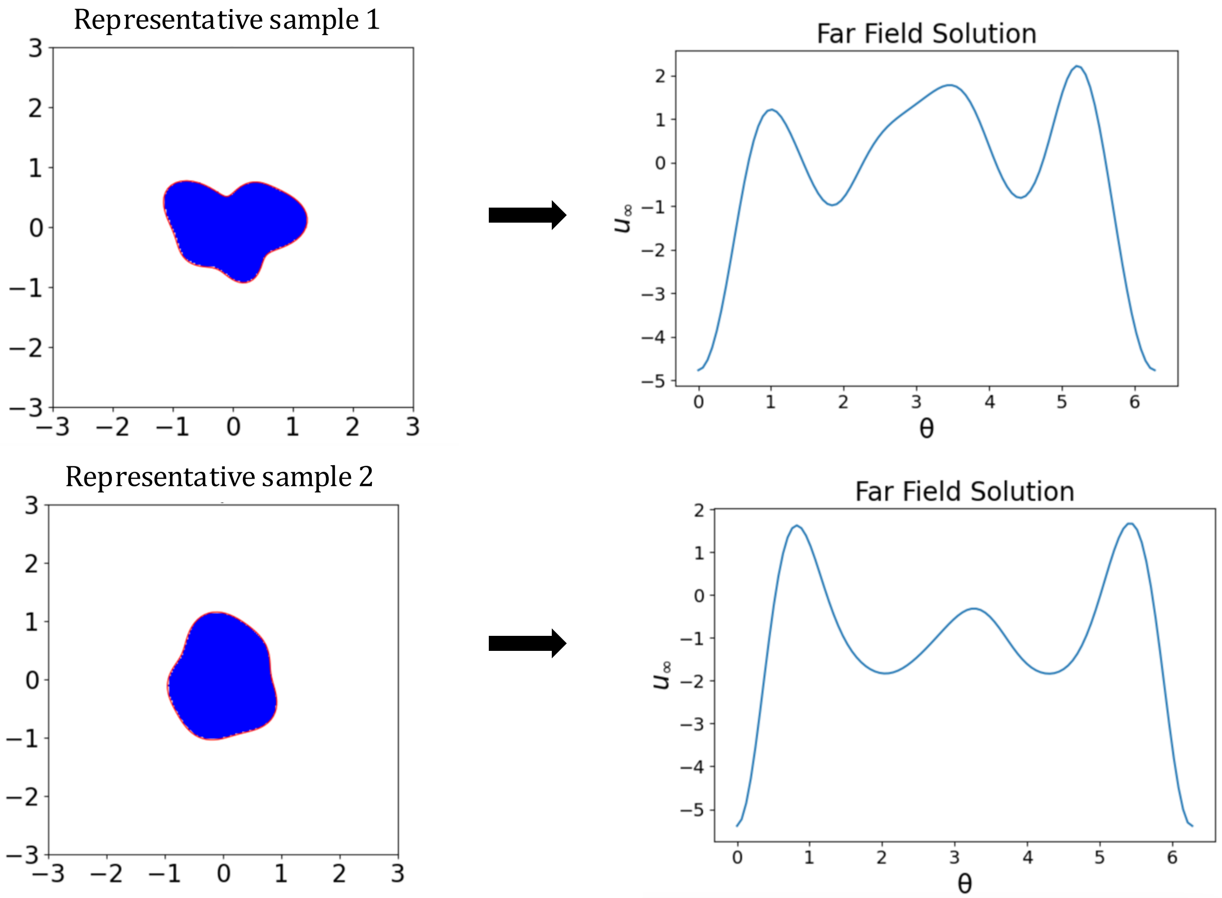
$$r(\theta) = r\_base * (1 + shape\_param * (0.2 * \cos(2 * \theta) + 0.15 * \sin(3 * \theta) + 0.1 * \cos(5 * \theta)))$$

$r\_base \sim N(1, 0.2)$

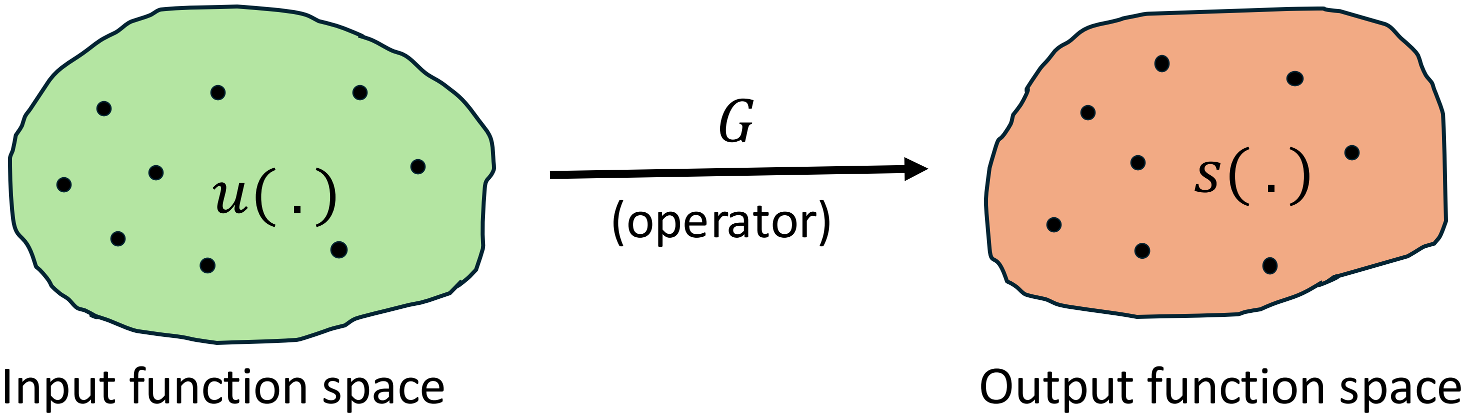
$shape\_param \sim N(0, 1)$

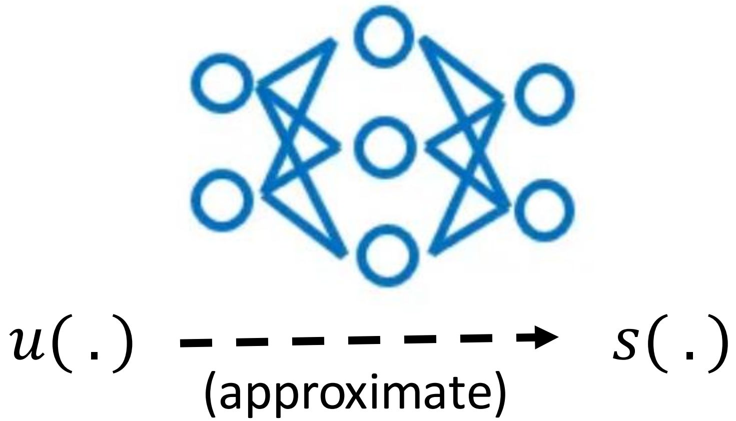
$\lambda \sim N(5, 1.5)$

$\#$ of Samples: 2000



Operator learning framework

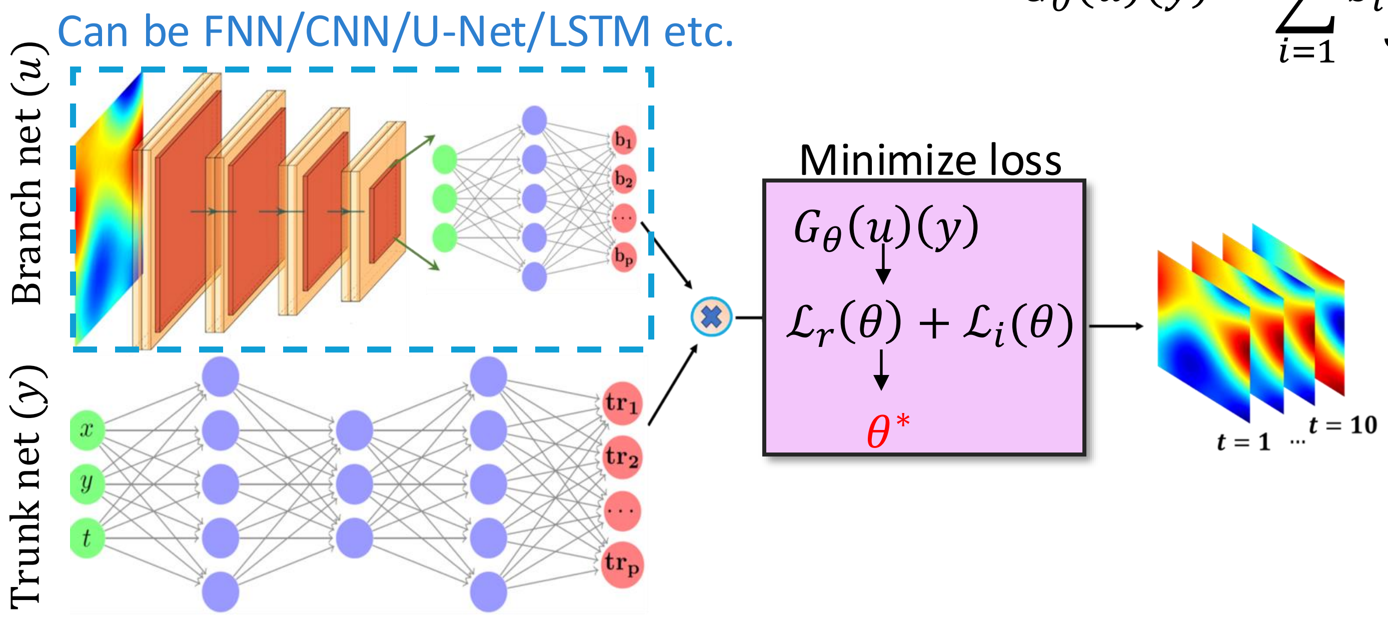




Deep Operator Networks (DeepONet)

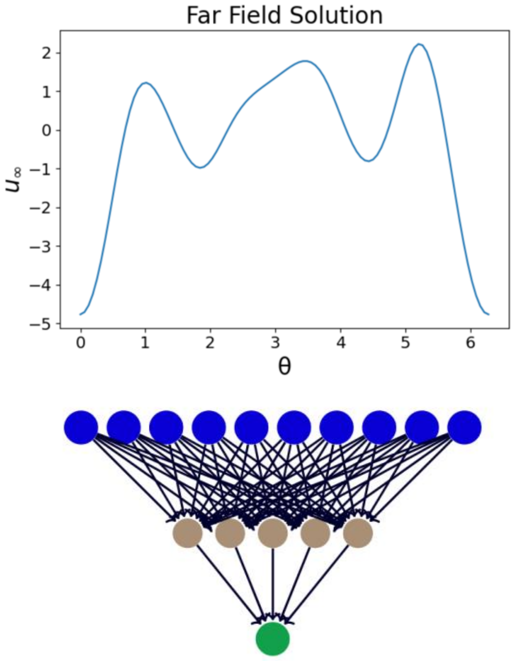
- Generalized Universal Approximation Theorem for Operator [Chen '95, Lu et al. '19]
- **Branch net**: Input $\{u(x_i)\}_{i=1}^{m}$, output: $\left[b_1, b_2, .., b_p\right]^T \in \mathbb{R}^p$
- **Trunk net**: Input y, output: $\left[t_1, t_2, .., t_p\right]^T \in \mathbb{R}^p$
- Input u is evaluated at the fixed locations $\{y_i\}_{i=1}^{m}$

$$G_\theta(u)(y) = \sum_{i=1}^{p} \underbrace{b_i(u(x_1), u\,(x_2), \ldots, u\,(x_m))}_{\text{branch net}} \cdot \underbrace{tr_i(y)}_{\text{trunk net}}$$



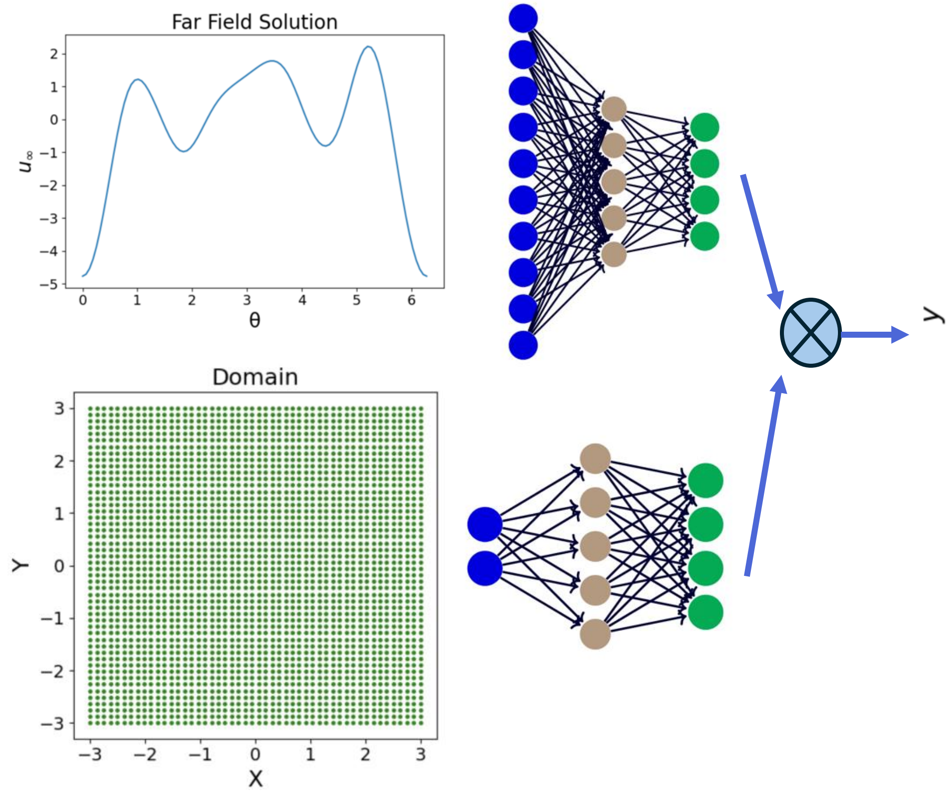
Our framework

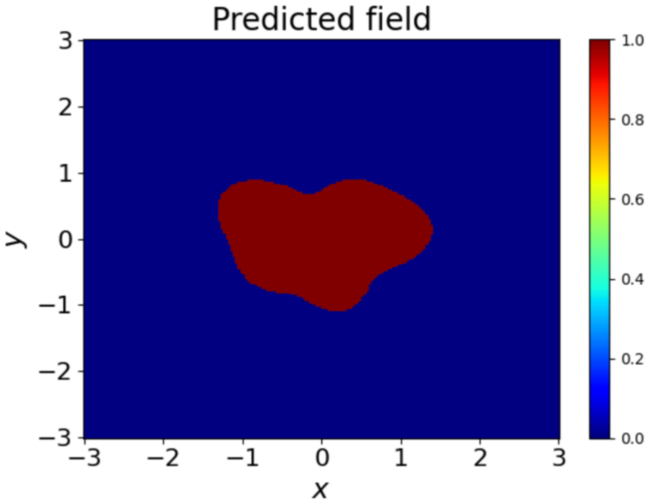
MLP



λ

DeepONet





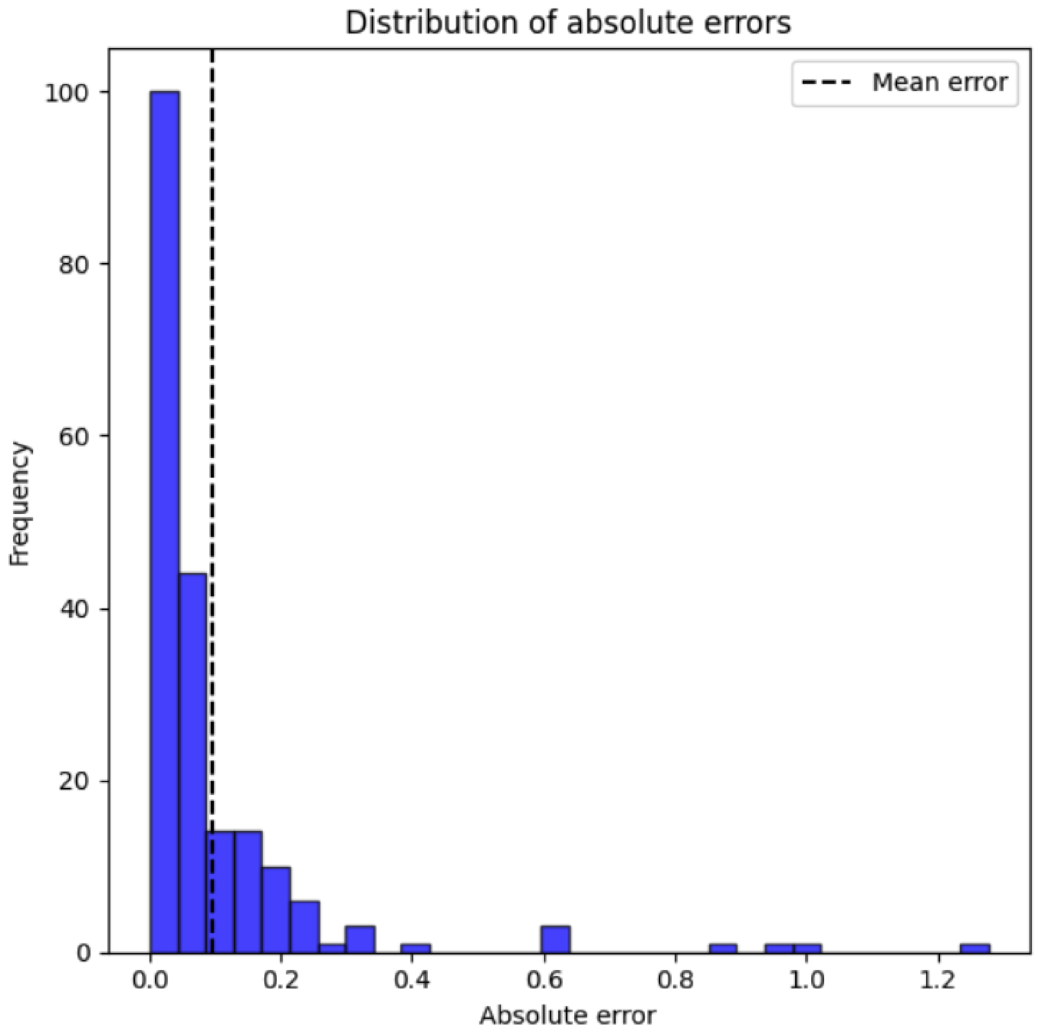
Phase density field

Loss: Mean Square Error (predicted field, Data)

Prediction:

if value < 0.5
value = 0
if value > 0.5
value = 1

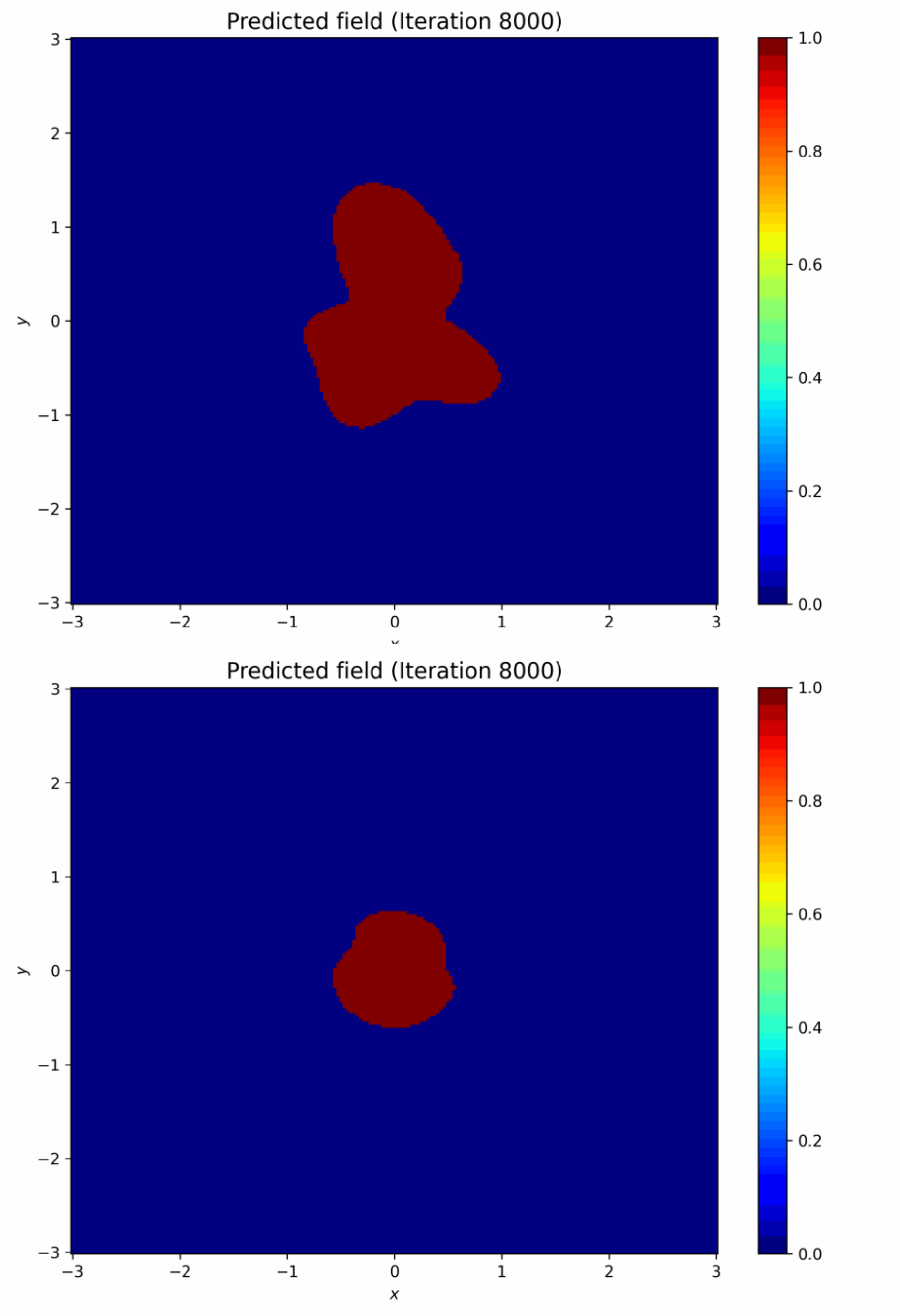
Impedance (λ) prediction



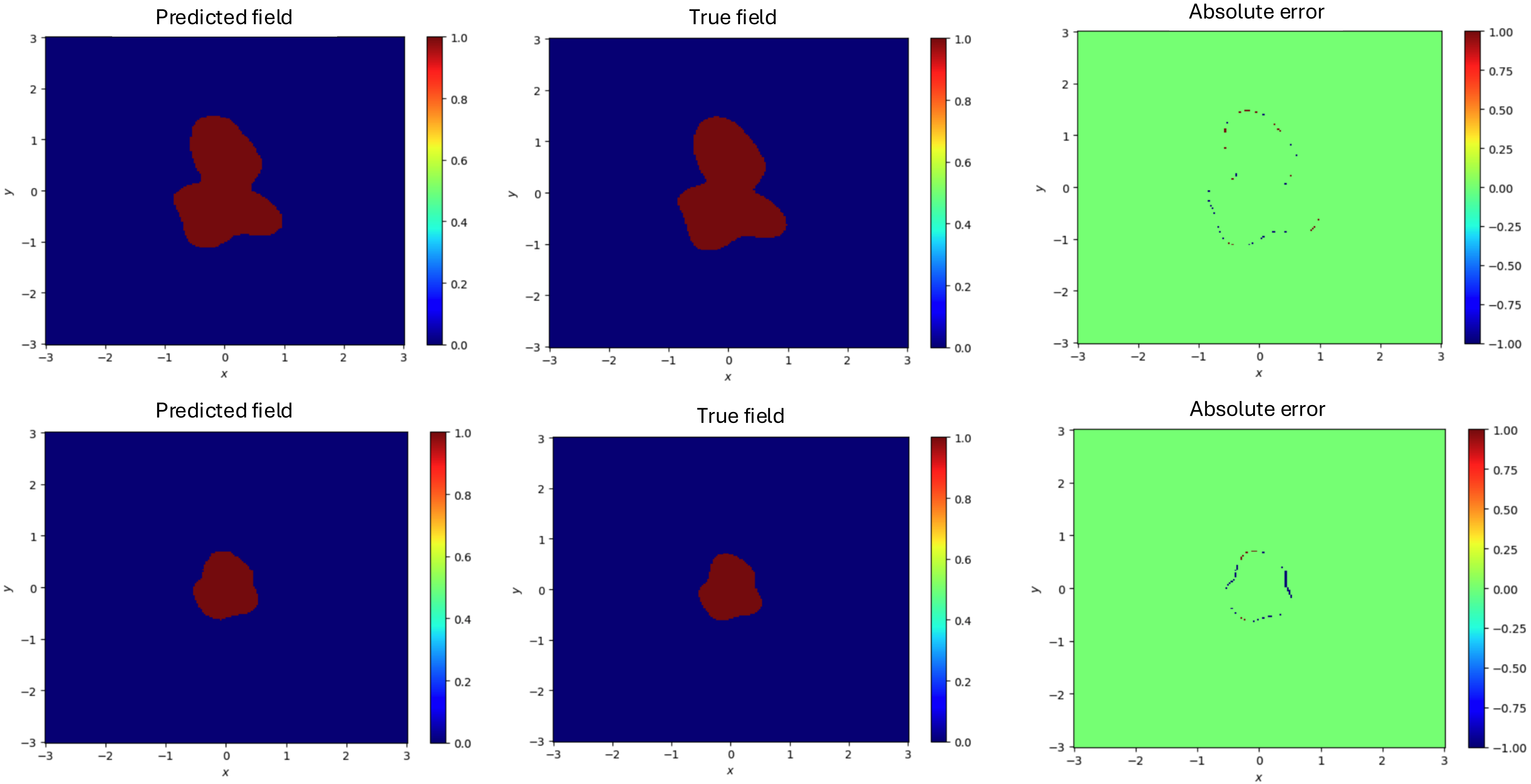
Best prediction λ
True value: 4.407370
Predicted value: 4.408105
Absolute error: 0.000735

Worst prediction λ
True value: 5.626672
Predicted value: 4.350400
Absolute error: 1.276272

Phase density prediction

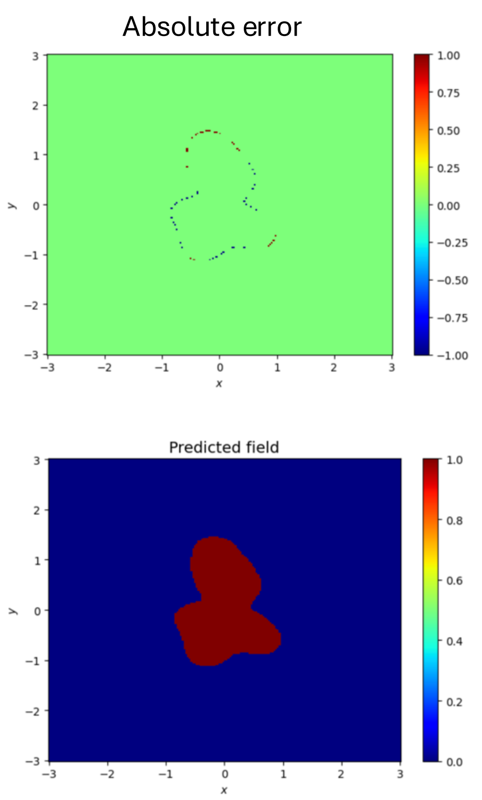
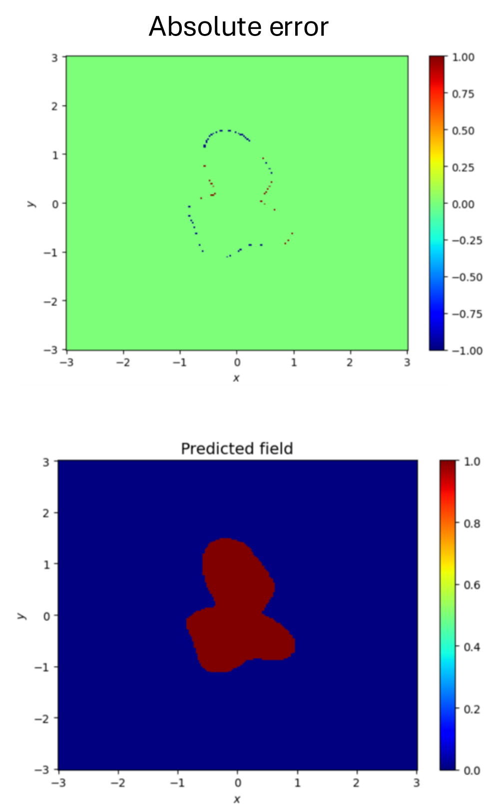
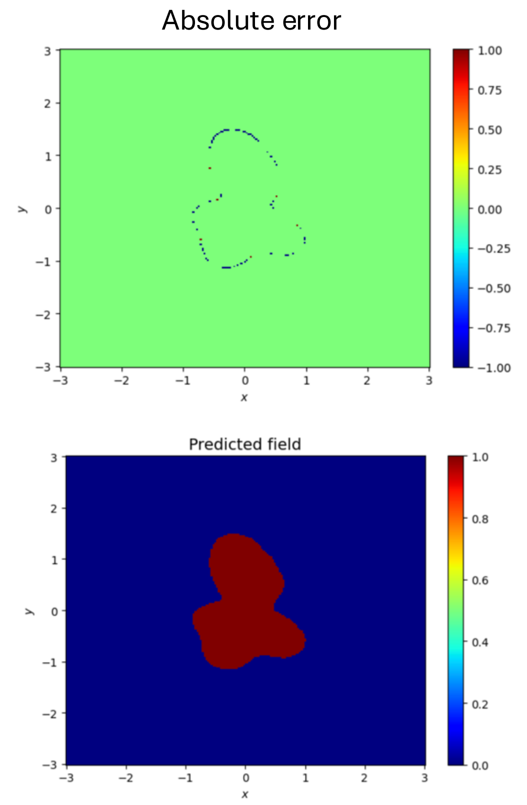
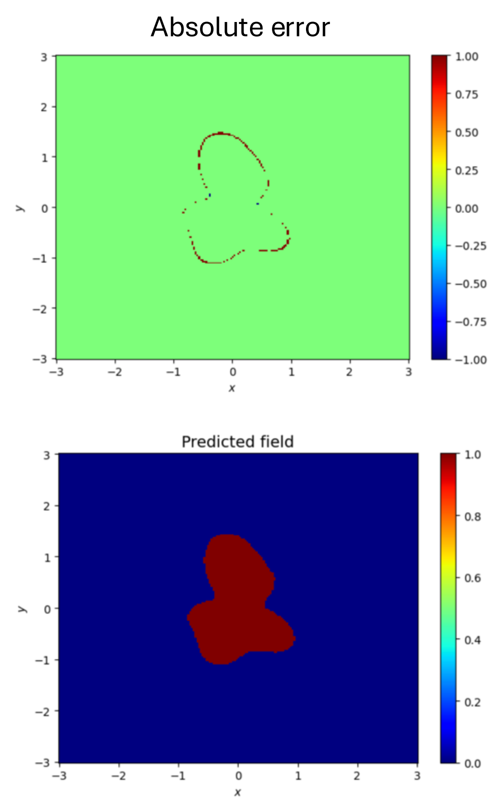
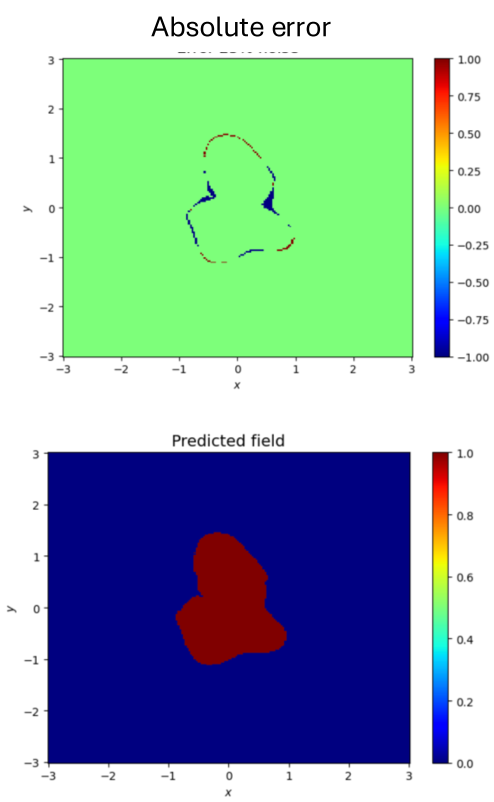


Phase density prediction



MSE across 200 test samples = $4.04 \times 10^{(-3)}$

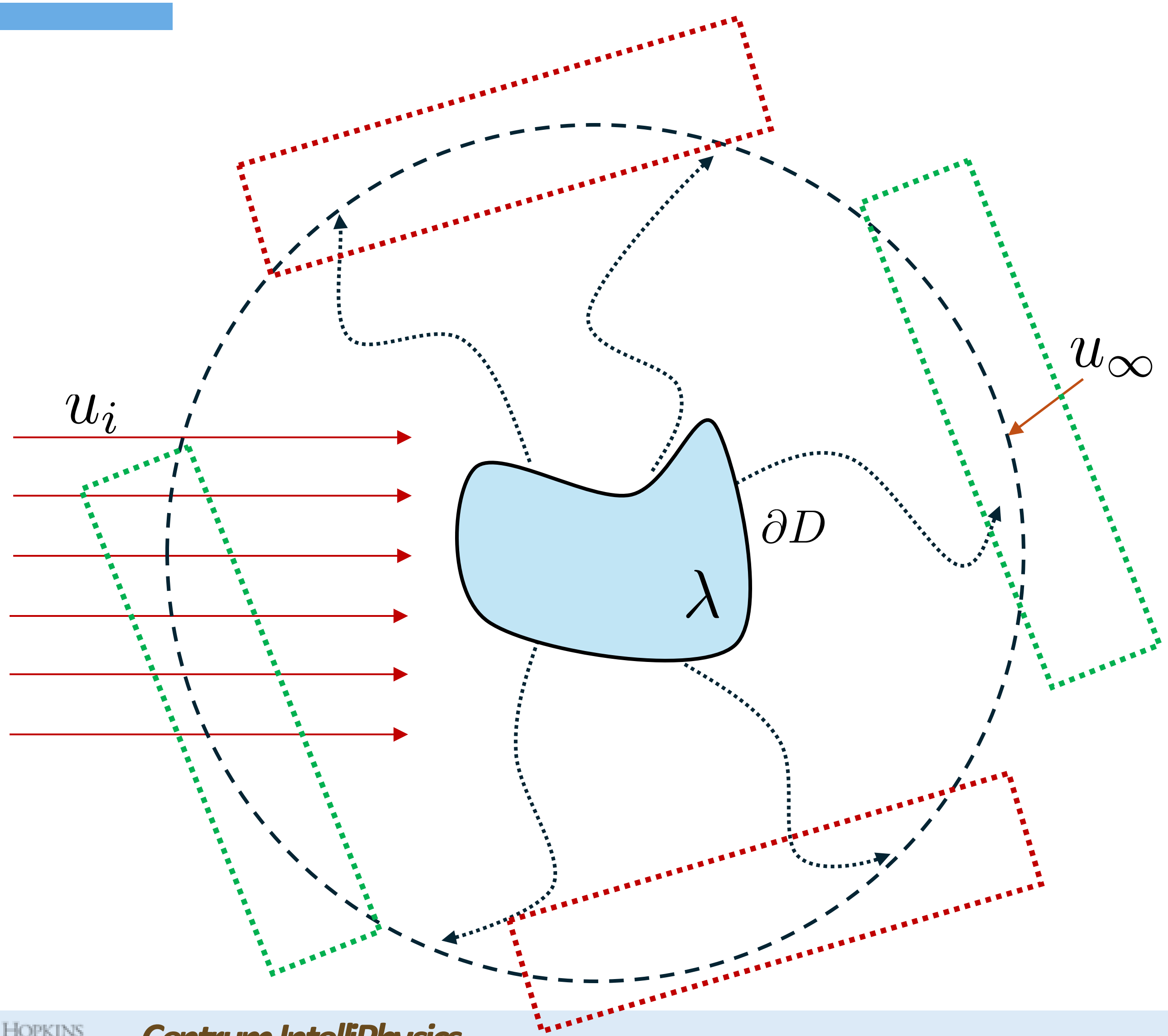
Phase density prediction (with noisy data)

5% noise	10% noise	15% noise	20% noise	25% noise
				

Summary

- We generated data by solving wave scattering problem.
- Utilized DeepONet to learn the inverse mapping between the far field pattern to the obstacle boundary and impedance.
- The framework is robust against 20% Gaussian noise.

Future work



- Mapping of partial far field pattern measurement to the phase density?
- Solve the problem in 3D.
(3D reconstruction)

Thank you