

# SVR based nonlinear PA equalization in MIMO system with Rayleigh channel

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**Abstract.** Power amplifier (PA) nonlinearity has been one of the crucial constraints to the performance of radio frequency (RF) communication systems. The distortion caused by amplitude-phase modulation (APM) results in severe performance degradation. In this paper, we study the effect of PA nonlinear distortion on bit error performance in Multiple-Input Multiple-Output (MIMO) wireless communication systems, and develop a new method based on Support Vector Regression (SVR) to compensate the nonlinear distortion. Under the condition that the receiver has no knowledge of PA distortion parameters, we propose a receiver compensation technique which involves estimating the points of the distorted AM-AM curve based on training using SVR. The proposed scheme can realize a model-free estimation. Simulation results show that, for  $4 \times 4$  MIMO with 16-QAM, the proposed scheme is effective to deal with the nonlinear distortion caused by PAs.

**Keywords:** Support Vector Regression, PA distortion, MIMO, Rapp model

## 1 Introduction

In wireless communication systems, PA is an indispensable component working at the end of transmitter. RF power amplifiers are usually nonlinear in practice, the output signal will be distorted by this nonlinearity. Besides, most modern wireless communication systems, including the fifth generation (5G) cellular systems, use multi-carrier or orthogonal frequency division multiplexing (OFDM) modulations whose signals have extremely high peak to average power ratio (PAPR). This makes it reckless to neglect the nonlinearity of PA.

In order to suppress the distortion, PAs in wireless transceivers usually have to work with high output backoff (OBO) in practice. However, PAs optimal power efficiency only lies in nonlinear saturated region. Driving the PA closer to its saturation point is appealing, since it would increase the energy efficiency and prolong the battery life of a mobile device. However, the nonlinear distortion makes it difficult to conduct symbol detection at the receiver. Therefore, it is attracting to carry out a method to enhance the power efficiency of PA and improve the accuracy of signal detection simultaneously.

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Various strategies have been investigated to solve this problem. First strategy is to linearize the PAs at transmitter. One of the popular methods today is to adopt the digital pre-distortion (DPD) before the PA. DPD techniques remove the nonlinear distortion by learning the nonlinearity parameters through a feedback path to characterize the inverse transfer function of PA for linearization. However, DPD is too complex and costly for small and low-cost 5G devices, especially for cost and battery limited device in massive machine-type communications or internet of things (IoT) in uplink transmissions.

So another strategy is to mitigate nonlinear PA distortion at the receivers via post-distorter equalization. This method does not require a feedback RF chain at the transmitter to learn the PA model and implement the predistorter, which reduce the hardware complexity in transmitter. Artificial neural networks (ANN) have also be studied for both nonlinear modeling of PAs and nonlinear equalization [1–3]. But some shortages also exist, that neural networks usually require relatively long training sequences to model the PA nonlinearity accurately. To deal with this problem, we consider taking advantage of the SVR algorithm to save the training sequences resource. SVR is an popular kernel-based supervised learning algorithm [4]. In contrast to ANN, SVR requires less training time and memory for modeling, allowing it to search for solutions more efficiently in a very high-dimensional space. Although SVR has been successfully applied for PA and some other microwave device modeling [5], previous work has not focused on PA nonlinear distortion equalization and recovery based on SVR as far as we know.

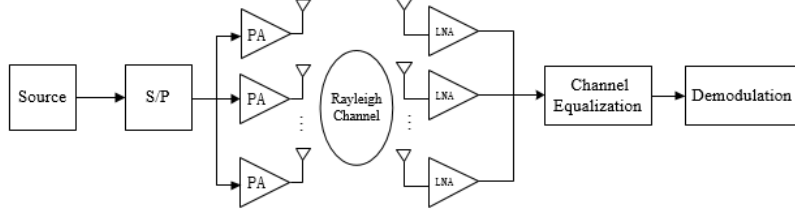
In this paper, we develop a nonlinear equalization scheme based on support vector regression algorithm to recover the PA distorted signals at the receiver. We take Rapp model [6], which is a widely accepted PA model of solid state power amplifiers (SSPA), as an example to construct the output signals distorted by nonlinear PA of a MIMO system. As the both the PA distortion model and parameters are unknown at the receiver in practice, we firstly propose a training-based nonlinearity estimation technique using SVR. The estimated nonlinearity model-free parameters are then used to eliminate the distortion at the receiver. It is noticeable that we just apply Rapp model as an example to validate the effectiveness of proposed model-free algorithm, which is demonstrated in this paper by simulations and experiments.

The rest of this paper is organized as follows: first, the MIMO channel model with channels nonlinearity is introduced in Section II. In Section III, the SVR-based nonlinear estimator and equalizer is explained. Then, some simulation results are given in Section IV. Finally, conclusions are shown in Section V.

## 2 MIMO System Model with Nonlinear Channel

Consider an MIMO system with  $n_t$  transmit antennas,  $n_t$  transmit RF chain, and  $n_r$  receive antennas. The MIMO transmitter with nonlinear PAs is shown in Fig.1. In this paper, we assume that channel is block fading and the block length is  $N$ . Let  $\mathbf{x} \in \mathcal{C}^{n_t \times N}$  denote the modulated signal matrix, where  $x_{i,n}, i =$

$1, \dots, n_t, n = 1, \dots, N$  denotes the complex signal in M-point signal constellation. We here adopt the 16-QAM signal constellation.



**Fig. 1.** MIMO system in Rayleigh channel with PAs.

An accurate nonlinear PA model is the Rapp model. The PA gain  $G(\cdot)$  can be expressed as

$$G(|x_{i,n}|) = \frac{A(|x_{i,n}|) \exp(j\phi(|x_{i,n}|))}{|x_{i,n}|}, \quad (1)$$

where the term  $|x_{i,n}|$  denotes the amplitude of  $x_{i,n}$ . Real amplifiers exhibit various magnitudes of nonlinearities. These are usually described by the amplitude transfer characteristics (also known as the amplitude modulation/amplitude modulation (AM/AM) conversion) and the phase transfer characteristics (also known as the amplitude modulation/phase modulation (AM/PM) conversion) of the amplifier. Function  $A(\cdot)$  and  $\phi(\cdot)$  represent the AM/AM conversion and the AM/PM conversion respectively. In the Rapp model for SSPA, the phase distortion is assumed to be small enough so that it can usually be neglected [7].

Therefore, the Rapp model can be characterized by the following AM/AM and AM/PM conversions  $g(A) = \frac{\alpha A}{[1 + (\frac{\alpha A}{\beta})^{2r}]^{\frac{1}{2r}}}$ ,  $\phi(r) = 0$ , where  $A = |x_{i,n}|$  is

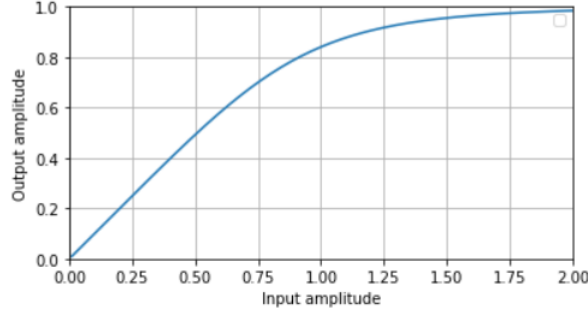
the amplitude of the PA input signal,  $\alpha$  is a small signal gain usually normalized to 1,  $\beta$  is the limiting output amplitude, and  $r$  controls the smoothness of the transition from linear operation to saturated operation. The AM/AM characteristic of such a model with  $\beta = 1, r = 2$  is shown in Fig.2.

Due to the MIMO transmitter has  $n_t$  transmit RF chains, we extend the PA distortion function  $G(\cdot)$  defined in (1) to vector functions as follows:

$$\forall \mathbf{v} \in \mathcal{C}^{n_t}, \mathcal{G}(\mathbf{v}) \triangleq \{G(v_i), i = 1, \dots, n_t\}. \quad (2)$$

Note that the map  $\mathbf{x} \mapsto \mathcal{G}(\mathbf{x})$  is one to one, then we see that  $\tilde{\mathbf{x}} = \mathcal{G}(\mathbf{x}), \tilde{\mathbf{x}} \in \mathcal{C}^{n_t \times N}$ .

Let  $\mathbf{H} \in \mathcal{C}^{n_r \times n_t}$  denote the MIMO channel gain matrix, where  $H_{ij}$  denotes the complex channel gain from the  $j$ -th transmit antenna to the  $i$ -th receive antenna. Both the real part and image part of the complex channel gains are assumed to be independent Gaussian with zero mean and unit variance. The received signal can be presented as  $\mathbf{y} = \mathbf{H}\tilde{\mathbf{x}} + \mathbf{n}$ , where  $\tilde{\mathbf{x}} \in \mathcal{C}^{n_t \times N}$  denotes the transmitted modulated signal which is distorted by PA nonlinearity. And



**Fig. 2.** AM/AM conversion characteristic of the SSPA model.

$\mathbf{n} \in \mathcal{C}^{n_r \times N}$  denotes the additive zero mean complex Gaussian noise vector with covariance matrix  $\sigma^2 \mathbf{I}_{n_r}$ . The average received signal-to-noise ratio (SNR) per receive antenna is given by  $\frac{P_{O,avg}}{\sigma^2}$ , where  $P_{O,avg}$  denotes the average power of the transmitted signal, which is the output of the PA.

### 3 SVR based nonlinear PA distortion equalizer

Linear approaches cannot achieve high estimation precision in the presence of nonlinear power amplifier, where the PAs amplitude characteristics perform heavy nonlinearity. Therefore, we adapt SVR derived from the SVM, which is a powerful tool in solving nonlinear, small samples and high dimensional regression problems. Thus, we map the input vector into a higher dimensional feature space  $\mathcal{H}$  (possibly infinity) by means of the nonlinear transformation  $\varphi(\cdot)$ . The following regression function is  $\hat{y} = \mathbf{w}^T \varphi(x) + b + e$ , where  $\mathbf{w}$  is the weight vector,  $b$  is the bias term, and residuals  $\{e_m\}$  account for the effect of both approximation errors and noise. In the SVM framework, the optimality criterion is a regularized and constrained version of the regularized Least Squares criterion. In general, SVM algorithms minimize a regularized cost function of the residuals, usually the Vapniks  $\varepsilon$  - *insensitivity* cost function.

To improve the performance of the estimation algorithm, a robust cost function is introduced which is  $\varepsilon$ -Huber robust cost function, given by

$$l_\varepsilon(e) = \begin{cases} 0, & |e| \leq \varepsilon \\ \frac{1}{2\gamma}(|e| - \varepsilon)^2, & \varepsilon < |e| \leq e_C \\ C(|e| - \varepsilon) - \frac{1}{2}\gamma C^2, & e_C < |e| \end{cases} \quad (3)$$

where  $e_C = \varepsilon + \gamma C$ ,  $\varepsilon$  is the parameter which is positive scalar that represents the insensitivity to a low noise level, parameters  $\gamma$  and  $C$  control essentially the trade-off between the regularization and the losses. In fact, concerning errors overhead  $e_C$ , the cost function is linear, however, it is quadratic for errors among  $\varepsilon$  and  $e_C$ . It should be noted that, errors lower than  $\varepsilon$  are disregarded in the  $\varepsilon$  - *insensitivity* zone.

Now, the SVR primal problems can be formulized as

$$\min_{w,b} \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^m l_{\varepsilon}(f(\mathbf{x}_i) - y_i). \quad (4)$$

By introducing relaxation variable  $\xi_i$  to allow the existence of classification errors within a certain range and improve the generalization ability of learning methods, the original problem can be optimized as

$$\begin{aligned} \min_{\mathbf{w}, b, \xi_i, \hat{\xi}_i} & \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^m (\xi_i + \hat{\xi}_i) \\ \text{s.t.} & \quad f(\mathbf{x}_i) - y_i \leq \varepsilon + \xi_i, \\ & \quad y_i - f(\mathbf{x}_i) \leq \varepsilon + \hat{\xi}_i, \\ & \quad \xi_i \geq 0, \hat{\xi}_i \geq 0, i = 1, 2, \dots, m \end{aligned} \quad (5)$$

Considering the Lagrange multiplier  $\alpha$ , we can convert the above problems into dual ones by convex optimization method, which can be expressed as

$$\begin{aligned} \max_{\alpha, \hat{\alpha}} & \sum_{i=1}^m y_i(\hat{\alpha}_i - \alpha_i) - \varepsilon(\hat{\alpha}_i + \alpha_i) - \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m (\hat{\alpha}_i - \alpha_i)(\hat{\alpha}_j + \alpha_j) \mathbf{x}_i^T \mathbf{x}_j \\ \text{s.t.} & \quad \sum_{i=1}^m (\hat{\alpha}_i - \alpha_i) = 0, \\ & \quad 0 \leq \alpha_i, \hat{\alpha}_i \leq C \end{aligned} \quad (6)$$

By solving the convex problem, we can get the  $\alpha_i, \hat{\alpha}_i$  and  $b$ , then the solution of the SVR problem can be presented as

$$f(\mathbf{x}) = \sum_{i=1}^m (\hat{\alpha}_i - \alpha_i) \mathbf{x}_i^T \mathbf{x} + b. \quad (7)$$

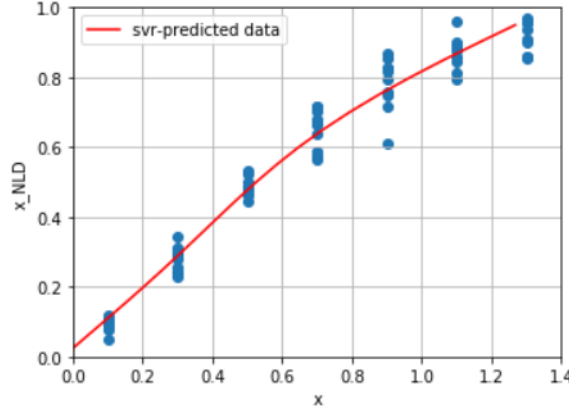
Let  $K(\mathbf{x}_i, \mathbf{x}_j) = \langle \varphi(\mathbf{x}_i), \varphi(\mathbf{x}_j) \rangle$  be a Mercers kernel which we here choose the RBF kernel. The RBF kernel function can be presented as  $K(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\sigma^2}\right)$ , where  $\|\cdot\|$  represent the euclidean norm,  $\sigma$  is the scaling factor, which could be optimized by cross-validation or by particular priori knowledge. By introducing the following kernel function we can get the final solution of regression problem.

$$f(\mathbf{x}) = \sum_{i=1}^m (\hat{\alpha}_i - \alpha_i) K(\mathbf{x}, x_i) + b, \quad (8)$$

where  $x_i$  is the input of training samples, and  $\mathbf{x}$  represents the signal amplitude recovered from distortion based on SVR. For PA nonlinearity is much more obvious for large signal amplitude, we can apply small-amplitude training sequences  $x_i$  to estimate the channel matrix  $\mathbf{H}$  and then recover nonlinear distortion after channel equalization.

## 4 Simulation Results

In this section, we present the simulation results for an  $N_t = 4, N_r = 4$  MIMO system with 16-QAM. The Rapp model of the SSPA with the following nonlinear distortion parameters is  $\beta = 1, r = 2$ . Figure 3 shows the SVR-predicted amplitude curve at the receiver with nonlinear PA distortion at the MIMO transmitter. The amplitude of the transmitted signal is normalized to 1. This figure represents the training process of the SVR based nonlinear distortion estimation. The blue points represent the noisy training data accumulated by receiver, which is used to train the SVR-based nonlinearity estimator. These training data firstly go through the MIMO channel equalization process, which we assume that the channel matrix  $\mathbf{H}$  is perfectly estimated by pilots whose amplitude is small enough to avoid the PA nonlinear distortion. The red curve represents the predicted PA nonlinear AM-AM curve. With the increase of SVR training size, the fitting curve becomes more accurate and show the tendency of convergence.

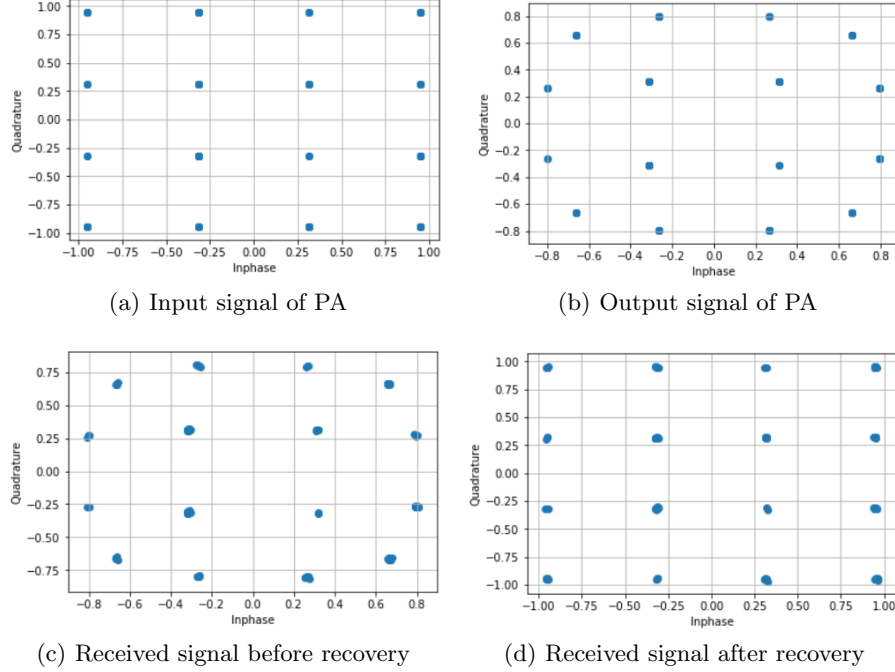


**Fig. 3.** Trained predicting curve based on SVR with noisy training data.

Figure 4(a) and 4(b) shows the points of the 16-QAM signal constellation at the input and output of the PA for the selected PA model and parameters above-mentioned. The AM/AM nonlinear distortion of PAs cause the amplitudes of both the in-phase and the quadrature components of the complex signal no longer maintain the original ratio for amplitude levels. We see that the points with amplitude levels farther from zero are much more effected by distortion, and the minimum Euclidean distance among the constellation points at the PA output gets reduced, which results in the degradation in BER performance, which is highlighted in Fig.5.

Figure 4(c) and 4(d) show the received data before and after SVR-based nonlinear distortion recovery at  $E_b/N_0 = 25dB$ . It is obvious that after the nonlinear equalization process with proposed scheme, the constellation points

return to normal in general, which the effectiveness of the proposed method is corroborated from the perspective of constellation points.

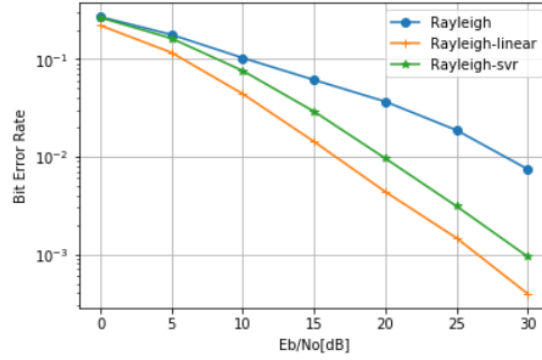


**Fig. 4.** Constellation diagrams before and after equalization.

Figure 4 shows the BER performance with varying  $E_b/N_0$  in different scenarios. For the blue curve in figure 5, no compensation is done to deal with the PA distortion. Undoubtedly, the performance degrades in this scenario comparing with the MIMO system with ideal PAs, which is shown by the yellow curve as the contrast analysis. The green curve represents the BER performance for proposed scheme at the receiver. We can see that the SVR-based method performs close to within 4dB of the performance of the MIMO system with ideal PAs.

## 5 Conclusions

The paper develops a model-free nonlinear PA distortion equalization scheme in MIMO communication system based on support vector regression. The simulation results show that this method is efficient to deal with the nonlinear distortion at the receiver after training. As we only choose one PA nonlinear model and 16-QAM modulation to simulate in this paper, more research may be carried out with other common PA model and modulation scheme. Besides,



**Fig. 5.** Ber performance for MIMO system in different scenarios.

more interest may concentrate on how to diminish the training data size and make it much faster to converge.

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