# Design and Modeling of Control System of Tilt Rotor Mechanism of a VTOL UAV

Mr. Ubair Ejaz Butt
Department of Aeronautics & Astronautics
Institute of Space Technology,
Islamabad, Pakistan
ubairbutt@yahoo.com

Mr. Haseeb Farrukh
Department of Aeronautics & Astronautics
Institute of Space Technology,
Islamabad, Pakistan
lgsjtchessclub@gmail.com

Mr. Izhar Hussain Kazmi
Department of Aeronautics & Astronautics
Institute of Space Technology,
Islamabad, Pakistan

Mr. Saad Riffat Qureshi
Department of Aeronautics & Astronautics
Institute of Space Technology,
Islamabad, Pakistan

Mr. Aseem-Ul-Islam
Department of Aeronautics & Astronautics
Institute of Space Technology,
Islamabad, Pakistan.
jamdurak@gmail.com

Abstract—This paper includes the design and control of the tilt rotor mechanism that is used in vertical take-off and landing UAV's. The design of Tilt Rotor is modeled on Creo2.0 and it involves the mathematical modeling of the control system so that transition from vertical to horizontal flight involves no loss in altitude with no change in attitude.

#### 1. INTRODUCTION

A vertical takeoff & landing unmanned air vehicle take off and land vertically. First it takeoff from the ground vertically after that the *tilt rotor mechanism* is used for the transition from vertical to horizontal flight. The VTOL UAV consist of four rotors to lift the fixed wing aircraft. Two propellers are mounted on a distance of 5.5 in. from the leading edge of the wing and the back two rotors are mounted upside down on a distance of 8 in. from the trailing edge of the wing to counter the pitching moments of the aircraft in the vertical flight. This paper includes the design and mathematical modeling of the control of the tilt rotor mechanism. Following is the schematic shown in which the propellers are mounted on the fixed wing aircraft.

As shown in the Fig 1, four motors are used to provide a vertical thrust to the aircraft. The angle between the three shafts connecting the motors is  $120^0$  and the distances are set as all the motors are balancing the pitching moments produced.

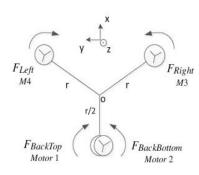


Fig 1: Rotors Configuration 1

The main focus of the research is the design and mathematical modeling that is of the tilt rotor mechanism. The design is such that when the aircraft takeoff vertically it has a thrust in the vertical direction, now to move in the horizontal flight, the front two propellers are tilted using the tilt rotor mechanism in the positive 90°(counter clockwise). The design and the mathematical modeling of this mechanism is discussed in the proceeding sections.

#### 2. DESIGN OF THE MECHANISM

The design of the mechanism is modeled on Creo2.0. The mechanism consists of the following;

- Pylons
- Bracket
- Servo
- Horns
- Push-rod

The mechanism is shown as:

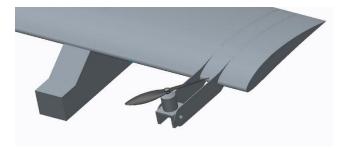


Fig 2: Tilt Mechanism

The servo is mounted on the inner side of the pylon which is connected to the bracket by means of a push rod. The push rod is connected at both ends with horns which on one side is attached to the servo and to the bracket on the other side.

Initially the bracket is at  $0^{0}$  As the servo has a rotation of  $90^{0}$ , the push rod rotates the bracket by an angle of  $90^{0}$  and in this way the bracket is aligned in the forward direction. A full 3D Creo model is shown below:

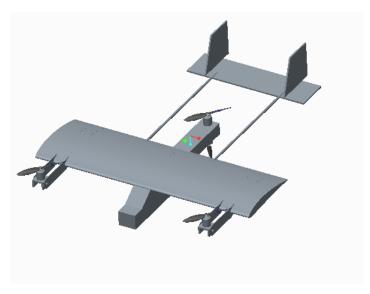


Fig 3: Full 3D Model

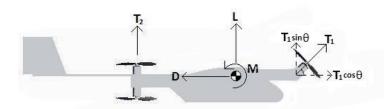
As shown in the above figure, the front two propellers will tilt using the tilt rotor mechanism and this is how the UAV accelerates in the forward direction thus moving to horizontal flight.

# 3. MATHEMATICAL MODELING OF CONTROL OF TILT ROTOR

The mathematical modeling of the tilt rotor mechanism is done by using the equilibrium equations for the x-direction of the aircraft, which is the forward direction. The main equation that is used for modeling is the Newton's second law as:

$$\Sigma F = ma = m\ddot{x}$$

When the front two rotor tilts, the forces that act on the aircraft are shown in the diagram below:



Using the above equation (Newton second law), we can find the equations of motion in x and z direction, with  $\Theta$  positive counter clockwise. [1]

Sum of forces along x axis-  $\Sigma F_x$ :

$$T_1 cos\theta - qSC_D = m\ddot{x} (eq. 1)$$

Sum of forces along z axis-  $\Sigma F_z$ :

$$T_2 + T_1 \sin\theta + qSC_L = W (eq. 2)$$

Sum of moments about y axis-  $\Sigma M_v$ :

$$8.5(T_1 \sin\theta) + q \operatorname{Sc}C_m = 17(T_2) (eq. 3)$$

Where,

Distance of front propeller from the C.G of aircraft = 8.5 in.

Distance of back propeller from the C.G of aircraft =17 in.

Simultaneously solving eq.1, eq. 2 and eq. 3 gives us the following equation:

$$tan\theta = \frac{0.66W + 0.33qSC_L - 0.039qScC_m - qSC_L}{m\ddot{x} + qSC_D} (eq. 4)$$

We will now calculate the maxima by taking the time derivative of the above equation and putting it equal to zero, because we require x(t) such that  $\theta(t)$  is at its minimum value at each 't'. The quotient rule is applied of derivatives on eq. 4. This leads us with one trivial solution that is:

$$cos^2\theta = 0$$

and from this we get  $\theta = 90^{\circ}$  (not requitred)

Solving the other equation by putting it equal to zero and substituting:

$$q = 0.5 \rho \dot{x^2}$$

And,

$$\dot{q} = \rho \dot{x} \ddot{x}$$

Putting these values and simplifying we get:

$$A\ddot{x^2}\dot{x} + B\ddot{x}\dot{x^3} + C\ddot{x} + D\dot{x^2}\dot{x} + E\dot{x}\ddot{x}$$
 (eq. 5)

The equation obtained above is a third order non-linear ordinary differential equation and its solution is calculated using the numerical techniques. We used the Runge-Kutta Method [2] to obtain the solution of the above non-linear differential equation.

Where,

$$A = -0.66 \rho SC_L m - 0.039 \rho ScC_m m$$

$$B = -0.019 \rho^2 ScC_m SC_D + 0.33 \rho^2 C_m S^2 cC_D$$

C = -0.66mW

$$D = 0.33 \rho SC_L + 0.019 m \rho ScC_m$$

$$E = -0.66 \rho SC_D W$$

And,

$$S = 0.2935m^2$$

$$C_D = 0.0085$$

$$C_L = 0.8$$

$$C_m = 0.01$$

$$m = 2.5 \, kg$$

$$c = 0.3048 m$$

Using Matlab's ODE45 function (Runge-Kutta), the numerical solution of the differential equation is obtained which is substituted into eq.4 to five a function of  $\theta$ .

Applying equilibrium in z direction and moment, the thrust required at theta 90 for our aircraft was calculated. Now at  $\theta = 90^{\circ}$ , the thrust generated by the front two propellers, that is:

$$T_1 = 16.25N$$

Assuming  $T_1$  constant at this angle, the thrust of the back propellers is calculated using the variation of  $\theta$ . To balance this thrust and the moments produced by this, the back propellers have to produce a thrust equal to:

$$T_2 = 8.16N$$

Which will be verified by the graphs plotted by the solution obtained from the Matlab's ODE45 function.

#### 4. RESULTS

The equations are solved using Matlab's ODE45, and the following graphs are obtained, which show us the variation of  $T_2$  and  $\theta$  with time while keeping  $T_1$ constant at a value of 16.25N.

### Variation of Position, Velocity and Acceleration with time:

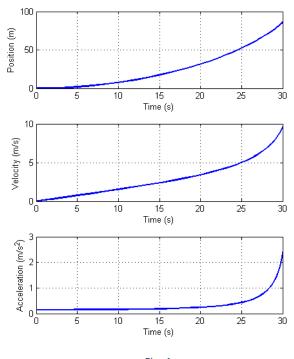


Fig. 4

#### Variation of $\theta$ with time:

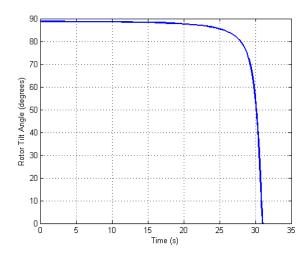
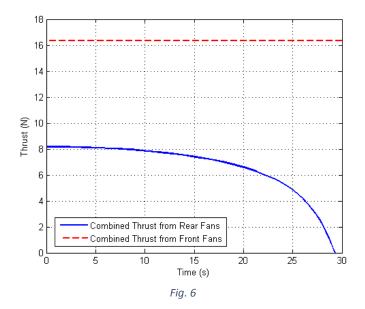


Fig. 5

## Variation of T<sub>2</sub> with time:



# 5. CONCLUSION

From the above obtained graphs, it is concluded that as per our requirement, we are getting the thrust that is half the thrust of the front propellers at each value of theta to balance out the pitching moments and thus keeping the aircraft in the horizontal stable direction.

Also we can see from the graph of theta that how slowly the value of theta changes from 90, when it is in the vertical flight and then at almost 30 seconds time, the tilt is

complete to the vertical direction, hence moving the aircraft in the forward direction and keeping it stable too.

The Velocity variation graph shows us that in the vertical position, aircraft has no horizontal velocity that is the reason this graph starts from zero and then once it starts moving in the horizontal direction, the velocity keeps on increasing linearly with time.

It is quite evident from the results obtained that our solutions are verifying the modeled equations.

#### **ACKNOWLEDGEMENTS**

First of all we would like to thank ALLAH ALMIGHTY as no work can be done without HIS consent. We would also like to thank our supportive juniors who helped us whenever we needed them. We would like to thank Sir Izhar Kazmi and Sir Saad Qureshi for their endless support and guidance during this research.

#### REFERENCES

- [1] J. D. Anderson, Aircraft Performance & Design, McGraw Hill, 1999.
- [2] S. Bhatti and N. Bhatti, A first course in Numerical Analysis with C++, 2002.