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### **Traffic flow in a cellular automaton model of single- and double-lane freeway**

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## Abstract

**English/Engelsk:** The goal with this report is to research and understand the motion of traffic and the formation of traffic jams in highways with one and two lanes. To do this we have used a modified version of Nagel–Schreckenberg model in order to simulate traffic. The key modifications are that we added velocity dependent randomization and that in expanding to a two lane model we included a rule for changing lanes.

The key findings in the report is that in general for both single and double lane cases once the density becomes high enough for traffic to form the flux of the road doesn't increase with higher maximal velocity. Specifically it was found that in a two lane road where the order with which the lane was decided was random raising the maximal velocity beyond the simulations equivalent to 110km/h didn't have any effect no matter the density. In general we saw that doubling the amount of lanes increased the flux more or less in the same way as had the system just had two separated single lanes dependent on how optimized the road selection was. Another key finding was that for roads where the cars had different internal speed limit the outer lane would only be dominated by the faster cars as long as the road was in free flow, once traffic started to form the internal maximal velocity once again stopped mattering.

In general we found that the results of the simulations generally were in line with the expectations and observations we had from the real world giving us the impression that the simulations in general were successful.

**Danish/Dansk:** Formålet med denne rapport var at undersøge trafikens opførsel samt opstandelsen af trafikpropper, på veje med både en og to baner. Til dette formål brugte vi en modificeret version af Nagel-Schreckenberg modellen, en relativ simpel cellulær model. De større ændringer var at tilføje en velocitets afhængig tilfældighed, og indfører regler for at skifte mellem de to baner. Vi fandt fra vores simulationer at efter vej-densiteten når et vist punkt, vil en øget max hastighed ikke have nogen effekt på trafikens flux. Specifikt fandt vi at fluxen ikke steg yderligere efter bilerne var givet en max hastighed svarende til 110 km/t i to-bane simuleringer hvor vejbanen var tilfældigt bestemt, ligegyldigt densiteten. Generelt så vi at en fordobling af vejbanerne ledte en tilsvarende fordobling i fluxen, nogenlunde som hvis man i stedet havde to separate enkelt bandede vejlængder, dog afhængigt af hvor veloptimeret vejbane valget var. Yderligere fandt vi at den ydre bane var domineret af biler med en høj maximumhastighed så længe at vejen var i frit flow i to bane systemet hvor bilerne havde varierende individuelle maximumhastigheder. Det stoppede dog med at gøre nogen forskel efter at trafik begyndte at opstå.

Generelt lå vores simulationer inden for vores forventninger og observationer fra den virkelige verden, hvilket understøtter at de har været succesfulde.

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# 1 Introduction

Traffic is an essential part of daily life, and many have attempted to model it to different extents. Such models have several potential applications, from predicting the flow of traffic at certain point of the day, the effect of accidents and other disturbances, to optimization of the behavior of self-driving vehicles.

## 1.1 The traffic problem

One major point of interest in any model of traffic is the formation and perpetuation of traffic jams. No human driver is perfect, nor perfectly able to time and judge their speed and acceleration in relation to other vehicles. This means that a minor disturbance on the road can have long lasting effects on the flow of traffic as each car in turn reacts to the car ahead.

## 1.2 Different models

There are many approaches to modeling and simulating the flow of traffic, each with its own advantages and disadvantages. A macroscopic model might be formulated by using the density field and velocity field as time and position dependent variables, and applying the hydrodynamic equations that dictate their behavior. Such a model could be used to analyze the statistical properties and mass action of large numbers of vehicles, but may experience trouble with smaller numbers. It would also be computationally heavy, as even simplified fluid behavior can range into the complex.

A microscopic model, in which mobile units and their behavior are individually simulated presents its own decisions and challenges. One such class of models might describe the equation of each individual car as a differential equation dependent on the cars immediate speed, the distance to, and the speed of the cars in front of it.

Another example of a microscopic model is the cellular automaton approach. It describes the road as a lattice of cells, to which you assign values corresponding to the car that might occupy it. Time is discretized into steps that progress forward. For each of these the cells are updated depending on the state of the cells around it, and certain rules you set out from the onset. This method allows you to ascribe distinct behavior to specific vehicles as they are individually described. It is also more easily computable than the other models mentioned.

In general for the microscopic simulations, the question arises of how to update

your model for each increment of time. As the movement of each car is dependent on the position of others, you may in some cases experience that two or more cars overlap if you elect to update the position of all cars simultaneously. Updating the cars in sequence resolves this problem, yet that also has its troubles.

### 1.3 Our Motivation

We decided to expand on a cellular automaton model, the Nagel-Schreckenberg model, by giving the vehicles another degree of freedom in adding another lane, as well as lane-shifting behavior to our simulation, as the model rarely have been used for such. We wanted to examine how this change would affect such things as the flux of traffic and the formation and resolution of traffic jams in a closed loop system. We further wanted to test the effects of varying densities and velocities in both systems

## 2 Methods

### 2.1 The classic Nagel-Schreckenberg model

Despite the many different ways of modeling traffic we have decided to use one of the simpler models which still has room for improvements, the Nagel-Schreckenberg model. This model is an example of a cellular automaton model in which the cells can either contain a car or be empty. The basic model thus consists of four rules explaining how to update the cellular system of  $n$  cars with individual velocity  $v_n$  and position  $x_n$ . When repeated many times the four rules lay the foundation for a basic simulation of traffic on a single road. The four rules are [Sch02]

#### 1. *Acceleration*

If  $v_n < v_{max}$  increase the velocity by one.

$$v_n = \min(v_n + 1, v_{max})$$

#### 2. *Deceleration (braking due to other cars)*

If the distance to the next car  $d_n \leq v_n$  reduce the velocity to  $d_n - 1$ .

$$v_n = \min(d_n - 1, v_n)$$

#### 3. *Randomization*

If  $v_n > 0$  there a chance with probability  $P_n$  to reduce the velocity by one where  $P_n$  is the same for all cars

With probability  $P_n : v_n = \min(0, v_n - 1)$

#### 4. *Movement*

Move the car  $v_n$  steps forward on the road

$$x_n = x_n + v_n$$

In this classic version of the model it is assumed that every step is performed simultaneously on all  $n$  cars before moving on to the next step. While this is not possible in a program where you have to check cars one after another the order with which you apply these four rules to your cars does matter as the result is the same whether you begin or end with car number 1.

The Nagel–Schreckenberg model(from now on NaSch) has the advantage that each of the four steps has a clear and simple interpretation to what it represents in real life traffic making it clear what our simulations mean. The first step is the drivers natural desire to reach the top speed allowed on the road, the second step represents a certain safety distance to the car in front of it. The third step represents the inconsistencies in driving that may arise, be it a car that brakes too much, an overestimation of safety distance or a general distraction. This step is the key part of forming traffic jams and if it wasn't there the road would reach an equilibrium where every car is evenly spaced out. The fourth step is the physical representation of the cars driving moving along the road. An assumption made by these rules is that a driver cannot determine how fast the car in front is driving causing them to always have a speed less than the distance to the next car. Whether this assumption is correct or not can be discussed, but it is the interpretation we have decided on. A technical detail to mention is that our road loops meaning that there is no entering and exiting the system giving a constant amount of cars.

Finally one has to estimate what the units we are measuring in represent in the real life. How long is a time step, how wide is a cell and thus what speed does our  $v_{max}$  equal to on real roads. Considering we are modeling a highway the usual speed limit lies at 110km/h or about 30m/s makes for a good target for our maximum velocity. In our model the standard speed limit has been set at 5 cells/time step. If we see one time step as equivalent to one second we get that every cell on the road represents a 6 meter long section. Comparing this to an average car length of about 4m it seems that our units are reasonable enough when interpreting them as physical values.

## 2.2 Velocity Dependant Randomization

While we have covered the rules of the classical NaSch model and its interpretations the model still had room for improvement and changes. For the singular lane the only change we made was to introduce the concept of slow start which is a popular addition to the NaSch model often called Velocity Dependant Randomization(VDR). In order to add VDR to our model we need an addition of a rule 0 making  $P_n$  not the same for all cars [Sch02].

### 0. *Determination of P*

If  $v_n = 0$  increase the probability of randomly braking by a factor of  $\kappa$ .

$P_n = P_0$  if  $v_n > 0$  else if  $v_n = 0$  then  $P_n = \kappa \cdot P_0$   $\kappa > 1$

The reason for the addition of this rule is that it simulates the difficulty real drivers have in accelerating from a full-stop compared to accelerating when already in motion. Because rule 1 prevents any cars from having velocity 0 unless they are blocked in step 2 it becomes necessary to decide the individual probability before any of the other steps. The effect of this rule is a general widening of the traffic jams as they will be slower to resolve.

## 2.3 Method of Analysis

In analyzing the different models there are quite a few different tools that we can use to compare the efficiency and behavior of different simulations. Some of the key terms when analyzing traffic is the *density* and the *flux*. The density is simply how filled the road is and can be found as  $\rho = \frac{\text{Cars}}{\text{RoadLength}}$ . The flux on the other hand is an indication of how much the cars move at a given moment of time. While there are different ways to find the flux we have a point on the road that counts every car passing. When dividing this total with the time it has been measuring we get a  $J = \frac{\text{Car}}{\text{Timeunit}}$  value which is our given flux. The flux is also called flow.

However before using these tools to analyze the system it is important to make sure that the system is in what is called *steady state*. Steady state is the most optimal flow state in which no changes occur to the system other than the stochastic fluctuation due to rule 3. This means for a  $P=0$  scenario the flow would be just about constant as all the cars has been evenly spread out driving around the road. Once this criteria has been fulfilled you can make the most essential graph in analyzing traffic, the (density,flow) graph which is called the fundamental diagram. This graph easily depicts the maximal flow achievable along with the optimal amount of cars allowed on the road under different conditions. Furthermore the graph itself can be broken

down into three different phases which can help us understand the behavior of the traffic. The three phases are; the free flow phase, the transition phase and the traffic jam phase.

In the *free flow phase* there is not enough cars on the road for them to meaningfully interact with each other resulting in every car reaching the maximal velocity. In the case where  $P$  is zero that will give us the expected expression  $flow = \rho \cdot v_{max}$  since a new car is

In the *traffic jam phase* however there are too many cars to have them all reach maximum velocity as a result of them having to brake. Once the system has reached a steady state however one can assume that the spacing between the cars is more or less even and that the average space between cars become  $\frac{RoadLength}{Cars}$  or  $\frac{1}{\rho}$ . Seeing that cars can no longer hit  $v_{max}$  the effective maximum velocity they can hit becomes the distance to the next car, minus one. As such we get that  $V_{max}^{effective} = \frac{1}{\rho} - 1$  and that the total flux, which now is found as  $flow = \rho \cdot v_{max}^{effective}$  equals  $\rho \cdot (\frac{1}{\rho} - 1) = 1 - \rho$ . The transition phase is a mixture between the two phases dependent on the variable  $P$ .

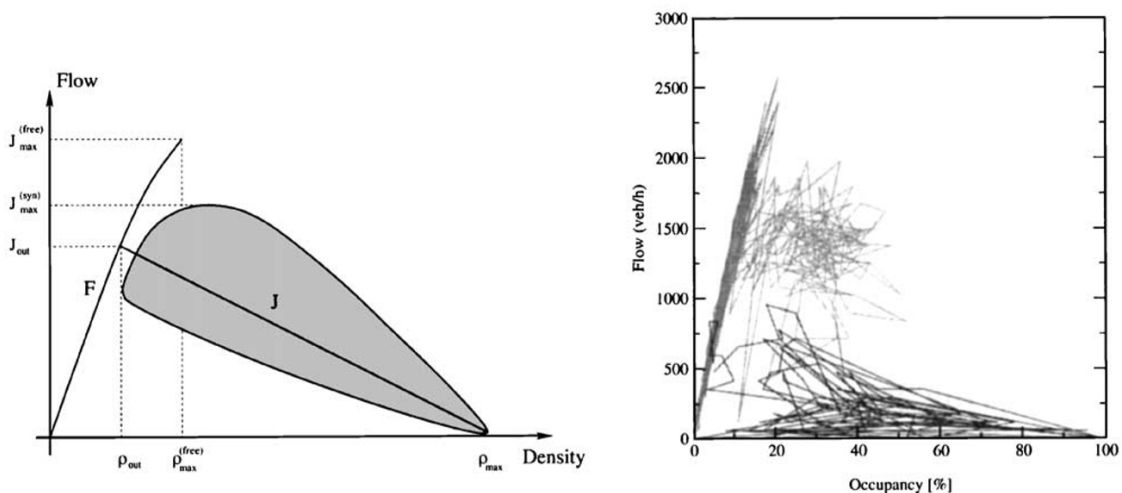


Figure 1: Left: We see a theoretical depiction of the different phases of a single lane simulation. F shows the free flow state where J shows the traffic jammed state. Right: Empirical diagram with clear parallels to the theoretical, although more chaotic.[Sch02]

### 3 One lane case

In this section we will look at the characteristics of the classical NaSch model with VDR added and how different values of  $v_{max}$  and  $P$  will affect the flow of the traffic.



### 3.1 Results and characteristics

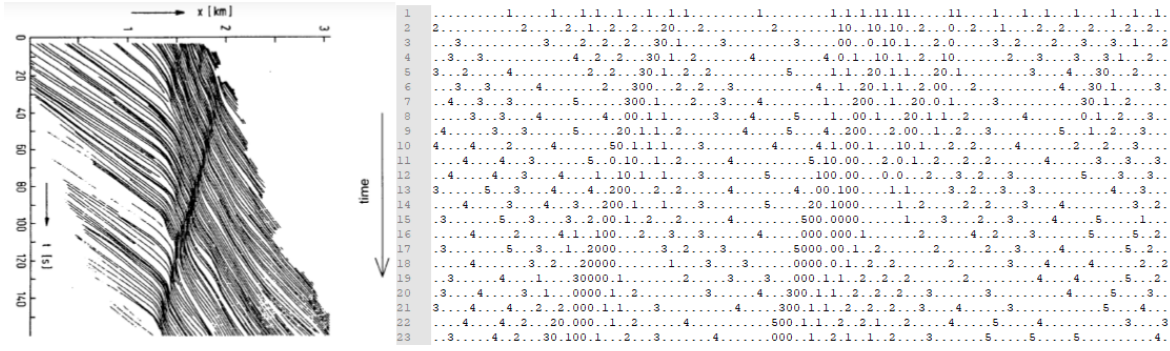
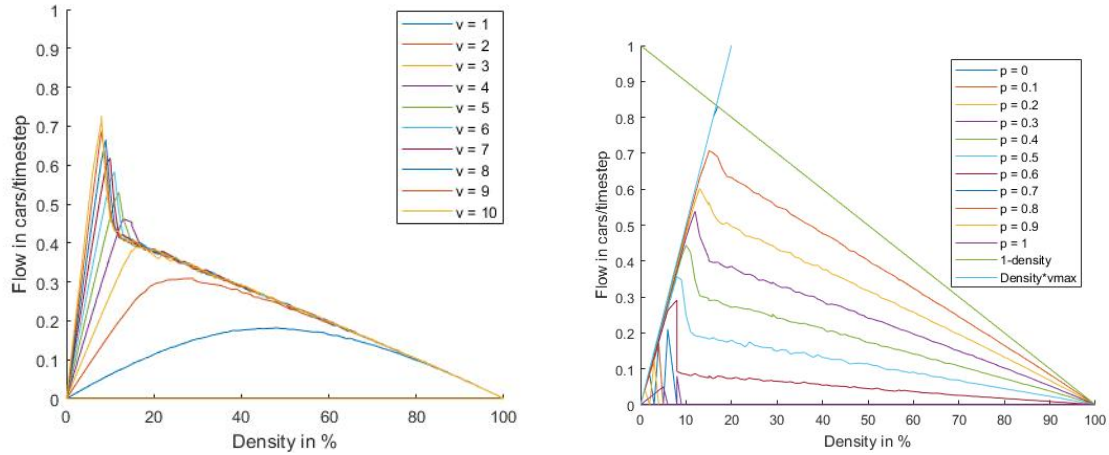


Figure 2: Left: Empirical data showing trajectory lines of individual cars place as a function of time [Sch02], Right: Our simulation at different time steps with each number representing a car with the given velocity.  $P=0.2$ ,  $\rho = 0.25$



(a)  $P = 30\%$ ,  $\kappa = 1.5$ , road length = 100

(b)  $v_{max} = 5$ ,  $\kappa = 1.5$ , road length = 100

Figure 3: The fundamental diagram showcasing different  $v_{max}$  (a) and different  $P$  values (b).

### 3.2 Analysis

We see that the (position, time) diagram in figure 2 is looking very similar to the empirical data which while hard to completely analyze brings us a qualitative verification that our simulation is working. When looking at our fundamental diagrams in figure 3(b) the first thing we notice is that our theoretical  $p=0$  predictions align perfectly with the results in our simulation granting us some validation that the simulation is actually functioning as intended. Furthermore we see that the while our system is in free flow state its slope doesn't change no matter the  $p$  value, meaning that the flow is probably always  $\rho \cdot v_{max}$  in a free flow state.

However while  $p$  doesn't seem to affect the flow in free flow state we do see that it

affects span of it, causing traffic jams to begin sooner when the randomization probability increases. This means that the traffic jam phase begins sooner and that the maximally achievable flux is reduced. The transition point, and hereby the general flux, is reduced until we notice that the graph completely collapses once our value exceeds 0.7 at which point the flow becomes non existent for almost all densities. This collapse at high  $P$  values is also another sign that the code works as it is a result of us adding VDR. In these examples our  $\kappa$  values are 1.5 meaning that once  $P_0$  exceeds 67% the resulting probability for a stopped car is above 1 preventing it from ever starting again once it has come to a stop, resulting in a complete stop of flow. As to why the graph fluctuates so much for high  $P$  low  $\rho$  values it is because the system here has a higher probability of never having a car hit full stop meaning some flow can be achieved. However if the starting conditions are "unlucky" a car will start with no speed preventing any flow. As such the early part will vary from simulation to simulation.

Another interesting phenomenon we notice is that once the  $P$  value and the maximum velocity are high enough the smooth curve that characterizes the transition phase disappears. If we look at the examples for  $v_{max} < 4$  in figure 3(a) the graphs smooths out as the transition state is reached, just as expected, whereas on the other hand when we look at the high values for  $P$  or  $v_{max}$  we see that the flow almost instantaneously falls to a significantly lower value creating some sort of drop. The reason behind this apparent drop can be found in our road length. When running the simulation with larger road lengths this phenomenon disappears as seen in appendix. This behavior tells us that the road length plays an role in the forming or traffic jams. A possible explanation to the reason for these spikes can be that the small road size doesn't grant enough freedom for the system to change effectively "trapping" it in a free flow state for longer than it naturally would be in causing the system to seemingly overshoot until traffic does occur. This is not immediately intuitive as one could think that in a circular road the length wouldn't matter but as we can see from the results it actually has a quite significant effect.

A final thing to note is that while raising the maximum velocity beyond 5 increases the maximum flow for the system, it only does so while in free flow state. As seen in figure 3(a) once fully transitioned to the traffic jam phase all the graphs flat out to the exact same flow. From this we are led to think that a maximum velocity beyond that of 5 is not really beneficial for the flow of our system as the density rarely will be low enough to generate a benefit. This result once again aligns well with the theory considering that at certain densities it is not the actual  $v_{max}$  but the effective  $v_{max}$  that counts.

## 4 Two lane case

Another addition done to the NaSch model is the addition of a second lane. The original model is designed to model highway traffic in a single lane but as stated our goal with report is to take a closer look at traffic with multiple lanes in order to see what difference this additional dimension creates.

### 4.1 New rules

To add an extra lane all the previous steps for the NaSch(VDR) model are the same, simply repeated for both lanes, however we need to add a new rule to determine how the exchange of cars between the lanes will be handled. This new rule is called rule 1.5 and is as follows.

#### 1.5 Lane selection

If a lane switch is possible the car will pick the outer lane only if it can achieve greater velocity there after step 2.

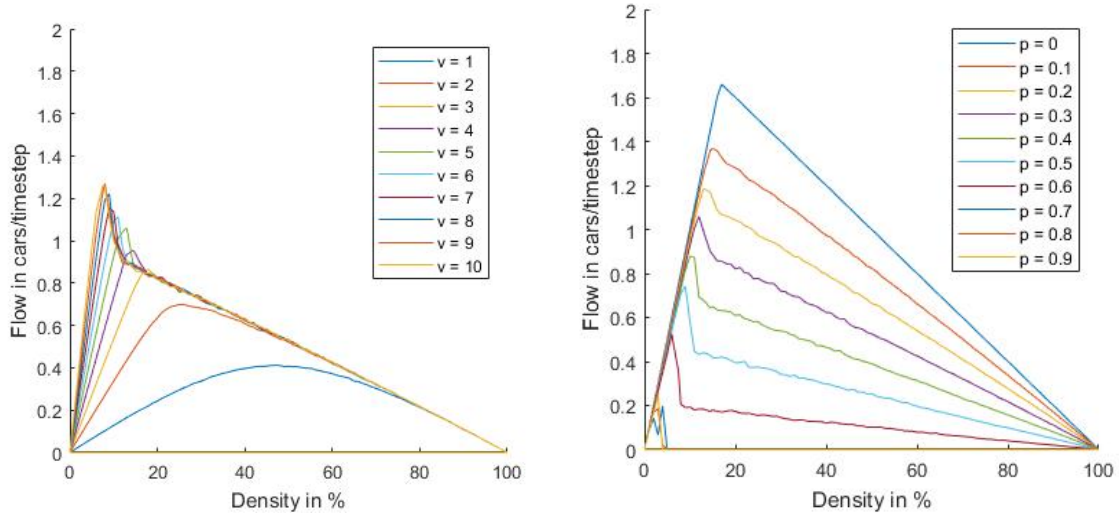
If  $road_1(x_n) = 1$  and  $road_2(x_n) = 1$  then  $L_n = L_n$

If not then  $L_n = 2$  if  $\min(v_n, d_2) > \min(v_n, d_1)$  and  $L_n = 1$  if  $\min(v_n, d_2) \leq \min(v_n, d_1)$

A few new variables has been added which needs explaining.  $road_i(x)$  is a function which determines whether there is a car at the  $x$ 'th space of road  $i$ , returning 1 if filled and 0 if empty. The value of  $L_n$  indicates the lane the  $n$ 'th car is in, 1 or 2. As for interpretation of this rule it is quite simple, like a normal highway there is a fast and a slow lane so making the car take the inner lane even if it can achieve the same speed in the outer simulates this phenomenon. One could think that it should count as moving a block if a car swap lanes but having it not count as a move is in our eyes somewhat equivalent to cars speeding up a bit when trying to overtake another car and pick the fast lane.

An interesting effect of the lane swapping rule is that it now becomes evident that the order in which we update the cars suddenly begin to matter since the decisions are based on its surroundings which change in the same step. One can imagine the situation where the backmost car decides that the outer lane would be fastest, thereafter the car in front of it also decides to move to the outer lane nullifying the first decision. This is a feature we intend to explore in order to see how a randomized decision order which might resemble reality more, compares to the theoretically optimized order. Additionally we will look at a case where cars have different maximal velocity to simulate the difference in drivers comfortableness with speed.

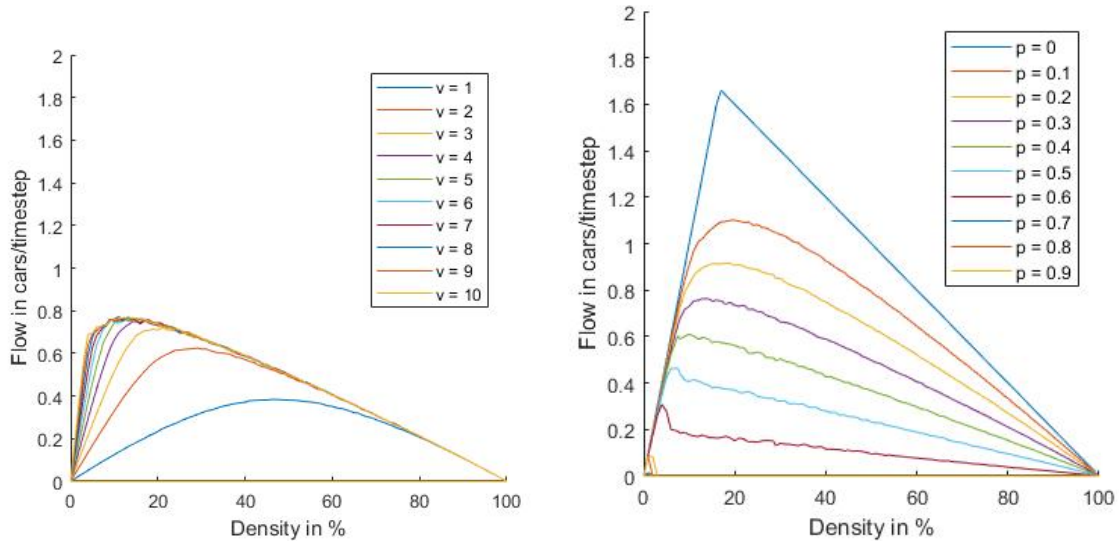
## 4.2 Results



(a)  $P = 30\%$ ,  $\kappa = 1.5$ , road length = 100

(b)  $v_{max} = 5$ ,  $\kappa = 1.5$ , road length = 100

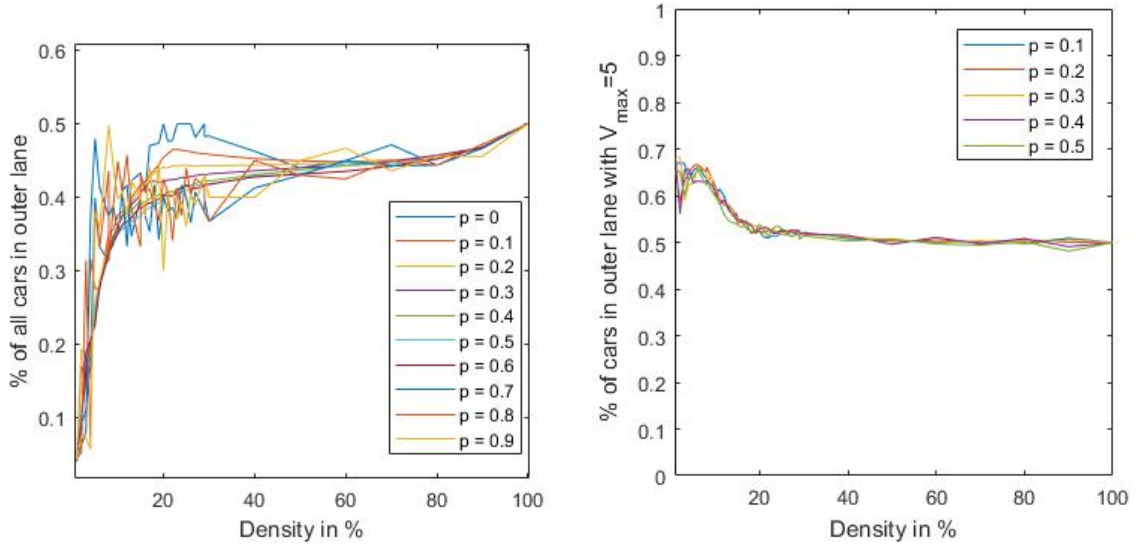
Figure 4: The fundamental diagram of an optimized two lane road showcasing different  $v_{max}$  (a) and different  $P$  values (b).



(a)  $P = 30\%$ ,  $\kappa = 1.5$ , road length = 100

(b)  $v_{max} = 5$ ,  $\kappa = 1.5$ , road length = 100

Figure 5: The fundamental diagram of a two lane road with random road selection showcasing different  $v_{max}$  (a) and different  $P$  values (b).



(a) 50%  $v_{max} = 5$  and 50%  $v_{max} = 3$ ,  $\kappa = 1.5$ , (b) 50%  $v_{max} = 5$  and 50%  $v_{max} = 3$ ,  $\kappa = 1.5$ , road length = 100. Optimized decisions

Figure 6: (a). Shows the average percentage of total cars in outer lane as a function of density, (b) Shows the percentage of cars in the outer lane with the maximal velocity of 5 as a function of density. After  $\rho = 30\%$  there is only data from every 10% point of density.

### 4.3 Analysis

In this two lane case we have a couple of different results too look at. If we start with the optimized case of figure 4 we see that the behavior is very similar to the single lane case. When looking at the  $P=0$  case from (b) we see that the maximum velocity is simply the double of what it was in the single lane case while all the critical points happen at the same time density. This makes sense since there are twice as many cars and road space and so this tells us that our optimized system very much behaves like the sum of two single lanes. Interestingly enough the drops caused by overshooting still occurs as heavily in this optimized case despite there being two roads which one would assume was preventing traffic jams and preventing the system from being locked.

Taking a look at the  $v_{max}$  graph for the optimized case in figure 4(b) the general behavior is still the same as in the single lane case however here the comparison is not quite as simple as for the  $P$  graph in fig(b). In figure 3(a) for the single lane case we saw that while the free flow flux topped at around 0.7 for high velocities once the graphs collapsed to the same slope at around  $\rho = 20\%$  the flux was 0.4 and went down from there. In our two lane case seen in figure 4(a) the maximal flux is actually a fair bit lower than the expected  $2 \cdot 0.7 = 1.4$ , only about 1.25, which probably is caused by additional freedom to move preventing the system from be-

ing "locked" quite as hard and thus overshooting as badly. On the other hand we see that the collapse in return is less severe than in the single lane example and that they collapse to about 0,9 flow instead of the expected 0,8. This seems to show that two lanes with intersection allow for a higher flux at high densities than just having two individual roads. This result however, is apparently only evident when we look at the optimized selection order case.

Looking at the random selection order case in figure 5(a) the  $v_{max}$  results are much different. In extension of what we just discussed we see that in this case the "collapse point" is actually a little bit under 0.8 which is a bit less than what we expected from having double the number of lanes. However a new twist is that there simply is no overshoot to collapse from. Once the maximal velocity reaches values above 5 it has no impact on the flow of the system, not even while the system is in the free flow phase. This is very interesting as this behavior differs greatly from the one lane case leaving one to ponder how a randomized lane selection could cause such a phenomenon. Looking at the randomized P graph from figure 5(b) we see that the randomization didn't have any effect on the case where  $P=0$  however as soon as step 3 is introduced with  $P > 0$  we see great changes compared to the optimized case. The reason as to why the nullification of the overshooting happened is not completely clear but due to the lack of impact in the  $P=0$  case compared to  $P > 0$  we can assume that it has to do with the widening transition phase. One theory is that making the decision order random makes the system explore many different configurations without being locked in a special order. Compared to the optimized state where one configuration is always the best here many different possible arrangement of the cars are possible and the system doesn't get "locked" in a specific one. Seeing that the overshooting flux peaks came from being locked in free flow state one can imagine that the added freedom is the cause. This also explains the widened transition state as added randomness further increases the possibility for the system to be in either of the two main phases (free flow and traffic).

When looking at the slightly different simulation in figure 6 where we had half the cars have a fast maximum velocity of 5 and the other half a slow maximal velocity we notice some interesting behavioral patterns in the system. The first thing we looked at in figure 6(a) was strictly how often the outer lane was used and it seems that it is rarely used in low density scenarios after which it quickly flattens out to a bit over 40% for a long time until both roads gets completely filled at the high densities. This makes sense as our code enforces one lane as the primary lane and only uses the outer lane if cars could gain speed. In low density scenarios the cars can all drive more or less independently with their own  $v_{max}$  causing the enforced inner lane to be most popular. The little density that does arise is probably caused by

the difference in  $v_{max}$  causing the faster cars to catch up to the slower ones needing to use the outer lane to take over, much like the real world. When we look at figure 6(b) this behavior is further supported as we can see what percentage of the cars in the outer road are fast cars and what percentage is slow. While the difference is not staggering we see that while in free flow state faster cars are dominant in the outer lane but once the density is high the individual  $v_{max}$  is replaced with a  $V_{max}^{effective}$  for the system homogenizing the cars in the outer lane.

## 5 Discussion

### 5.1 Estimation of Error

Having looked at the simulations for both the single and double lane case we conveniently avoided any mention of error of the simulation despite having many seemingly high variance parameters included. This is because it turns out that the error for such simulations is actually very low which we will show in this section. The primary value we have found in our simulations is the flux of the system at different densities, as such we will calculate this error in a single lane case. When measuring the flux in our we have a single point where we measure whether a car has passed or not every time step. Due to the nature of our simulation the maximum amount of cars that can pass this point every time step is 1 meaning that we either get 1 or 0 as our flux every time step. This can be compared to flipping a potentially unfair coin giving two results every flip. We can then treat every 100 timesteps as had we conducted an experiment of 100 coin flips in order to determine how the coin was rigged. Since we have 7000 timesteps we can see it as had we conducted this experiment 70 times with 70 different results. Alternatively you can see it as one experiment with 7000 coins flipped. From these results it is simply to use the formula for standard error to find out how big the error on our estimated flux is.

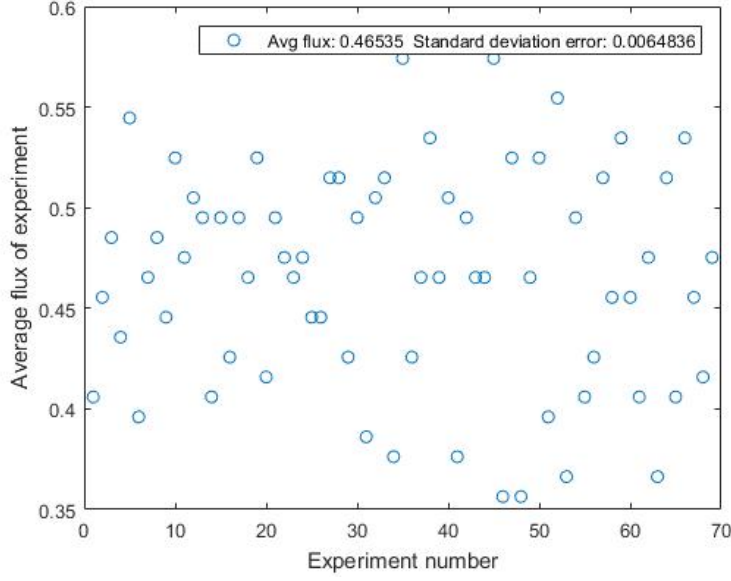


Figure 7: Error estimation for a simulation with 7000 timesteps treated as 70 independent experiments. The values  $\rho = 25\%$  and  $P=20\%$  were used resulting in an average flux of 0,465 and an error of 0,006.

As one can see in our example from figure 7 the error for our experiments would only be about 1% of the actual flux value confirming that our simulation is accurate enough to ignore potential errors while looking at the general behavior of the different systems. The reason one might want to split it up to resemble multiple experiments instead of seeing it as a single 7000 flip experiment is that a key assumption in finding the standard error for  $fx$  coin flips is that every result is independent of the other results. In our traffic scenario this key assumption isn't quite true as having one car pass through greatly decreases the chance of a car following right after. By splitting it up into small "blocks of experiments" these interactions between experiments will become increasingly negligible and the assumption can be seen as correct.

## 5.2 Comparison to the real world

Having confirmed that our simulations are probably correct we can now discuss our discoveries more thoroughly. One result is that was quite interesting was that the randomized lane sequencing for two lanes completely nullified the effect of an increased  $v_{max}$ . Considering that the randomized sequence is often the one that is compared to realistic behavior this would mean that maximum velocities above 5, or 110 km/h would have little to no effect on the actual flow with which the traffic can move, making one ponder whether or not this is part of the reason highways rarely have speed limits above these values.



An effective way our simulation could be used is to estimate the road capacity of different highways. Looking at the actual numbers we see that for a two lane example with random lane swapping you can maximally get between 0.8 and 1 flux dependent on what you assume  $P$  to be. This means that on at peak conditions only 3600 cars can pass a point every hour on a two-lane highway. If its a popular highway one could consider expanding with a third lane to increase the possible flux and expand our model to get a rough idea as to whether that would be enough or not. A problem our model has compared to the real world is that once we introduce random lane deciding the distance between cars is no longer optimized and as such there are scenarios where one car switches lane in front of another causing the car to go from  $v_{max}$  to 0 in a single time step which is somewhat equivalent to a car crash. In our model this has no real consequences which might be an indication that the models we have proposed aren't by any means complete or flawless and has much room for improvement.

### 5.3 Improvements

Seeing that our model has room for improvement this section will discuss what further work we could have done on the model in order to improve it and what the benefits and consequences of those changes could be. The first thing one could improve upon our model would be relevant both for single lane and double lane cases, it is the drivers ability to judge speed. In our original NaSch model rules we mentioned than an assumption made was that a car could not judge the other cars velocity and as such had to keep a very safe distance in order to avoid accidents. In real life drivers focus on a lot more than simply avoiding accidents and a key element is driving comfortably [Sch02]. If one attempts to emulate comfortable driving they would have to accord for the fact that drivers tend to anticipate the velocity of the surrounding cars since they don't expect car crashes or abrupt braking to happen. Additionally since drivers are often aware of their surroundings they might also slow down in anticipation of a slower car additionally changing the way they decide on when to brake. One could imagine implementing this would mean some large changes to rule 2. Similarly if a driver has found a comfortable driving speed they are not necessarily attempting to accelerate whenever possible, meaning one would have to alter step 1. The pros of adding these would be a more realistic model which might make more accurate small scale analysis possible, however it becomes much more complex and time consuming not only to create and understand the simulation but probably also to run it.

Another way to change the model would be to break the circle and let cars flow in and out of the road allowing the simulation to more closely resemble real roads

which are not circular but have entrances and exits. Doing this would be beneficial if we wished to compare the influx with the outflow of the road observing how a traffic jam would resolve and behave in another environment. The downside is again time and the difficulty of finding the right values for the influx of cars. While not impossible it was not prioritized in our simulation, although it was definitely considered. In general the code we had did its job of simulating one and two lane highways very well while being fast enough to repeat multiple times which was the result we desired once we started, additional improvements could be nice when looking deeper into the subject but it wasn't in our scope for this project.

## 6 Conclusion

In this report we have attempted to simulate highway traffic using the Nagel-Schreckenberg model which is an Cellular Automaton type model. We have looked at both single and double lane highways in our simulation trying to understand the difference. In the single lane example we saw that once the density of a road became high enough, often about 20%, the flow of the traffic didn't increase with higher speed limits. Furthermore we saw that short roads could cause the system to overshoot its flow compared to the expectations. These findings transitioned to the two lane example. In general we found that the flow for an optimized two lane scenario was slightly more than doubled at higher densities but in an unoptimized case where the order of selection was random the flow was slightly less than doubled. Another finding was that in the unoptimized two lane case overshooting due to short roads was not happening. Finally we examined the two lane road in a case where the cars had different maximal velocity. In this case we saw that the outer lane became increasingly popular with higher densities but that at lower densities it was primarily dominated by cars with a high internal speed limit. Finally we made an analysis of the error on a simulation and found that it was negligible due to the many time steps the simulation is run over. In general we found that many of these results aligned well with the behavior in the real world giving us the impression that the simulations generally were successful.

## References

[Sch02] Andreas Schadschneider. Traffic flow: a statistical physics point of view. *Physica A*, 313:153–187, 2002.

## A Appendix

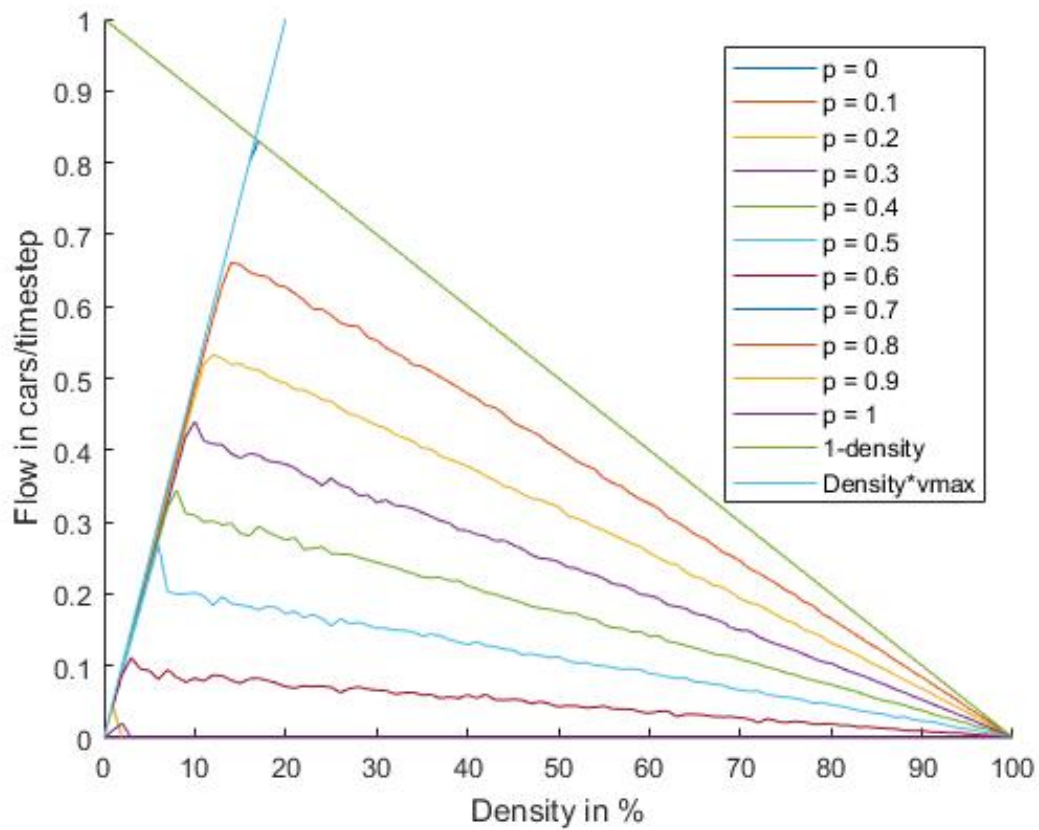


Figure 8: The fundamental diagram for different  $P$  values with a road length of 1000,