Inv Pend Math

Laplace Solution

```
clc
clear
syms Tau b theta(t) I_s I_w s g m t l
theta_dot = diff(theta,1,t);
theta_ddot = diff(theta_dot,1,t);
Is = I_s; Iw = I_w;
T = Tau;th = theta; thd = theta_dot;thdd = theta_ddot;
eqn1 = T - b*thd + g*m*l*th == Is*thdd
```

eqn1(t) =

$$-b\,\frac{\partial}{\partial t}\,\theta(t) + \mathrm{T} + g\,l\,m\,\theta(t) = I_s\,\frac{\partial^2}{\partial t^2}\,\theta(t)$$

```
eqn2 = laplace(eqn1,t,s)
```

eqn2 =

$$\frac{\mathrm{T}}{s} + b\,\left(\theta(0) - s\,\mathrm{laplace}(\theta(t),t,s)\right) + g\,l\,m\,\mathrm{laplace}(\theta(t),t,s) = -I_s\,\left(s\,\theta(0) + \left(\left(\frac{\partial}{\partial t}\,\theta(t)\right)\Big|_{t=0}\right) - s^2\,\mathrm{laplace}(\theta($$

```
Y = solve(eqn2,th(t))
```

Y =

$$\frac{I_s\,\theta(0)\,s^2 + b\,\theta(0)\,s + \mathrm{T}}{I_s\,s^2 + b\,s - g\,l\,m}$$

```
% Assume theta(0) = 0
Y = subs(Y,th(0),0)
```

Y =

$$\frac{\mathrm{T}}{I_s\,s^2 + b\,s - g\,l\,m}$$

```
TF = Y/T
```

TF =

$$\frac{1}{I_s\,s^2 + b\,s - g\,l\,m}$$

```
fprintf('The final transfer function utilized in the calculations is')
```

The final transfer function utilized in the calculations is

```
display(TF)
```

TF =

$$\frac{1}{I_s\,s^2 + b\,s - g\,l\,m}$$

State Space System

```
% Based off my calculations, Controllable Cannonical Form
A = [0 1;-g*m*l -b/Is]
```

A =

$$\begin{pmatrix} 0 & 1 \\ -g\,l\,m & -\frac{b}{I_s} \end{pmatrix}$$

```
C = [-1 0]
```

```
C = 1×2
    -1     0
```

```
B = [0;1]
```

```
B = 2×1
     0
     1
```

```
D = 0
```

```
D =
0
```

```
A = double(subs(A,[m g l b Is], [ 1 2 3 4 5])) % insert example components
```

```
A = 2×2
         0    1.0000
   -6.0000   -0.8000
```

```
rank(ctrb(A,B))
```

```
ans =
2
```

```
rank(obsv(A,C))
```

```
ans =
2
```

```
fprintf('The controlability and observailty matricies are full rank')
```

```
The controlability and observailty matricies are full rank
```