Obtain the Best-fit Orbital Parameters From the RV Data of an Exoplanet

DIY Problem 1

金秉诚

bc.jin@stu.pku.edu.cn

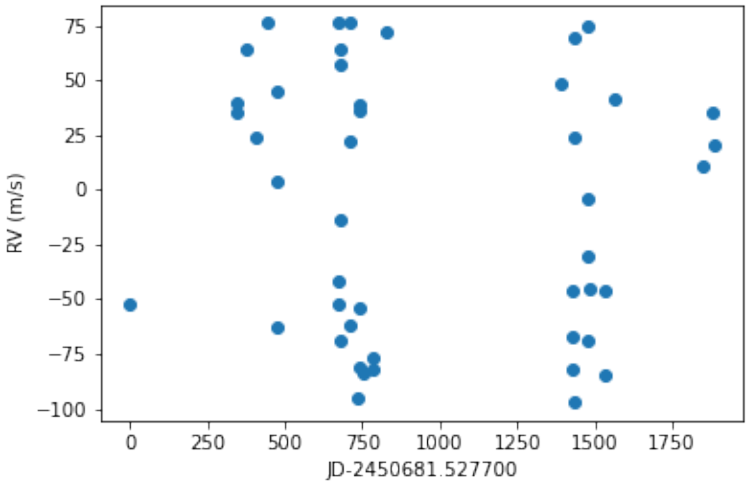
2023年3月17日

Please make sure you've read the Jupyter Notebook file "DIY1.ipynb" before starting this PDF.

Assuming the most simple model, we have a clear relation of RV and t :

$$\mathrm{RV}(t) = K\cos(2\pi t/P + \phi_0) + \gamma$$

where γ represents the center-of-mass velocity and $K = \frac{2\pi}{P}\frac{M_p}{M_* + M_p}a$ represents the amplitude of the radial velocity.



From the original data, we can infer that the system has a very large K, which implies a much more extreme scenario than the Solar system.

Assuming M_* to be a solar mass, and $M_* \gg M_p, M_{Jup}$,

Using Kepler's third law, we can derive $K = \sqrt{\frac{GM_p^2}{a(M_* + M_p)}}$ .Thus

$$\frac{K_p}{K_J} = (\frac{M_p}{M_J})(\frac{M_* + M_p}{M_* + M_J})^{-1/2}(\frac{a_p}{a_J})^{-1/2} \simeq \frac{M_p}{M_J}(\frac{a_p}{a_J})^{-1/2}$$

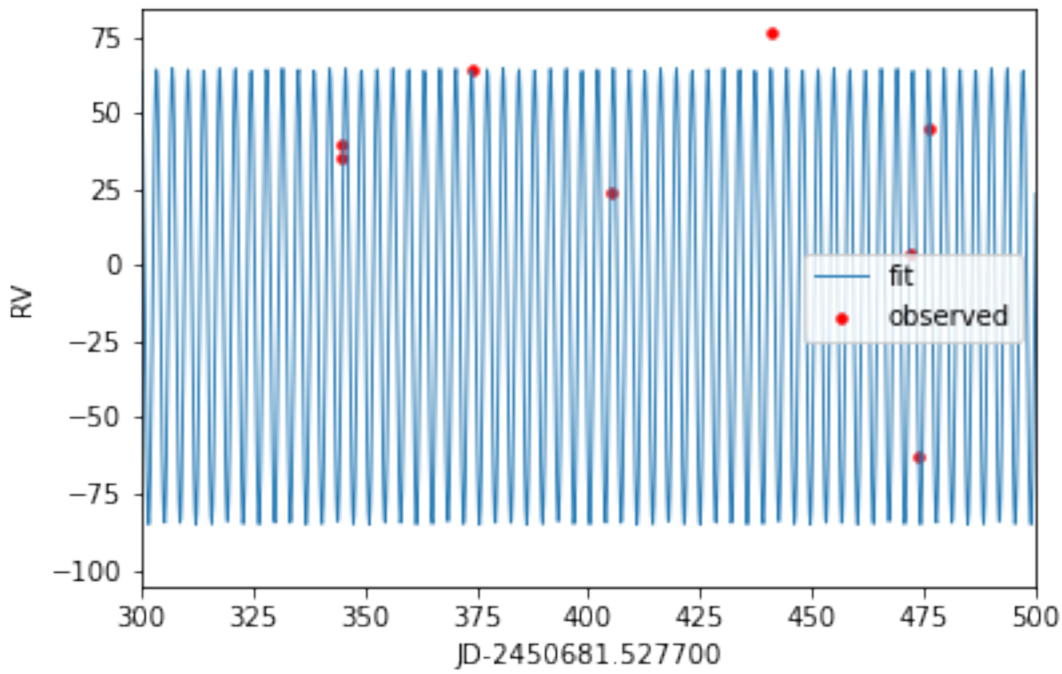
If M_p has a similar mass to M_J, the orbit must be very close to the star, I.e. $\frac{K_p}{12.6\,(\mathrm{m/s})} \simeq (\frac{a_p}{5.4\,(\mathrm{AU})})^{-1/2}$

With $K_p \approx 75\,(\mathrm{m/s})$ we have $a_p \approx 0.14\,(\mathrm{AU})$, which implies a very short period $P \simeq 20\,\mathrm{days}$. This would definitely help me to limit the range of the parameter P.

To run a code, the relation can be simplified as

$$\mathrm{RV}(t) = A\cos(2\pi t/P) + B\sin(2\pi t/P) + \gamma$$

I choose four parameters to span a parameter space:



$$\theta = \{A, B, \gamma, P\}$$

From the Jupyter Notebook file 'DIY1.ipynb' I've obtained my best-fit orbital parameters.

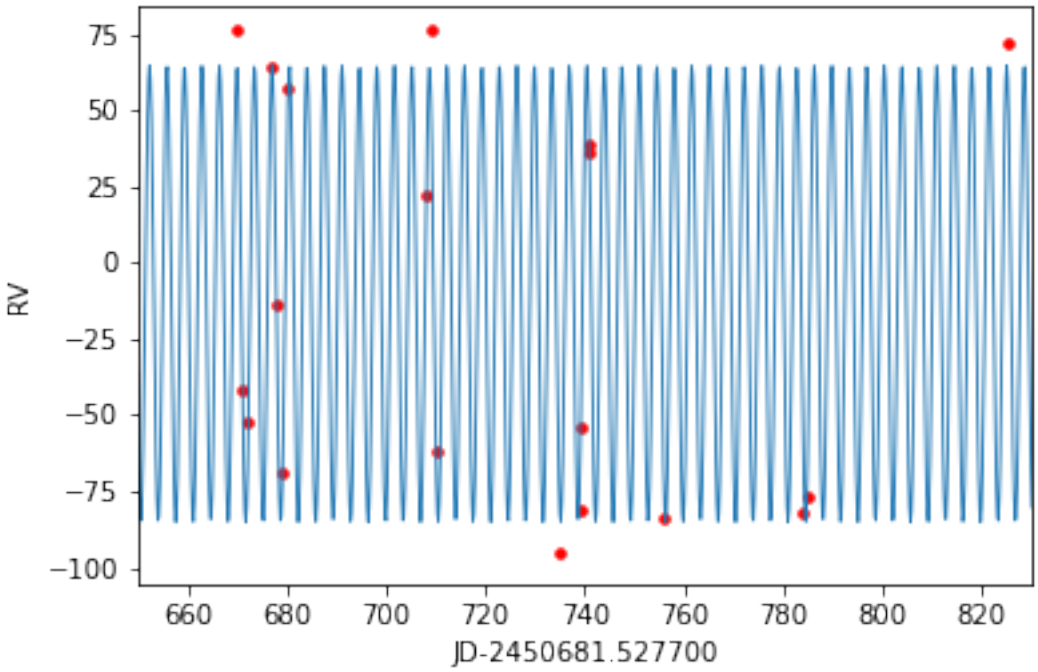
$$\theta = \{\ 43.28144626\ ,\ 61.00551206\ ,\ -10.28913155\ ,\ 3.52427464\ \}$$

The best fit parameters indicates $K = 74.7\ (\mathrm{m/s})$, and $P = 3.5243\ (\mathrm{days})$

The planet's mass can be roughly determined by the RV mass function:

$$M_p \sin i = K(\frac{PM_*}{2\pi G})^{1/3}$$

If compared to the Jupiter,



$$\frac{M_p \sin i}{M_J} = \frac{K}{12.6\ (\mathrm{m/s})}\left(\frac{P}{11.862\ (\mathrm{yr})}\right)^{1/3}\left(\frac{M_*}{M_s}\right)^{1/3} \simeq 0.55\left(\frac{M_*}{M_s}\right)^{1/3}$$

Knowing that the main sequence star has the same order of magnitude as our solar mass, the upper limit of the planet's mass should be about $0.55M_J$.

With a very short period, the planet can be really close to the star. From $\frac{K_p}{12.6\,(\mathrm{m/s})} \approx \left(\frac{a_p}{5.4\,(\mathrm{AU})}\right)^{-1/2}\frac{M_p}{M_J}$, we derive $a_p \approx 0.047\ (\mathrm{AU})$. This implies that maybe we could detect the transit thus obtaining more information.

The temperature could be extremely high on the planet. Assuming an effective temperature for the star of G and a planetary albedo of A. Because the planet has reached an equilibrium state, we have

$$(1-A)\sigma T^4 \frac{R_s^2}{a^2}\pi R_p^2 = \sigma T_p^4 4\pi R_p^2$$

The effective temperature of the planet is $T_p \approx 1200(1-A)^{1/4}\ (\mathrm{K})$