Portfolio Management

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Abstract

The COVID 19 pandemic caused widespread uncertainty, affecting all global economies and a shortage in supply side. The rose of the interest rate due to the Fed's goal to control inflation in 2022 and to slow spending. This study selected 6 USA stocks according to their market capitalization in S&P 500. By using the Monte Carlo simulation, this paper found the efficient frontier and the maximum Sharpe ratio portfolio. The result showed that the TSLA possesses the largest proportion of the maximum Sharpe ratio portfolio. The cumulative return of the optimal portfolios perform better than benchmark index with 1 Year Return -15.85% [2]. This result provides investors some idea for replicating the S&P 500 index.

1 Introduction

The COVID-19 pandemic impacted the U.S. and global economies greatly. To stimulate the economy, the U.S. government enacted a number of policies to provide fiscal stimulus. To complement the fiscal stimulus, the Fed increased money supply which leads to inflation. In 2022, to control the inflation, the Fed announced several rounds of rate hikes to keep the inflation about 2%. History shows the $S\&P$ 500 can climb following multiple interest rate hikes by the Federal Reserve. There were 17 total rate hikes in 2004, 2005, and 2006, yet the S&P 500 managed to gain in every year[1]. Therefore, the US stock market has taken a hit. Replicating S&P 500 index with optimized portfolio weight that maximizes return to risk ratio can enhance investors' gains from the trend of rate hiking and booming stock market. According to Harry Markowitz, the modern portfolio theory is a practical method for selecting investments to maximize their overall returns within an acceptable level of risk [4]. Markowitz argues that people achieve their best results by choosing an optimal mix of assets with high risk and high return or low risk and low return and adjusting portfolio weight based on an assessment of their individual tolerance to risk [4]. The Efficient Frontier is introduced to illustrate the different weights of assets that will provide the highest level of return given a level of risk. The Monte Carlo simulation is used to generate 10,000 portfolios consisted of the 6 stocks. Among these portfolios, the allocation with the maximum Sharpe ratio, the maximum return to risk ratio, is the optimal allocation weight. The result showed that the optimal portfolio outweighed the S&P 500 index greatly. A robustness check is conducted, proving rationality of the method and results to ensure validation.

2 Data

The Standard and Poor's 500, S&P 500, is a stock market index tracking the stock performance of 500 large companies listed on exchanges in the United States. This paper selects the top 6 representative stocks according to the market capitalization of the S&P 500—Apple (AAPL): 7.14 %, Microsoft (MSFT): 6.1%, Amazon (AMZN): 3.8%, Tesla (TSLA): 2.5%, Berkshire Hathaway Class B (BRK.B): 1.7%, Meta (META), formerly Facebook, Class A: 1.4% [2]. Adjusted closing prices from August 30th, 2021, to August 30th, 2022 are used for calculating the average return and covariance matrices to construct the efficient frontier. The selected stocks' performance is summarised by descriptive statistics, including mean, volatility, Sharpe ratio, Var, CVar, and Maximum Drawdown. This information on the ten chosen stocks is presented in Table 1, Table 2, and Figure 1. According to Table 1, TSLA has the highest average return and the highest Sharpe ratio, risk-adjusted return. Table 2 reveals that TSLA has the largest max drawdown and BRK-B has smallest max drawdown. TSLA also has highest absolute value in VaR(0.05), indicating that it has 5% chance earning less than -6.68% of return, with

an average of -8.75% of return. Figure 1 reveals that presents TSLA the highest cumulative return, whereas the cumulative return of META decreased with fluctuation.

Stocks	Mean	Vol	Sharpe
TSLA	1.67%	13.82%	12.07%
AAPL	0.47%	0.69%	6.97%
BRK-B	0.10%	4.21%	2.28%
MSFT	-0.41%	6.65%	-6.14%
AMZN	-0.89%	9.64%	-9.28%
META	-3.47%	11.84%	-29.28%

Table 1: Descriptive Statistics of the daily return of the six stocks.

Stocks	VaR(0.05)	CVaR	Max Drawdown
TSLA	-3.31%	-4.15%	-28.35%
AAPL	-3.72%	-6.47%	-29.08%
BRK-B	-4.34%	-2.67%	-44.64%
MSFT	-1.89%	-2.67%	-25.60%
AMZN	-5.08%	-7.88%	-59.22%
META	-6.68%	-8.75%	-48.93%

Table 2: Tail metrics of the daily stock return.

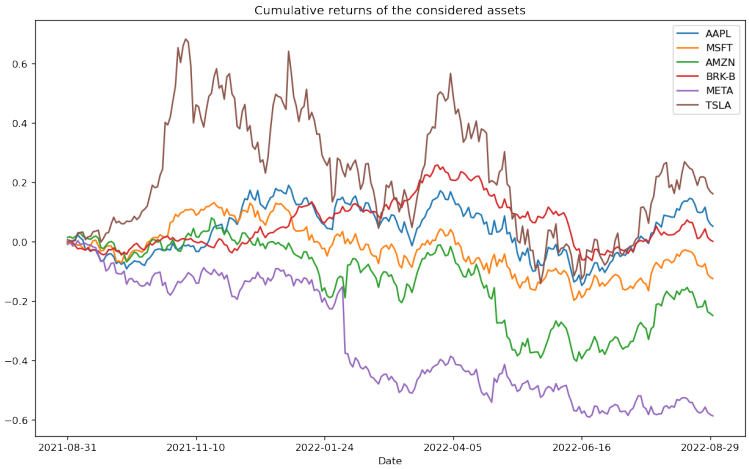

Figure 1: Cumulative Returns of the considered stocks

3 Methods

3.1 Return Forecasting

ARIMA Model is used to forecast returns. An ARIMA model requires time series to be differenced at least once to make it stationary and combine the autoregressive and moving average terms. Auto-Regressive (AR only) model is one where Y_t depends only on its own lags.

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + ... + \beta_p Y_{t-p} + \epsilon_1 \tag{1}$$

Since Auto-Regressive Model, a linear regression model, uses its own lags as predictors and works best when the predictors are not correlated and are independent of each other, the time series data needs

to be made stationary by differencing. The value of d in the ARIMA Model is the minimum number of differencing needed to make the series stationary. ADF test is used to find this d term that makes the series stationary. The null hypothesis of the ADF test is that the time series is non-stationary. So, if the p-value of the test is less than the significance level (0.05), then the null hypothesis is rejected and inferred that the time series is indeed stationary [3].

The moving Average (MA only) model is one where Y_t depends only on the lagged forecast errors, where the error terms are white noise errors of the autoregressive models of the respective lags.

$$Y_t = \alpha + \epsilon_t + \phi_1\epsilon_{t-1} + \phi_2\epsilon_{t-2} + ... + \phi_q\epsilon_{t-q} \tag{2}$$

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + ... + \beta_p Y_{t-p} + \phi_1\epsilon_{t-1} + \phi_2\epsilon_{t-2} + ... + \phi_q\epsilon_{t-q} \tag{3}$$

3.2 Mean Variance Optimization

To optimize the portfolio by altering the weight allocation, one needs to initialize the weights and calculate the initial metrics and use a random generator with a random state of 42 to generate weights for 100,000 portfolios. The weights must come from a uniform distribution from 0 to 1 (as weights denote percentage holdings). The Monte Carlo Simulation generates 100,000 portfolios with different weights and draws the efficient frontier. Let w_i be the weight of ith asset in the portfolio, μ_i be the expected return of the ith asset in the portfolio. The Expected portfolio return is computed as:

$$\mu_p = \sum_i w_i \mu_i \tag{4}$$

Let σ_i be the volatility of the ith asset's return, and ρ_{ij} is the correlation coefficient between the returns on assets i and j. The Expected portfolio return variance is computed as:

$$\sigma_p = \sum_i w_i^2 \mu_i^2 + \sum_i \sum_j \sigma_i \sigma_j w_i w_j \rho_{ij} \tag{5}$$

Since Sharpe Ratio is a good metric that provides how efficient a portfolio return is, concerning how risky its composition, the optimal choice is the portfolio with the largest Sharpe Ratio on the efficient frontier. The Sharpe Ratio is computed as:

$$SharpeRatio = \frac{\mu_p - \mu_{riskfree}}{\sigma_p} \tag{6}$$

4 Results

First, we perform the ADF test on ten stocks' daily returns; all the p-value are insignificant, so we difference the series (d=1), and now the series is stationary.
AAPL: AIC (1, 0) has the minimum score, and thus the order of the ARIMA model is (1,1,0). MSFT: AIC (3, 2) has the minimum score, and thus the order of the ARIMA model is (3,1,2).TSLA: AIC (3, 2) has the minimum score, and thus the order of the ARIMA model is (3,1,2). AMZN: AIC (3, 3) has the minimum score, and thus the order of the ARIMA model is (3,1,3). BRK.B: AIC (4, 3) has the minimum score, and thus the order of the ARIMA model is (4,1,3). META: AIC (3, 2) has the minimum score, and thus the order of the ARIMA model is (3,1,2)
The results of 10,000 simulations by Monte Carlo method forms the dots shown below (Figure 2), and the blue edge is the efficient frontier. The black markers on the plot indicate the risk and return position of a portfolio when it is formed by only one of the six stocks. The weight for best risk-return portfolio is the maximum Sharpe Ratio portfolio with a performance with returns: 21.05% , volatility: 41.81%, sharpe ratio: 50.34%. The portfolio allocation weights are AAPL: 5.15% MSFT: 3.44% AMZN: 0.33% BRK-B: 31.49% META: 0.55% TSLA: 59.04%. As a result, the optimal portfolio annual return is 21.05%, with a volatility of 41.81%, which is greater than the benchmark market index annual return of −15.85%.

Stocks	AIC	Ljung-Box
TSLA	AIC (3, 2)	0
AAPL	AIC(1,0)	0
BRK-B	AIC (4, 3)	0
MSFT	AIC (3, 2)	0
AMZN	AIC (3, 3)	0
META	AIC (3, 2)	0.04

Table 3: ADF test results

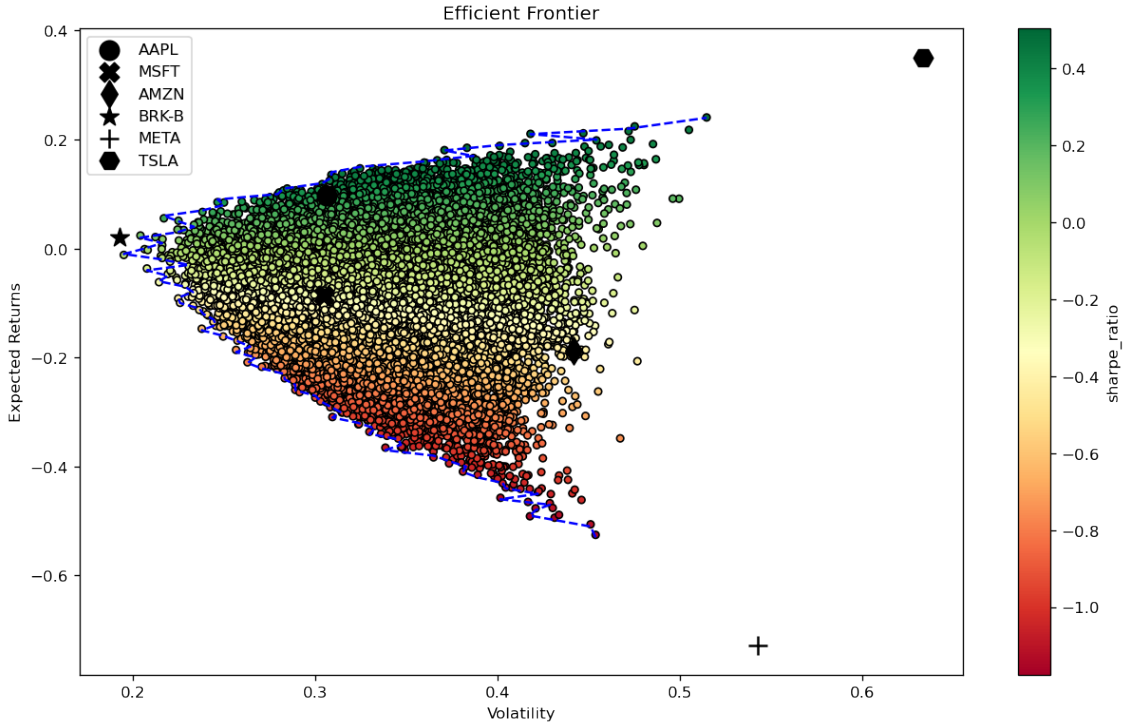

Figure 2: Efficient Frontier

Stocks	Weight
TSLA	59.04%
AAPL	5.15%
BRK-B	31.49%
MSFT	3.44%
AMZN	0.33%
META	0.55%

Table 4: Weight maximizing Sharpe Ratio of the portfolio

5 Conclusion

Based on the data from yahoo finance, the descriptive data and tail metrics of the 6 selected stocks are calculated from 2021-08-30 to 2022-08-30. Then, by Monte Carlo simulation, a 10000 portfolios based on these 6 stocks are generated to plot the efficient frontier. The maximum Sharpe ratio portfolio with highest return to risk ratio is chosen to be the optimal portfolio. This optimal portfolio return is much higher than the $S\&P$ 500 index return given the same period. Thus, the mean-variance optimization of portfolio is highly recommended to the investors.

References

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