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Mathematical Induction
Objective: The following by using principle of mathematical induction.
1. \(1+3+3^2+3^3+...+3^{n-1} = (3^n - 1)/2\)
2. \(1^2 + 2^2 + 3^2 + ... + n^2 = n(n+1)(2n+1)/6\)

(1) \(1+3+3^2+3^3+...+3^{n-1} = (3^n - 1)/2\)

Solution: Let \(P(n)\) be \(1+3+3^2+3^3+...+3^{n-1}\)
Step 1: \(P(1)\) is true
\(P(1) = 1\) and \((3^1 - 1)/2 = 1\)
So, \(P(1)\) is true

Step 2: Assume \(P(k)\) is true for some positive integer \(k\)
Step 3: We shall now prove that \(P(k+1)\) is true
$$P(k+1) = 1+3+3^2+3^3+...+3^{k-1} + 3^k$$
$$= (3^k - 1)/2 + 3^k \quad \quad \text{ [From (1)]}$$
$$= (3^k - 1 + 2.3^k)/2 \quad \quad$$
$$= (3^{k+1} - 1)/2$$
$$= (3^{(k+1)} - 1)/2$$
$$= P(k+1)$$
Hence \(P(n)\) is true for \(n = k+1\)

Therefore \(P(n)\) is true for all \(n \in N\)

(2) \(1^2 + 2^2 + 3^2 + ... + n^2 = n(n+1)(2n+1)/6\)

Solution: Let \(P(n)\) be \(1^2 + 2^2 + 3^2 + ... + n^2\)
Step 1: \(P(1)\) is true
\(P(1) = 1^2 = 1\) and \(1(1+1)(2(1)+1)/6 = 1\)
So, \(P(1)\) is true

Step 2: Assume \(P(k)\) is true for some positive integer \(k\)
Step 3: We shall now prove that \(P(k+1)\) is true
$$P(k+1) = 1^2 + 2^2 + 3^2 + ... + n^2 + (k+1)^2$$
$$= n(n+1)(2n+1)/6 + (k+1)^2 \quad \quad \text{ [From (1)]}$$
$$= (n(n+1)(2n+1) + 6(k+1)^2)/6$$
$$= (2n^3 + 3n^2 + n + 6k^2 + 12k + 6)/6$$
$$= (2n^3 + 3n^2 + 7n + 6k^2 + 12k)/6$$
$$= (n+1)(2n^2 + 5n + 6k^2 + 12k)/6$$
$$= (n+1)(n(2n+5) + 6(k(k+1)))/6$$
$$= (n+1)(n(2n+5) + 6(k(k+1) + 1 - 1))/6$$
$$= (n+1)(n(2n+5) + 6(k(k+1) + 1) - 6)/6$$
$$= (n+1)(2n^3 + 5n^2 + 6nk + 6n + 6k + 6 - 6)/6$$
$$= (n+1)(2n^3 + 5n^2 + 6nk + 6n + 6k)/6$$
$$= (n+1)(n(2n^2 + 5n + 6k) + 6(n+k))/6$$
$$= (n+1)(n(2n^2 + 5n + 6k) + 6(n+k+1) - 6)/6$$
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