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Mathematical Induction
Objective: The following by using principle of mathematical induction.
1. \(1+3+3^2+3^3+...+3^{n-1} = \frac{3^n-1}{2}\)
2. \(1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}\)

(1) \(1+3+3^2+3^3+...+3^{n-1} = \frac{3^n-1}{2}\)

Solution: Let \(P(n)\) be \(1+3+3^2+3^3+...+3^{n-1}\)
Step 1: \(P(1)\) is true
\(P(1) = 1\) and \(\frac{3^1-1}{2} = 1\)
So, \(P(1)\) is true

Step 2: Assume \(P(k)\) is true for some positive integer \(k\)
Step 3: We shall now prove that \(P(k+1)\) is true
$$P(k+1) = 1+3+3^2+3^3+...+3^{k-1} + 3^k$$
\(P(k)\) is true
$$1+3+3^2+3^3+...+3^{k-1} = \frac{3^k-1}{2}$$
$$3^k = 2(\frac{3^k-1}{2})$$
$$3^k = 3^k - 1$$
$$1 = 0$$
Hence \(P(k)\) is true for \(n=k\)

Step 4: We shall now prove that \(P(k+1)\) is true
$$P(k+1) = 1+3+3^2+3^3+...+3^{k-1} + 3^k$$
$$= (1+3+3^2+3^3+...+3^{k-1}) + 3^k \quad \text{ [From (1)]}$$
$$= \frac{3^k-1}{2} + 3^k \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (2)$$
$$= \frac{3^k-1+2.3^k}{2}$$
$$= \frac{3(3^k)-1}{2}$$
$$= \frac{3^{k+1}-1}{2}$$
$$= P(k+1)$$

Hence \(P(n)\) is true for all \(n\in N\)

(2) \(1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}\)

Solution: Let \(P(n)\) be \(1^2 + 2^2 + 3^2 + ... + n^2\)
Step 1: \(P(1)\) is true
\(P(1) = 1^2 = 1\) and \(\frac{1(1+1)(2.1+1)}{6} = 1\)
So, \(P(1)\) is true

Step 2: Assume \(P(k)\) is true for some positive integer \(k\)
Step 3: We shall now prove that \(P(k+1)\) is true
$$P(k+1) = 1^2 + 2^2 + 3^2 + ... + n^2 + (k+1)^2$$
\(P(k)\) is true
$$1^2 + 2^2 + 3^2 + ... + k^2 = \frac{k(k+1)(2k+1)}{6}$$
$$(k+1)^2 = k^2 + 2k + 1$$
$$1^2 + 2^2 + 3^2 + ... + k^2 + (k+1)^2$$
$$= \frac{k(k+1)(2k+1)}{6} + k^2 + 2k + 1$$
$$= \frac{k(k+1)(2k+1) + 6(k^2 + 2k + 1)}{6}$$
$$= \frac{2k^3 + 3k^2 + k + 6k^2 + 12