# Laboratory 1 - Fluids Labs

Sergio M. Vanegas A.
Department of Mathematics
Polimi—Politecnico di Milano
Milano, Italia

October 21, 2021

**Abstract**

The first test case is the steady-state development of incompressible flow between two parallel plates in the laminar regime (Figure 1). The plates are considered infinite in the direction transversal to the flow. The flow develops from a condition of uniform velocity (rectangular profile) imposed at the inlet boundary, reaching a fully-developed state at a certain distance downstream of it. [1]

## 1 Introduction

The fully developed laminar flow between two parallel plates admits an analytical solution (plane Poiseuille flow):

$$\begin{cases} u(x,y,z) = u(y) = -\frac{\delta^2}{2\mu}\frac{dp_e}{dx}\frac{y}{\delta}(2-\frac{y}{\delta})\ v(x,y,z) = 0\ w(x,y,z) = 0 \\ \frac{dp_e}{dx} = const < 0 \\ \tau_{yx}(x,y,z) = -2\mu\frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = -\mu\frac{du}{dy} = \frac{dp_e}{dx}(\delta - y) = \tau_{yx}(y) \end{cases} \tag{1}$$

Where $\mu$ is the dynamic viscosity of the fluid, $\delta$ is the half-distance between the plates, $u, v, w$ are the velocity components along directions $x, y, z$ (Figure 1), $p_e$ is the excess pressure with respect to the hydrostatic component, and $\tau_{yx}$ is the only nonzero shear stress. [1]

The configuration of the problem is as follows:

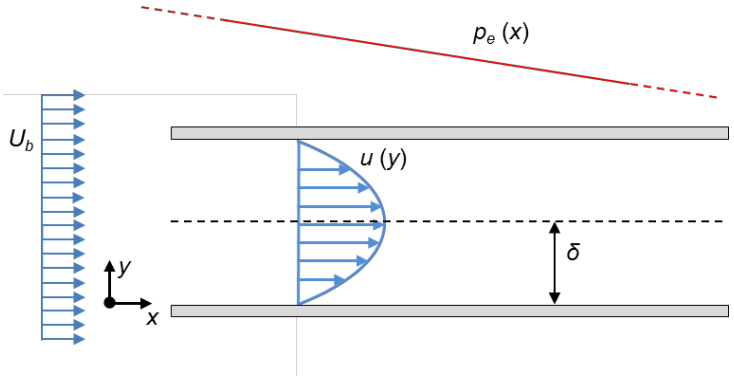


Figure 1: Sketch of the Case

- Length $L$, $L_p$ to be defined,
- Half-channel height $\delta = 5\ mm$,
- Bulk velocity $U_b = 5\ mm/s$,
- Fluid: Water at $20°C$ $(\rho = 998.23\ kg/m^3$, Kinematic Viscosity $\nu = 1.006E-6\ m^2/s)$.

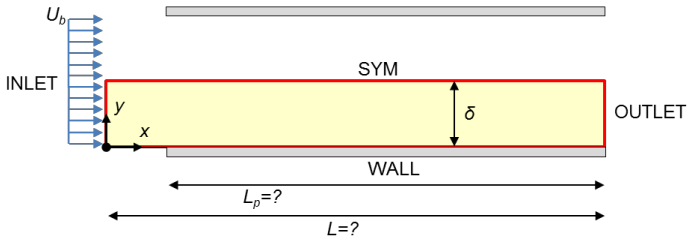


Figure 2: Domain and boundary conditions

**Outline** The remainder of the report is organizedas follows: Section 2 provides some gross estimate of the required $L_p$ to achieve a fully-developed flow in terms of channel height; Section 3 shows how a suitable configuration of the cartesian computational mesh was found; Section 4 compares the simulated solution with the analytical model both graphically and numerically; Finally, Section 5 provides some analysis regarding the vorticity profile on both the developing and fully-developed region.

## 2 Fully-developed flow conditions

## 3 Grid independence study

## 4 CFD solution validation

## 5 Vorticity

# References

[1] Prof. G. V. Messa and Dr. G. Ferrarese. Test case 1: Laminar flow development between two parallel plates. Lab Guide, Fluid Labs, 2021.