Laboratory 1 - Fluids Labs

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Abstract

The first test case is the steady-state development of incompressible flow between two parallel plates in the laminar regime (Figure 1). The plates are considered infinite in the direction transversal to the flow. The flow develops from a condition of uniform velocity (rectangular profile) imposed at the inlet boundary, reaching a fully-developed state at a certain distance downstream of it. [1]

1 Introduction

The fully developed laminar flow between two parallel plates admits an analytical solution (plane Poiseuille flow):

$$\begin{cases} u(x,y,z) = u(y) = -\frac{\delta^2}{2\mu}\frac{dp_e}{dx}\frac{y}{\delta}(2-\frac{y}{\delta}) \; v(x,y,z) = 0 \; w(x,y,z) = 0 \\ \frac{dp_e}{dx} = const < 0 \\ \tau_{yx}(x,y,z) = -2\mu\frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = -\mu\frac{du}{dy} = \frac{dp_e}{dx}(\delta - y) = \tau_{yx}(y) \end{cases} \tag{1}$$

Where μ is the dynamic viscosity of the fluid, δ is the half-distance between the plates, u, v, w are the velocity components along directions x, y, z (Figure 1), p_e is the excess pressure with respect to the hydrostatic component, and τ_{yx} is the only nonzero shear stress. [1]

The configuration of the problem is as follows:

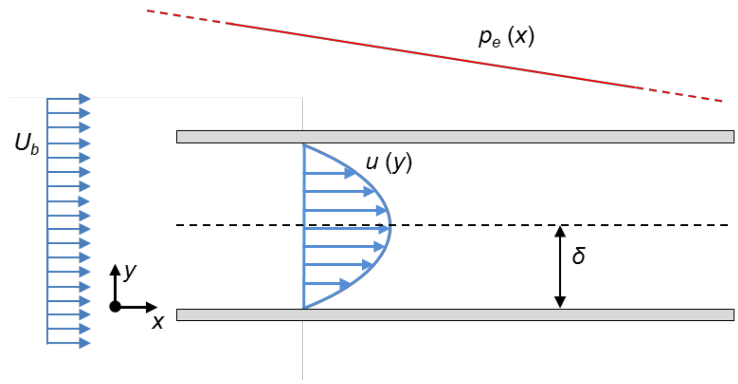


Figure 1: Sketch of the Case

- Length $L = 10\,cm$,
- Length $L_p = 9\,cm$,
- Half-channel height $\delta = 5\,mm$,
- Bulk velocity $U_b = 5\,mm/s$,
- Fluid: Water at $20°C$ ($\rho = 998.23\,kg/m^3$, Kinematic Viscosity $\nu = 1.006E - 6\,m^2/s$).

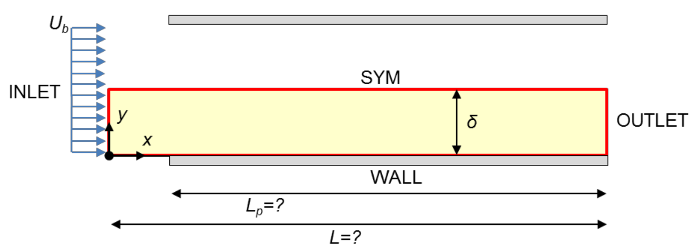


Figure 2: Domain and boundary conditions

Outline The remainder of the report is organizedas follows: Section 2 provides some gross estimate of the required L_p to achieve a fully-developed flow in terms of channel height; Section 3 shows how a suitable configuration of the cartesian computational mesh was found; Section 4 compares the simulated solution with the analytical model both graphically and numerically; Finally, Section 5 provides some analysis regarding the vorticity profile on both the developing and fully-developed region. It is worth noting that all simulations had a relative convergence tolerance of $1E-3$ for all variables taken into consideration.

2 Fully-developed flow conditions

The following profiles were taken from a simulation with a 40-by-40 mesh.

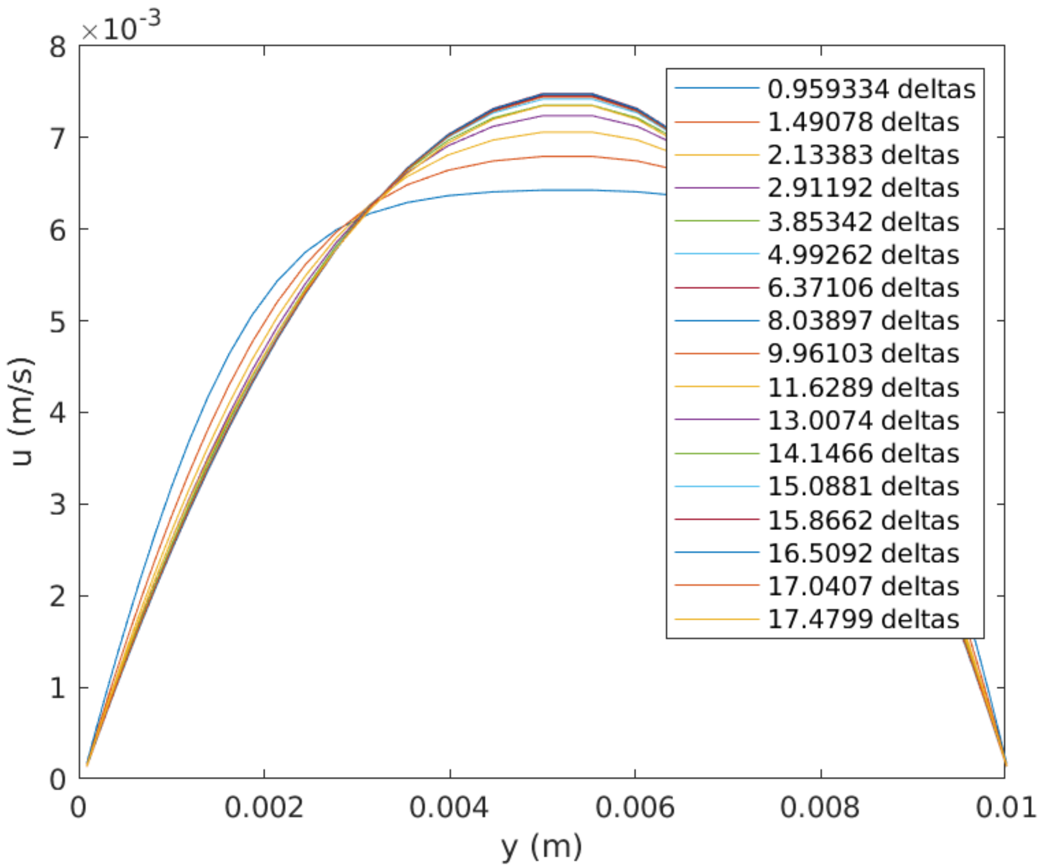


Figure 3: X-velocity Y-profile per delta-step

As we can see in Figure 3, the X-velocity Y-Profile stabilizes after roughly

4 deltas (2 cm), making our choice of an 18-delta channel with a 2-delta margin at the beginning (10 cm total) more than enough for the case of study. From now on, unless said otherwise, all X-specific data was taken after 12 deltas into the actual channel (7 cm from the origin of the X-axis).

3 Grid independence study

The following Grid-Independence study was performed by fixing 40 cells either along the X or Y axis, and then progressively increasing the amount of cells on the other axis until both X-velocity profile and Pressure-gradient convergence was observed.

As we can see in Figure 4, X-refinement is pretty much irrelevant for the X-velocity derivative profile; nevertheless, such is not the case for the Pressure-gradient which, as we can see in Figure 5, did not stabilize after at least 35 cells across the X-axis.

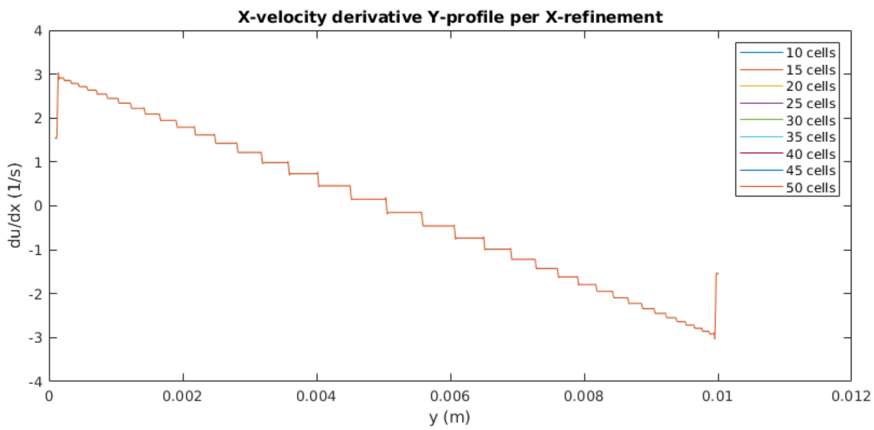
In the case of Y-refinement, 25 cells were enough to stabilize both the X-velocity derivative profile and average Pressure-gradient. Therefore, a 40-by-40 mesh kept being used for the remainder of the laboratory, since simulation times were low enough for a wide Y-refinement margin to not be a problem.

4 CFD solution validation

5 Vorticity

References

[1] Prof. G. V. Messa and Dr. G. Ferrarese. Test case 1: Laminar flow development between two parallel plates. Lab Guide, Fluid Labs, 2021.



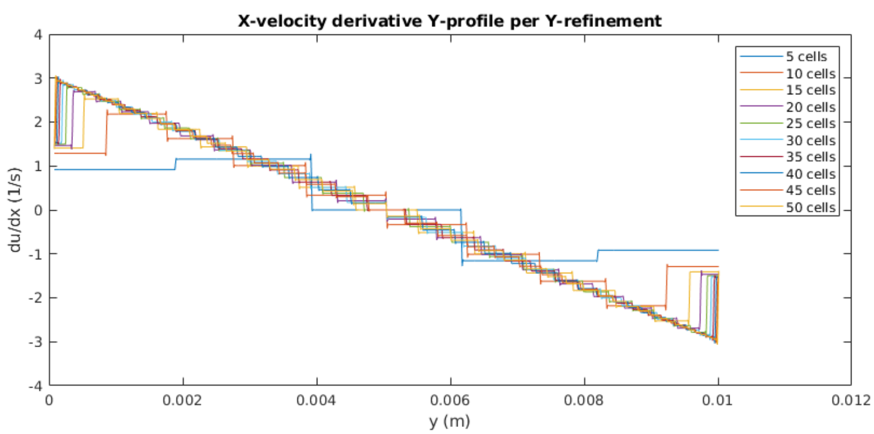


Figure 4: X-velocity Y-profile per cell amount

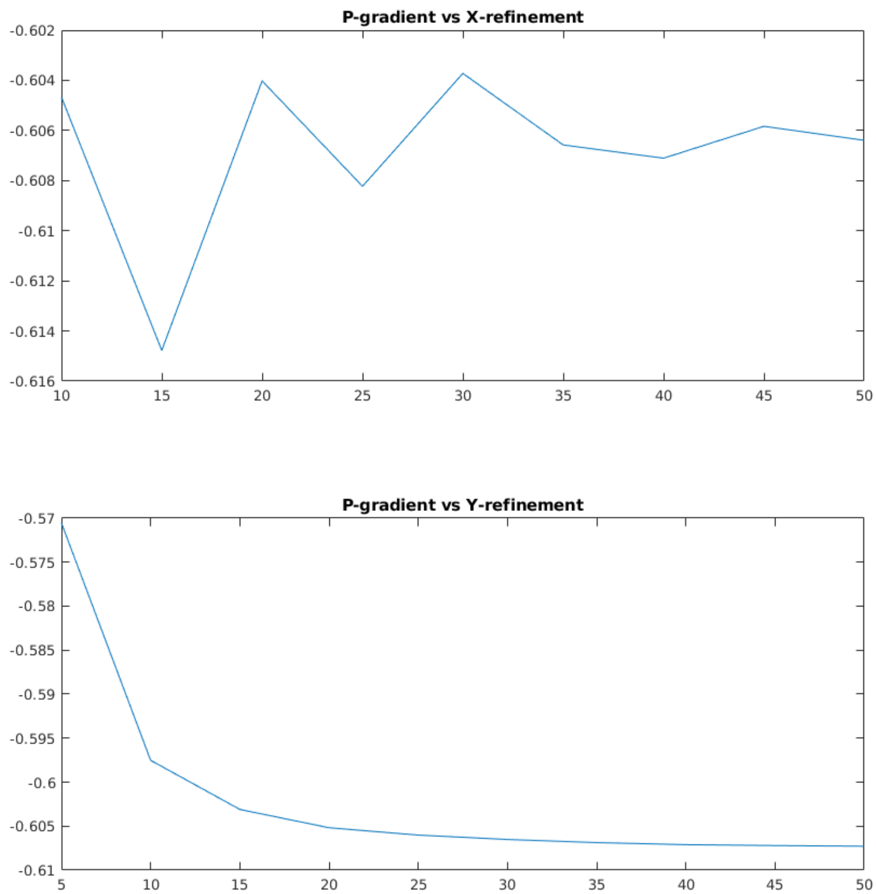


Figure 5: P-gradient vs cell amount