Expanding CertiGraph: Dijkstra, Prim, and Kruskal

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NUS Programming Language and Verification Seminar
December 16, 2020

Verified
Software
Toolchain

Certifying Graph-Manipulating C Programs via Localizations within Data Structures

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VST + CompCert + <u>CertiGraph</u>

A Coq library to verify executable code
against realistic specifications
expressed with mathematical graphs

This Work

Verified
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We verify Dijkstra, Prim, Kruskal

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In doing so, we:

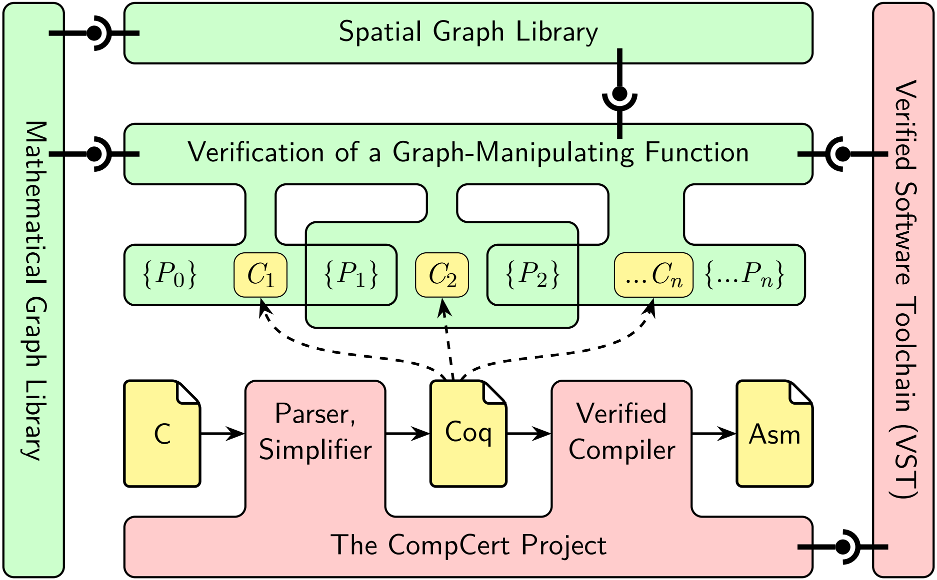
Test existing features [Dijk labels edges]
Expand into undirectedness [Prim, Krus]
Make nontrivial calls to verified methods [Krus calls UF]

Challenges

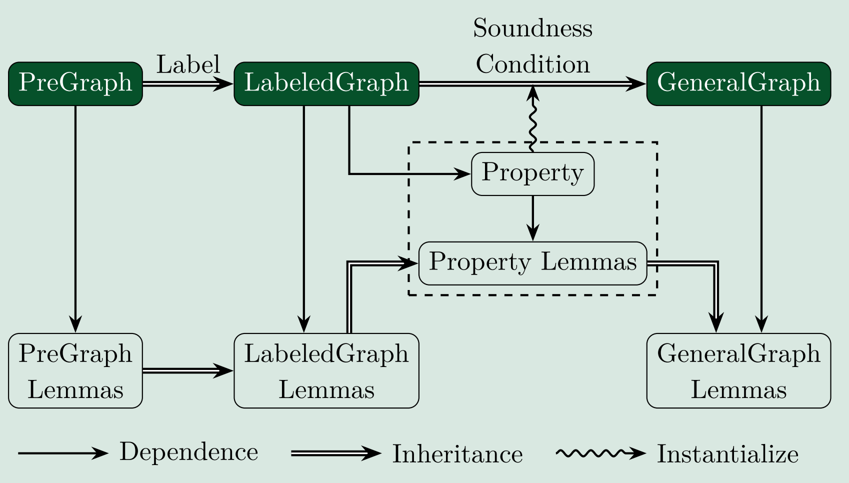
Using CompCert C, which
  is executable and realistic
  but has real-world complications

Aiming for full functional correctness

Maintaining modularity and reuse


Mathematical Graph Library
Spatial Graph Library
Verification of a Graph-Manipulating Function
$\{P_0\}$
C_1
$\{P_1\}$
C_2
$\{P_2\}$
$\ldots C_n$
$\{\ldots P_n\}$
Verified Software Toolchain (VST)
C
Parser, Simplifier
Coq
Verified Compiler
Asm
The CompCert Project

Math Graph Architecture



A PreGraph is a hextuple ($\texttt{VType}, \texttt{EType}, \texttt{vvalid}, \texttt{evalid}, \texttt{src}, \texttt{dst}$)

$$
\textbf{Dijk\_PG}(\gamma) \stackrel{\text{def}}{=}
\begin{array}{l}
\texttt{VType := Z} \\
\texttt{EType := VType * VType} \\
\texttt{src := fst} \\
\texttt{dst := snd} \\
\forall v.\ \texttt{vvalid}(\gamma, v) \Leftrightarrow 0 \leqslant v < \texttt{size} \\
\forall s, d.\ \texttt{evalid}(\gamma, (s, d)) \Leftrightarrow \texttt{vvalid}(\gamma, s) \wedge \texttt{vvalid}(\gamma, d)
\end{array}
$$

Instantiating DijkGraph

A LabeledGraph is a quadruple (PreGraph, `VL`, `EL`, `GL`)

$$\mathbf{Dijk\_LG}(\gamma) \stackrel{\text{def}}{=} \text{Dijk\_PG as shown}$$

```
VL := list EL
EL := Z
GL := unit
```

Instantiating DijkGraph

A GeneralGraph adds arbitrary soundness conditions

$$
\begin{aligned}
\textbf{DijkGraph}(\gamma) \stackrel{\text{def}}{=}\ & \text{Dijk\_LG as shown, and} \\
& \mathit{FiniteGraph}(\gamma) \wedge \\
& \forall i, j.\ \texttt{vvalid}(\gamma, i) \wedge \texttt{vvalid}(\gamma, j) \Rightarrow \\
& \qquad i = j \Rightarrow \texttt{elabel}(\gamma, (i, j)) = 0 \wedge \\
& \qquad i \neq j \Rightarrow 0 \leqslant \texttt{elabel}(\gamma, (i, j)) \leqslant \lfloor \texttt{MAX/size} \rfloor \wedge \\
& \ldots
\end{aligned}
$$

A GeneralGraph adds arbitrary soundness conditions

$$\begin{aligned}\textbf{DijkGraph}(\gamma) \stackrel{\text{def}}{=} & \text{ Dijk\_LG as shown, and} \\ & \mathit{FiniteGraph}(\gamma) \wedge \\ & \forall i, j.\ \texttt{vvalid}(\gamma, i) \wedge \texttt{vvalid}(\gamma, j) \Rightarrow \\ & \qquad i = j \Rightarrow \texttt{elabel}(\gamma, (i, j)) = 0 \wedge \\ & \qquad i \neq j \Rightarrow 0 \leqslant \texttt{elabel}(\gamma, (i, j)) \leqslant \lfloor \texttt{MAX/size} \rfloor \wedge \\ & \ldots\end{aligned}$$

$$\mathsf{list\_rep}(\gamma, i) \stackrel{\text{def}}{=} \texttt{data\_at array graph2mat}(\gamma)[i]\ \texttt{list\_addr}(\gamma, i)$$

$$\mathsf{graph\_rep}(\gamma) \stackrel{\text{def}}{=} \mathop{\Huge\ast}_{\texttt{vvalid}(\gamma, v)} v \mapsto \mathsf{list\_rep}(\gamma, v)$$

Representing DijkGraph in Memory

$$\mathsf{list\_rep}(\gamma, i) \stackrel{\text{def}}{=} \texttt{data\_at array graph2mat}(\gamma)[i] \ \texttt{list\_addr}(\gamma, i)$$

$$\mathsf{graph\_rep}(\gamma) \stackrel{\text{def}}{=} \mathop{\text{\Large$\ast$}}_{\texttt{vvalid}(\gamma, v)} v \mapsto \mathsf{list\_rep}(\gamma, v)$$

Relies on restrictions placed at the Math level

```
void dijkstra (int **graph, int src, int *dist,
               int *prev, int size, int inf {
```

$\{\mathsf{AdjMat}(\gamma) * \mathsf{array}(\mathtt{dist}, \_) * \mathsf{array}(\mathtt{prev}, \_)\}$

Code and Specification

```
void dijkstra (int **graph, int src, int *dist,
               int *prev, int size, int inf {
```

$\{\mathsf{AdjMat}(\gamma) * \mathsf{array}(\mathtt{dist}, \_) * \mathsf{array}(\mathtt{prev}, \_)\}$

```
 int pq = init(size); int i, j, u, cost;
 for (i = 0; i < size; i++)
 {  dist[i] = inf; prev[i] = inf; push(i, inf, pq);  }
 dist[src] = 0; prev[src] = src; dec_key(src, 0, pq);
```

```
void dijkstra (int **graph, int src, int *dist,
               int *prev, int size, int inf {
```

$\{\mathsf{AdjMat}(\gamma) * \mathsf{array}(\texttt{dist}, \_) * \mathsf{array}(\texttt{prev}, \_)\}$

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 int pq = init(size); int i, j, u, cost;
 for (i = 0; i < size; i++)
 {  dist[i] = inf; prev[i] = inf; push(i, inf, pq);  }
 dist[src] = 0; prev[src] = src; dec_key(src, 0, pq);
```

$$\left\{\begin{array}{l}\exists\, dist, prev, popped.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\texttt{pq}) * \mathsf{array}(\texttt{dist}, dist) * \\ \quad \mathsf{array}(\texttt{prev}, prev) \wedge dijk\_correct(\gamma, \texttt{src}, popped, prev, dist)\end{array}\right\}$$

```
 // big while loop
```

Code and Specification

$$\left\{\begin{array}{l}\exists \mathit{dist}, \mathit{prev}, \mathit{popped}.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\texttt{pq}) * \mathsf{array}(\texttt{dist}, \mathit{dist}) * \\ \quad \mathsf{array}(\texttt{prev}, \mathit{prev}) \wedge \mathit{dijk\_correct}(\gamma, \texttt{src}, \mathit{popped}, \mathit{prev}, \mathit{dist})\end{array}\right\}$$

Code and Specification

$$\left\{\begin{array}{l}\exists\, dist, prev, popped.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \mathsf{array}(\mathtt{dist}, dist) * \\ \ \mathsf{array}(\mathtt{prev}, prev) \wedge dijk\_correct(\gamma, \mathtt{src}, popped, prev, dist)\end{array}\right\}$$

```
while (!pq_emp(pq)) {
 u = popMin(pq);
```

Code and Specification

$$\left\{\begin{array}{l}\exists\, dist, prev, popped.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \mathsf{array}(\mathtt{dist}, dist) * \\ \ \mathsf{array}(\mathtt{prev}, prev) \wedge dijk\_correct(\gamma, \mathtt{src}, popped, prev, dist)\end{array}\right\}$$

```
while (!pq_emp(pq)) {
  u = popMin(pq);
```

$$// \left\{\begin{array}{l}\exists\, dist', prev', popped', i.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \\ \ \mathsf{array}(\mathtt{dist}, dist') * \mathsf{array}(\mathtt{prev}, prev') \wedge \\ \ dijk\_correct\_weak(\gamma, \mathtt{src}, popped', prev', dist', i, \mathtt{u})\end{array}\right\}$$

$$\left\{\begin{array}{l}\exists\, dist, prev, popped.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \mathsf{array}(\mathtt{dist}, dist) * \\ \ \mathsf{array}(\mathtt{prev}, prev) \wedge dijk\_correct(\gamma, \mathtt{src}, popped, prev, dist)\end{array}\right\}$$

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while (!pq_emp(pq)) {
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```
 for (i = 0; i < size; i++) {
 cost = getCell(graph, u, i);
 if (cost < inf) {
  if (dist[i] > dist[u] + cost) {
   dist[i] = dist[u] + cost; prev[i] = u;
   dec_key(i, dist[i], pq);
}}} // for
```

Code and Specification

$$\left\{\begin{array}{l}\exists \mathit{dist}, \mathit{prev}, \mathit{popped}.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \mathsf{array}(\mathtt{dist}, \mathit{dist}) * \\ \ \mathsf{array}(\mathtt{prev}, \mathit{prev}) \wedge \mathit{dijk\_correct}(\gamma, \mathtt{src}, \mathit{popped}, \mathit{prev}, \mathit{dist})\end{array}\right\}$$

```
while (!pq_emp(pq)) {
 u = popMin(pq);
```

$$// \left\{\begin{array}{l}\exists \mathit{dist'}, \mathit{prev'}, \mathit{popped'}, i.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \\ \ \mathsf{array}(\mathtt{dist}, \mathit{dist'}) * \mathsf{array}(\mathtt{prev}, \mathit{prev'}) \wedge \\ \ \mathit{dijk\_correct\_weak}(\gamma, \mathtt{src}, \mathit{popped'}, \mathit{prev'}, \mathit{dist'}, i, \mathtt{u})\end{array}\right\}$$

```
 for (i = 0; i < size; i++) {
 cost = getCell(graph, u, i);
 if (cost < inf) {
  if (dist[i] > dist[u] + cost) {
   dist[i] = dist[u] + cost; prev[i] = u;
   dec_key(i, dist[i], pq);
}}} // for
```

$$\left\{\begin{array}{l}\exists \mathit{dist''}, \mathit{prev''}.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \mathsf{array}(\mathtt{dist}, \mathit{dist''}) * \\ \ \mathsf{array}(\mathtt{prev}, \mathit{prev''}) \wedge \mathit{dijk\_correct}(\gamma, \mathtt{src}, \mathit{popped'}, \mathit{prev''}, \mathit{dist''})\end{array}\right\}$$

Code and Specification

$$\left\{\begin{array}{l}\exists dist, prev, popped.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \mathsf{array}(\mathtt{dist}, dist) * \\ \mathsf{array}(\mathtt{prev}, prev) \wedge dijk\_correct(\gamma, \mathtt{src}, popped, prev, dist)\end{array}\right\}$$

```
while (!pq_emp(pq)) {
 u = popMin(pq);
 for (i = 0; i < size; i++) {
 cost = getCell(graph, u, i);
 if (cost < inf) {
  if (dist[i] > dist[u] + cost) {
   dist[i] = dist[u] + cost; prev[i] = u;
   dec_key(i, dist[i], pq);
 }}} // for
```

Code and Specification

$$\left\{ \begin{array}{l} \exists \mathit{dist}, \mathit{prev}, \mathit{popped}.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \mathsf{array}(\mathtt{dist}, \mathit{dist}) * \\ \mathsf{array}(\mathtt{prev}, \mathit{prev}) \wedge \mathit{dijk\_correct}(\gamma, \mathtt{src}, \mathit{popped}, \mathit{prev}, \mathit{dist}) \end{array} \right\}$$

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while (!pq_emp(pq)) {
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 for (i = 0; i < size; i++) {
 cost = getCell(graph, u, i);
 if (cost < inf) {
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   dec_key(i, dist[i], pq);
 }}} // for
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```

$$\left\{\begin{array}{l}\exists \mathit{dist}, \mathit{prev}, \mathit{popped}.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \mathsf{array}(\mathtt{dist}, \mathit{dist}) * \\ \mathsf{array}(\mathtt{prev}, \mathit{prev}) \land \mathit{dijk\_correct}(\gamma, \mathtt{src}, \mathit{popped}, \mathit{prev}, \mathit{dist})\end{array}\right\}$$

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 for (i = 0; i < size; i++) {
 cost = getCell(graph, u, i);
 if (cost < inf) {
  if (dist[i] > dist[u] + cost) {
   dist[i] = dist[u] + cost; prev[i] = u;
   dec_key(i, dist[i], pq);
 }}} // for
} // while
```

$$// \left\{\begin{array}{l}\exists \mathit{dist}^{\circ}, \mathit{prev}^{\circ}, \mathit{popped}^{\circ}.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \\ \mathsf{array}(\mathtt{dist}, \mathit{dist}^{\circ}) * \mathsf{array}(\mathtt{prev}, \mathit{prev}^{\circ}) \land \\ \mathit{all\_popped}(\mathit{popped}^{\circ}) \land \mathit{dijk\_correct}(\gamma, \mathtt{src}, \mathit{popped}^{\circ}, \mathit{prev}^{\circ}, \mathit{dist}^{\circ})\end{array}\right\}$$

Code and Specification

$$\left\{\begin{array}{l}\exists\, dist, prev, popped.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \mathsf{array}(\mathtt{dist}, dist) * \\ \ \mathsf{array}(\mathtt{prev}, prev) \wedge dijk\_correct(\gamma, \mathtt{src}, popped, prev, dist)\end{array}\right\}$$

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   dec_key(i, dist[i], pq);
 }}} // for
} // while
```

$$//\ \left\{\begin{array}{l}\exists\, dist^{\circ}, prev^{\circ}, popped^{\circ}.\ \mathsf{AdjMat}(\gamma) * \mathsf{PQ}(\mathtt{pq}) * \\ \ \mathsf{array}(\mathtt{dist}, dist^{\circ}) * \mathsf{array}(\mathtt{prev}, prev^{\circ}) \wedge \\ \ all\_popped(popped^{\circ}) \wedge dijk\_correct(\gamma, \mathtt{src}, popped^{\circ}, prev^{\circ}, dist^{\circ})\end{array}\right\}$$

```
freePQ (pq); return;
}
```

$$
\begin{aligned}
&\mathit{dijk\_correct}(\gamma, \mathit{src}, \mathit{popped}, \mathit{prev}, \mathit{dist}) \stackrel{\text{def}}{=} \\
&\quad \forall d.\ \mathit{vvalid}(\gamma, d) \Rightarrow \\
&\qquad \big(d \in \mathit{popped} \ \Rightarrow \\
&\qquad\qquad \exists \mathit{path}.\ \mathit{path\_correct}(\gamma, \mathit{prev}, \mathit{path}, \mathit{src}, d) \wedge \\
&\qquad\qquad \mathit{path\_globally\_optimal}(\gamma, \mathit{dist}, \mathit{path})\big) \wedge \\
&\quad \big(\mathit{dist}[d] < \texttt{inf} \ \Rightarrow \\
&\qquad\qquad \texttt{let}\ \ m\ \ \texttt{:=}\ \ \mathit{prev}[d]\ \ \texttt{in}\ \ m \in \mathit{popped}(\mathit{priq}) \wedge \\
&\qquad\qquad \forall m' \in \mathit{popped}(\mathit{priq}).\ \mathit{cost}(\mathit{p2m}\texttt{+}(m, d)) \leqslant \mathit{cost}(\mathit{p2m'}\texttt{+}(m', d))\big) \wedge \\
&\quad \big(\mathit{dist}[d] = \texttt{inf} \ \Rightarrow \\
&\qquad\qquad \forall m \in \mathit{popped}(\mathit{priq}).\ \mathit{cost}(\mathit{p2m}\texttt{+}(m, d)) = \texttt{inf}\big)
\end{aligned}
$$

dijk_correct

$$
\begin{aligned}
&\mathit{dijk\_correct}(\gamma, \mathit{src}, \mathit{popped}, \mathit{prev}, \mathit{dist}) \stackrel{\text{def}}{=} \\
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dijk_correct

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& \qquad \big(d \in \mathit{popped} \ \Rightarrow \\
& \qquad\qquad \exists \mathit{path}.\ \mathit{path\_correct}(\gamma, \mathit{prev}, \mathit{path}, \mathit{src}, d) \land \\
& \qquad\qquad \mathit{path\_globally\_optimal}(\gamma, \mathit{dist}, \mathit{path})\big) \land \\
& \quad \big(\mathit{dist}[d] < \texttt{inf} \ \Rightarrow \\
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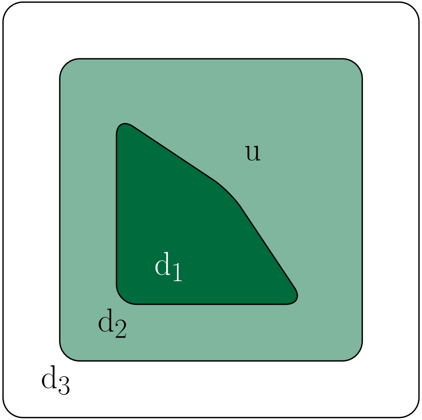
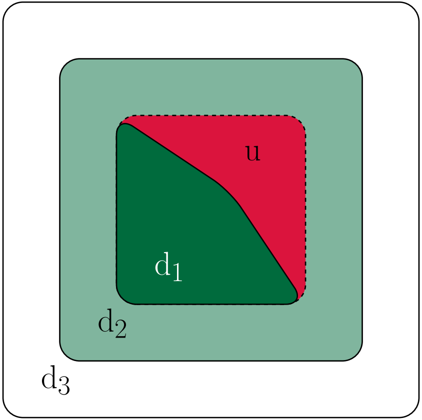
\end{aligned}
$$

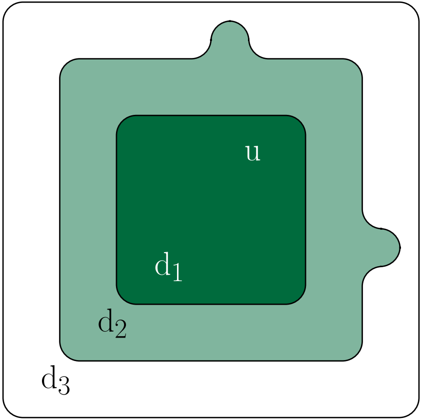
Key Transformation: Growing the Subgraph



Key Transformation: Growing the Subgraph



Key Transformation: Growing the Subgraph



dijk_correct_weak

$$
\begin{aligned}
& dijk\_correct\_weak(\gamma, src, popped, prev, dist, i, u) \stackrel{\text{def}}{=} \forall d. \\
& \quad \big(vvalid(\gamma, d) \;\Rightarrow\; d \in popped \;\Rightarrow\; \ldots \big) \wedge \\
& \quad \Big(0 \leqslant dst < i \;\Rightarrow\; \big(dist[d] < \texttt{inf} \;\Rightarrow \ldots \big) \wedge \big(dist[d] = \texttt{inf} \;\Rightarrow \ldots \big) \Big) \wedge \\
& \quad \Big(i \leqslant dst < size \;\Rightarrow \\
& \qquad \big(dist[d] < \texttt{inf} \;\Rightarrow \ldots \wedge m \neq u \wedge m' \neq u \big) \wedge \\
& \qquad \big(dist[d] = \texttt{inf} \;\Rightarrow \ldots \wedge m \neq u \big) \Big)
\end{aligned}
$$

$$
\begin{aligned}
& \mathit{dijk\_correct\_weak}(\gamma, \mathit{src}, \mathit{popped}, \mathit{prev}, \mathit{dist}, i, u) \overset{\text{def}}{=} \forall d. \\
& \quad \left(\mathit{vvalid}(\gamma, d) \Rightarrow d \in \mathit{popped} \Rightarrow \ldots\right) \land \\
& \quad \Big(0 \leqslant \mathit{dst} < i \Rightarrow \left(\mathit{dist}[d] < \texttt{inf} \Rightarrow \ldots\right) \land \left(\mathit{dist}[d] = \texttt{inf} \Rightarrow \ldots\right)\Big) \land \\
& \quad \Big(i \leqslant \mathit{dst} < \mathit{size} \Rightarrow \\
& \qquad \left(\mathit{dist}[d] < \texttt{inf} \Rightarrow \ldots \land m \neq u \land m' \neq u\right) \land \\
& \qquad \left(\mathit{dist}[d] = \texttt{inf} \Rightarrow \ldots \land m \neq u\right)\Big)
\end{aligned}
$$

$$
\begin{aligned}
&\mathit{dijk\_correct\_weak}(\gamma, \mathit{src}, \mathit{popped}, \mathit{prev}, \mathit{dist}, i, u) \stackrel{\text{def}}{=} \forall d. \\
&\quad \big(\mathit{vvalid}(\gamma, d) \;\Rightarrow\; d \in \mathit{popped} \;\Rightarrow\; \ldots\big) \wedge \\
&\quad \Big(0 \leqslant \mathit{dst} < i \;\Rightarrow\; \big(\mathit{dist}[d] < \texttt{inf} \;\Rightarrow \ldots\big) \wedge \big(\mathit{dist}[d] = \texttt{inf} \;\Rightarrow \ldots\big)\Big) \wedge \\
&\quad \Big(i \leqslant \mathit{dst} < \mathit{size} \;\Rightarrow \\
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\end{aligned}
$$

dijk_correct_weak

$$
\begin{aligned}
& \mathit{dijk\_correct\_weak}(\gamma, \mathit{src}, \mathit{popped}, \mathit{prev}, \mathit{dist}, i, u) \stackrel{\text{def}}{=} \forall d. \\
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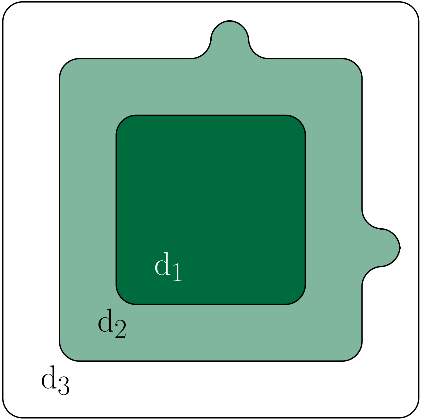
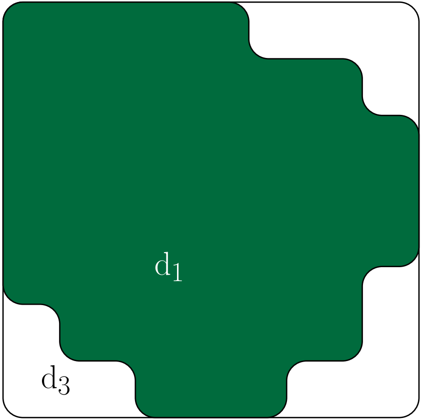
& \quad \Big(0 \leqslant \mathit{dst} < i \Rightarrow \big(\mathit{dist}[d] < \texttt{inf} \Rightarrow \ldots\big) \wedge \big(\mathit{dist}[d] = \texttt{inf} \Rightarrow \ldots\big)\Big) \wedge \\
& \quad \Big(i \leqslant \mathit{dst} < \mathit{size} \Rightarrow \\
& \qquad \big(\mathit{dist}[d] < \texttt{inf} \Rightarrow \ldots \wedge m \neq u \wedge m' \neq u\big) \wedge \\
& \qquad \big(\mathit{dist}[d] = \texttt{inf} \Rightarrow \ldots \wedge m \neq u\big)\Big)
\end{aligned}
$$

Postcondition



Postcondition



Overflow Strikes Again

The longest optimal path has `size-1` links

Overflow Strikes Again

The longest optimal path has `size-1` links
so say we set `elabel`'s upper bound to $\lfloor \texttt{MAX/(size-1)} \rfloor$

Overflow Strikes Again

The longest optimal path has `size-1` links
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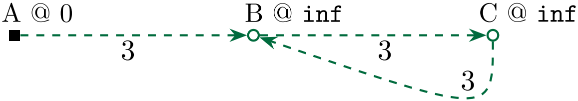
$\texttt{MAX} = 7$, $\texttt{size} = 3$, so $0 \leqslant \texttt{elabel}(\gamma, e) \leqslant 3$.



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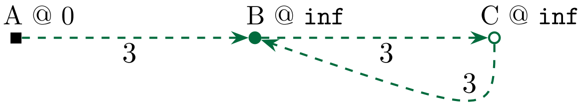

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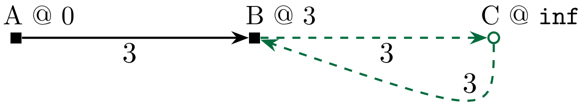

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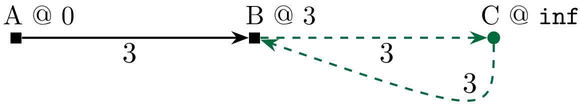

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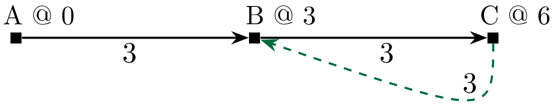

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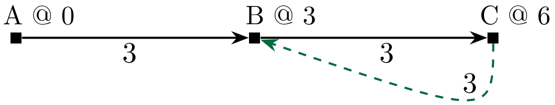

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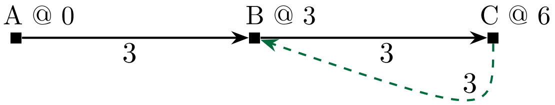

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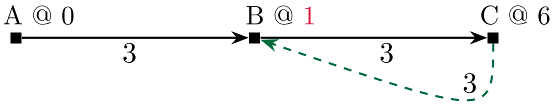

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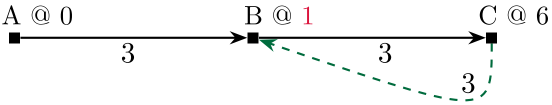

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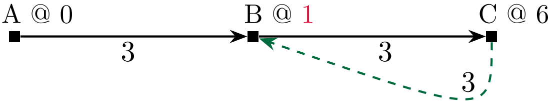

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One solution: Conservatively set upper bound to $\lfloor$`MAX/size`$\rfloor$

Max path cost is then $\lfloor$`MAX/size`$\rfloor$ `* (size-1) = MAX -` $\lfloor$`MAX/size`$\rfloor$

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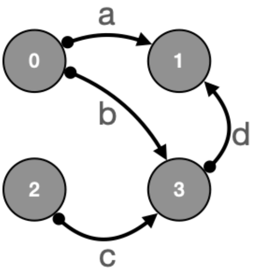
Sadly, intuition supports `inf = MAX`

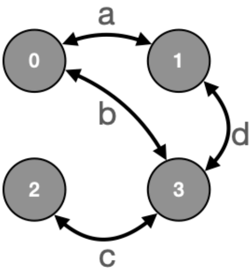
Prim: Extending to Undirectedness

Consider the AdjMat representation of a directed graph:



	0	1	2	3
0	0	a	inf	b
1	inf	0	inf	inf
2	inf	inf	0	c
3	inf	d	inf	0

Versus the AdjMat representation of an undirected graph:



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Prevent double-counting:

```
Class SoundUAdjMat (g: UAdjMatLG) := {
  sadjmat: @SoundAdjMat size inf g;
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```
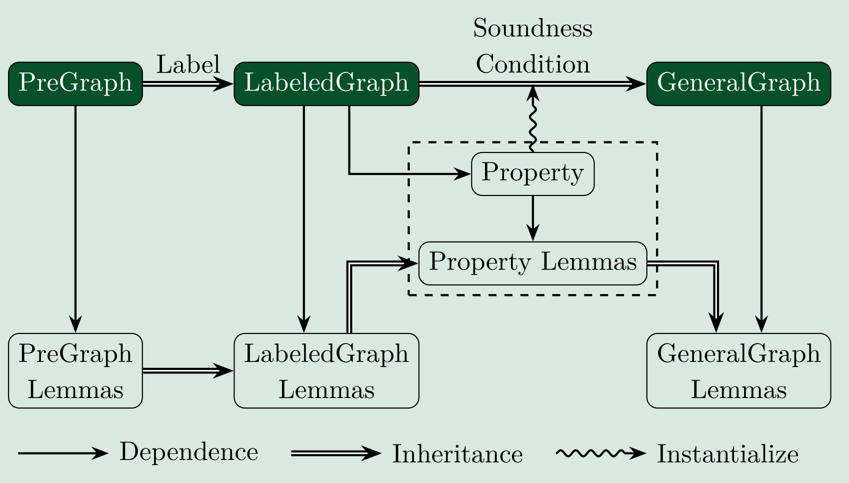

Build undirected idioms:

```
Definition adj_edge g e u v :=
  ((src g e = u /\ dst g e = v) \/
   (src g e = v /\ dst g e = u)).
```

Plus `upath`, `connected`, *etc.*

Recall: Math Graph Architecture



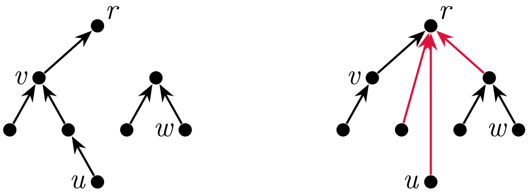
Kruskal: EdgeList Representation

Extend spatial support to accommodate EdgeList representation

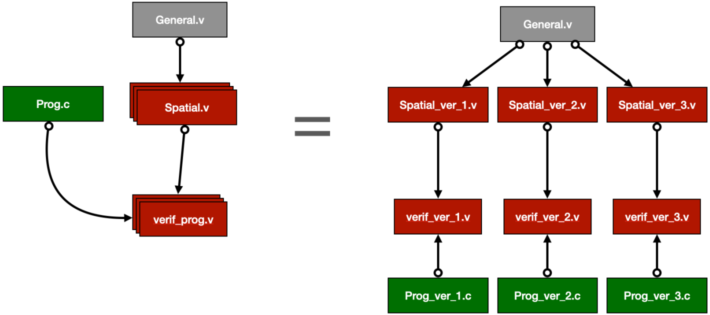
The double-counting restriction must be lifted:
an EdgeList-represented graph can have bona fide multi-connections

But the undirected idioms carry over

Consider performing `union u w`
Note: `reachable` is directed, `connected` is undirected



A Note on Modularity



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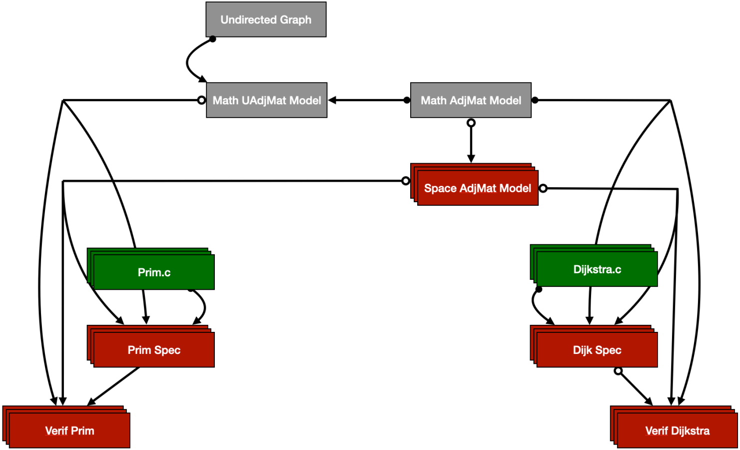

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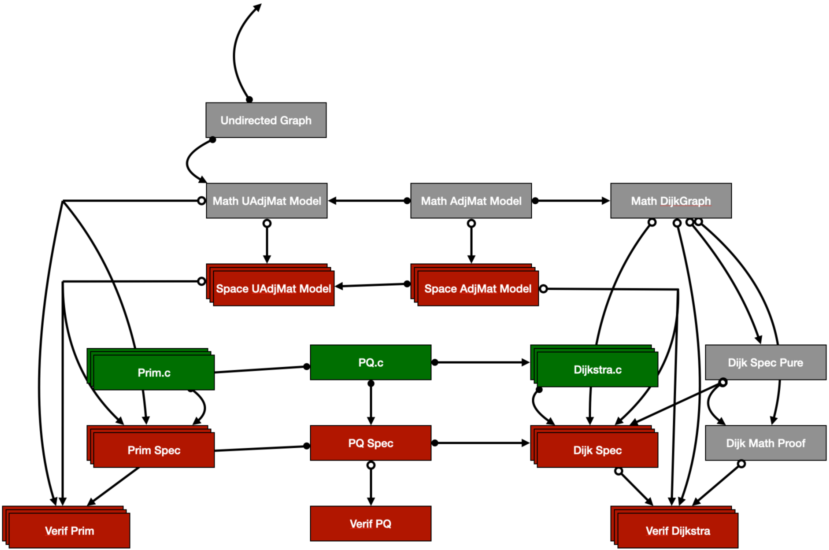

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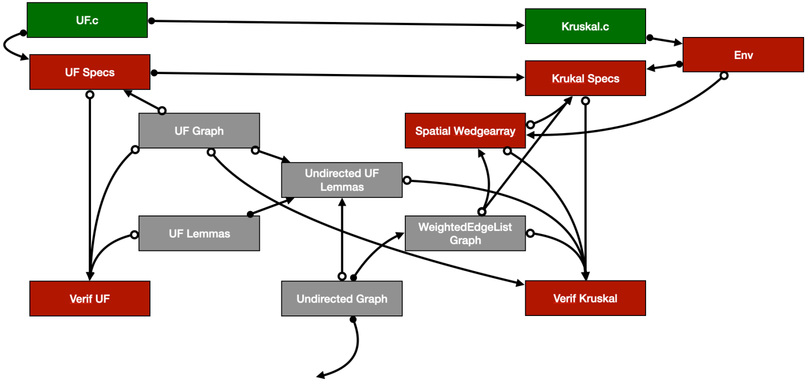

Possible Next Steps

Verify PQ with `decrease-key`
AdjList representation for Dijkstra, Prim
Plug into verified `malloc`
Floyd-Warshall using AdjMat
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