

A Verified Garbage Collector for Gallina

Shengyi Wang[†], Anshuman Mohan[†], Qinxiang Cao[‡], Aquinas Hobor[†]



APLAS NIER
December 1, 2019

Verify **graph-manipulating** programs
written in **executable C**
with **machine-checked** correctness proofs

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Ubiquitous in critical areas



Certifying Graph-Manipulating C Programs via Localizations within Data Structures

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VST + CompCert + 25000 LOC library



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Powerful enough to verify **executable code**
against **realistic specifications**
expressed with **mathematical graphs**



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[Wang *et. al.*, PACMPL OOPSLA 2019]



Gallina \rightsquigarrow CompCert C \rightsquigarrow Assembly



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Gallina assumes **infinite** memory
but CompCert C has a **finite** heap



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Solution: garbage collect the CompCert C code



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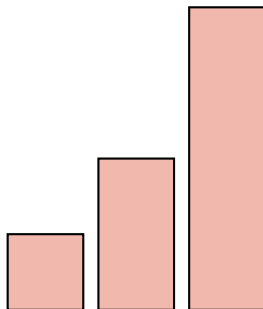
New problem: verify the garbage collector

Our Garbage Collector

GC has jurisdiction over the heap



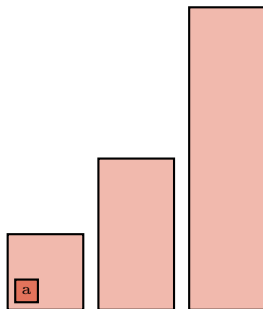
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Our Garbage Collector

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Mutator **allocs** in special subheap

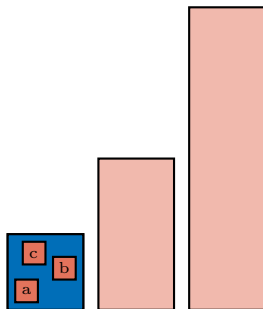


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Mutator **allocs** in special subheap

If subheap is full

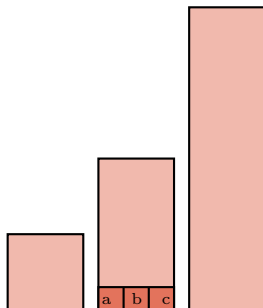


Our Garbage Collector

GC has jurisdiction over the heap

Mutator **allocs** in special subheap

If subheap is full **call GC**

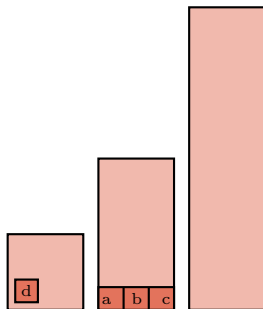


Our Garbage Collector

GC has jurisdiction over the heap

Mutator **allocs** in special subheap

If subheap is full **call GC** and try again

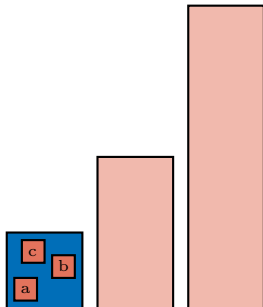


- 12 generations, doubling in size
- Functional mutator: no back pointers

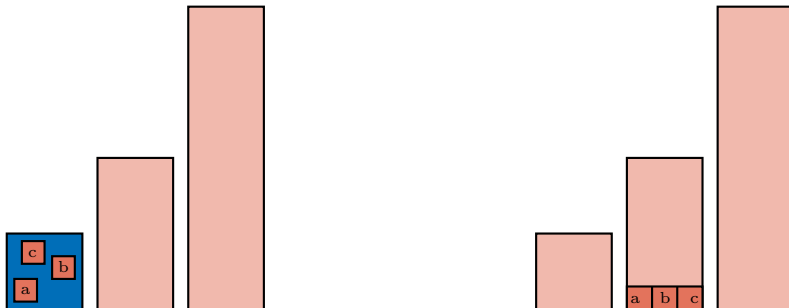
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- Cheney's mark-and-copy collects gen to next
- Potentially triggers cascade of pairwise collections

- 12 generations, doubling in size
- Functional mutator: no back pointers
- Cheney's mark-and-copy collects gen to next
- Potentially triggers cascade of pairwise collections
- Three key functions:
 - `forward` copies individual objects
 - `do_scan` repairs copied objects
 - `forward_roots` kick-starts the collection

Primum non nocere: first, do no harm

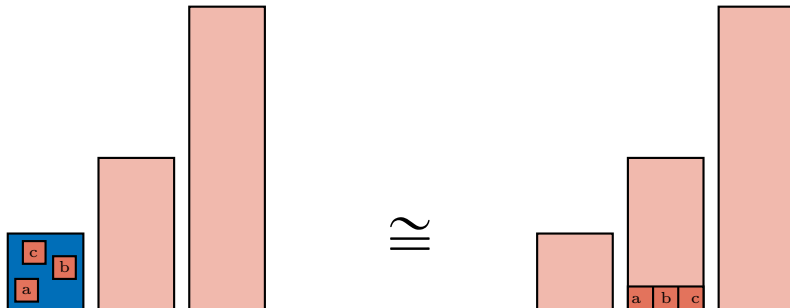


Primum non nocere: first, do no harm



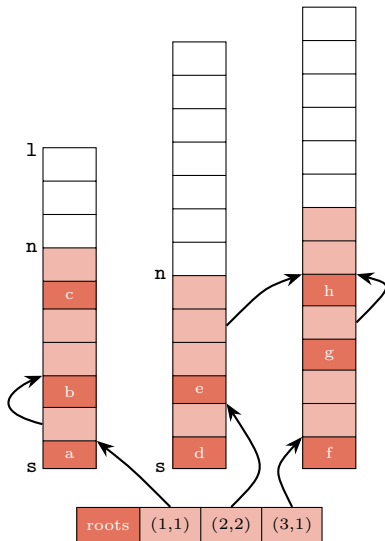
Intuitive Specification

Primum non nocere: first, do no harm



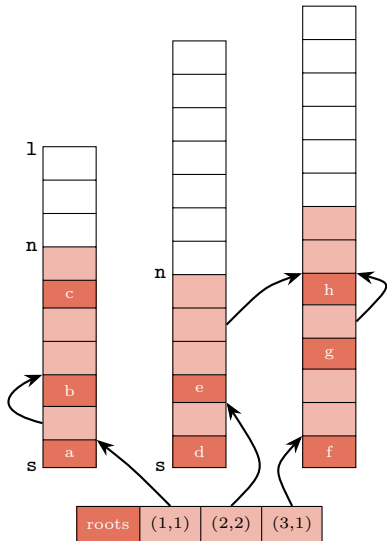
Overview of Operations

Nursery cannot fit `alloc`



Overview of Operations

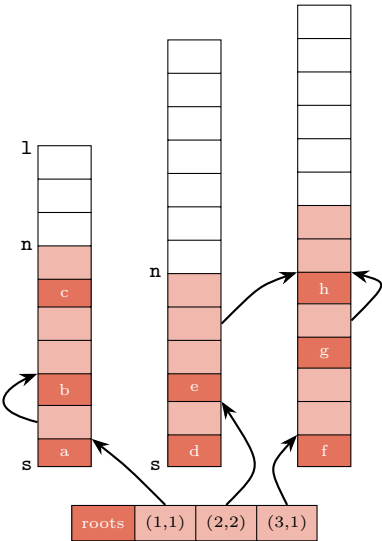
Nursery cannot fit `alloc`
`do_gen`



Overview of Operations

Nursery cannot fit alloc

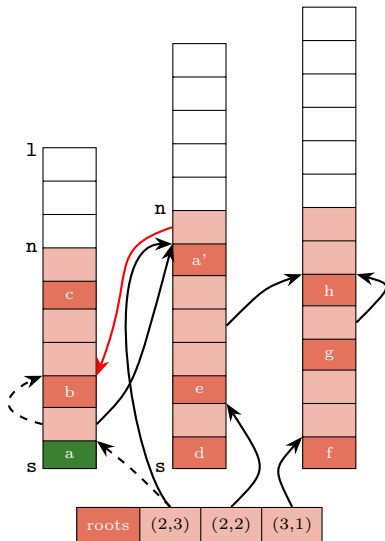
```
do_gen
  forward_roots
```



Overview of Operations

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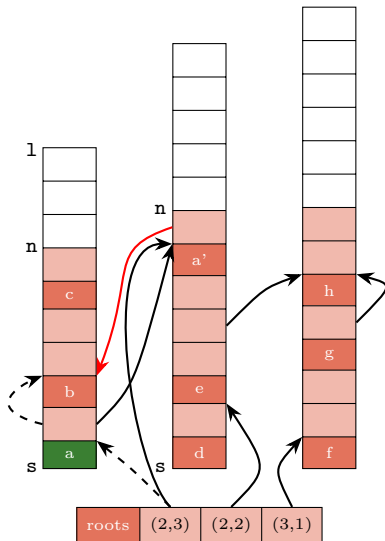
```
do_gen  
  forward_roots  
    forward
```



Overview of Operations

Nursery cannot fit alloc

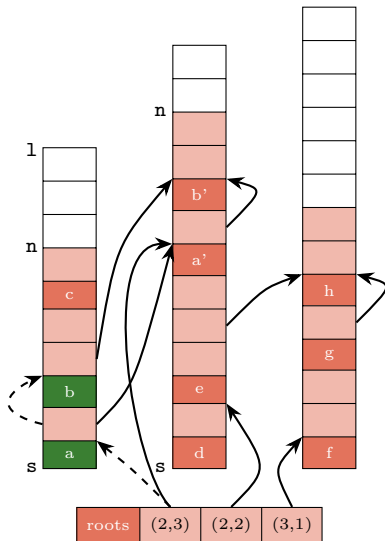
```
do_gen  
  forward_roots  
    forward  
do_scan
```



Overview of Operations

Nursery cannot fit alloc

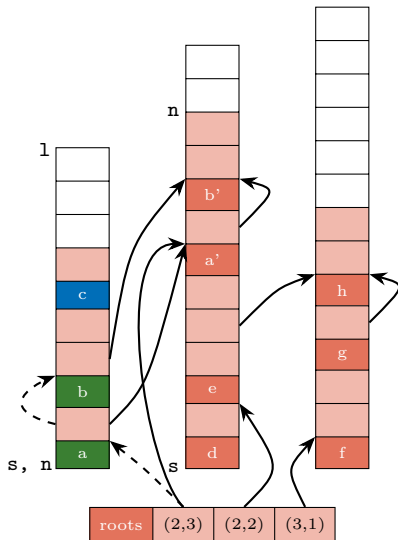
```
do_gen  
  forward_roots  
    forward  
do_scan  
  forward
```



Overview of Operations

Nursery cannot fit alloc

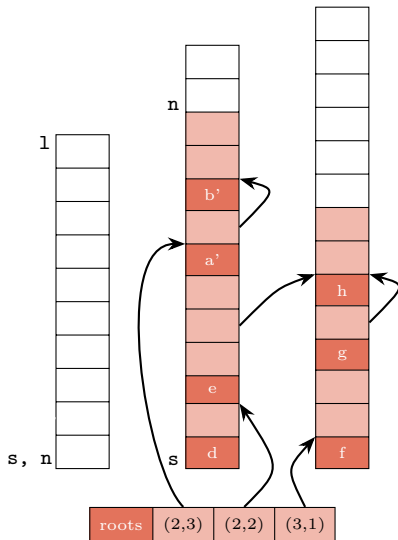
```
do_gen  
  forward_roots  
    forward  
do_scan  
  forward  
reset_gen
```



Overview of Operations

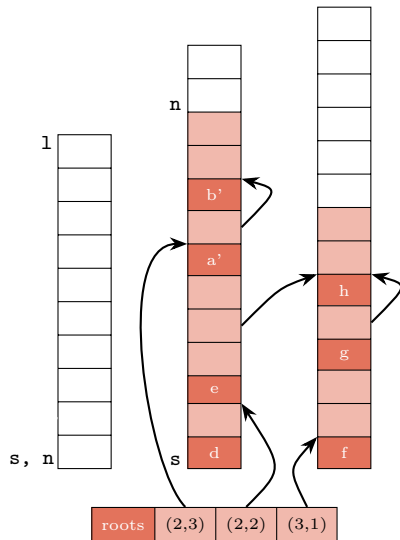
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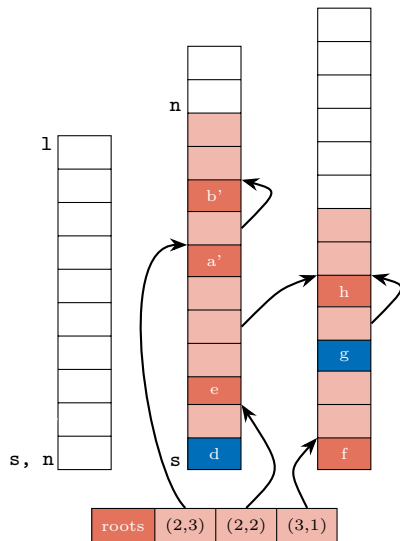
Non-Concerns



Overview of Operations

Non-Concerns

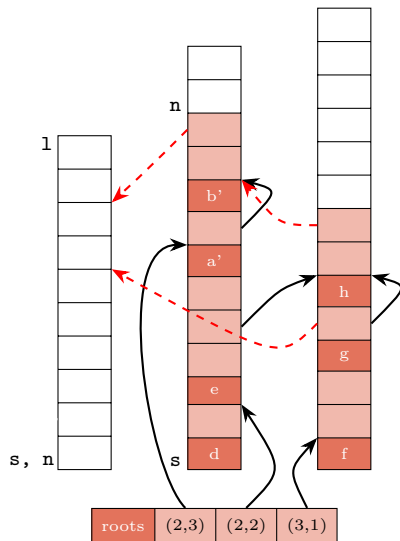
more garbage



Overview of Operations

Non-Concerns

- more garbage
- backward pointers

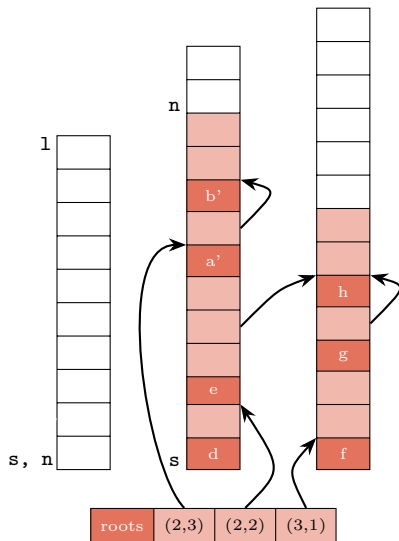


Overview of Operations

Non-Concerns

- more garbage
- backward pointers

Sources of Complexity



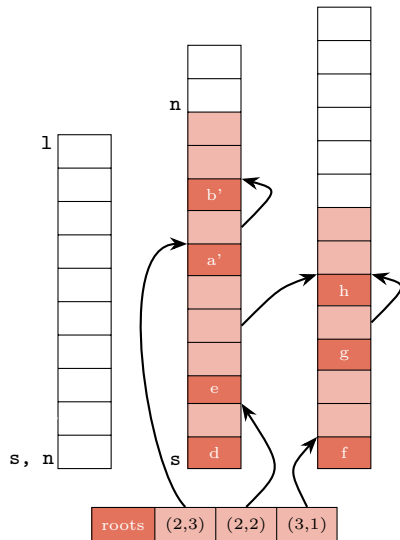
Overview of Operations

Non-Concerns

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Sources of Complexity

- variable-length objects



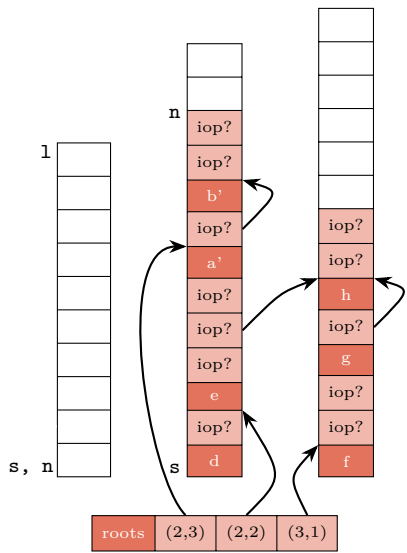
Overview of Operations

Non-Concerns

- more garbage
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Sources of Complexity

- variable-length objects
- disambiguate int/ptr



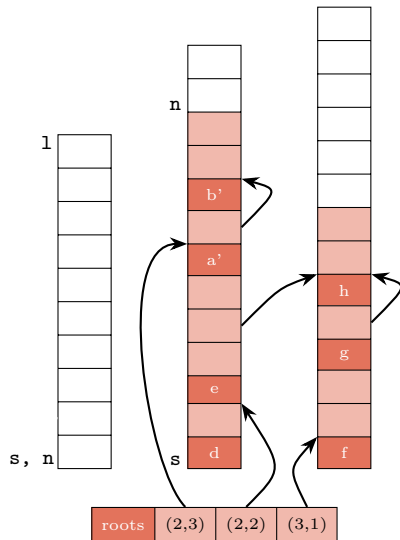
Overview of Operations

Non-Concerns

- more garbage
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Sources of Complexity

- variable-length objects
- disambiguate int/ptr
- determine v 's gen



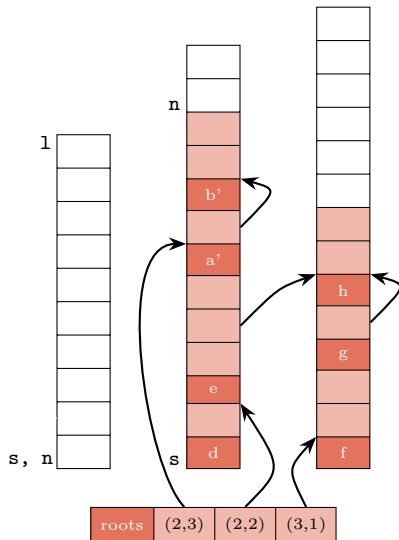
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Non-Concerns

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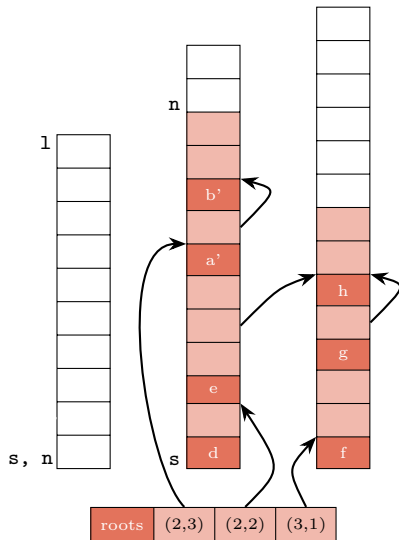
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- what if `malloc` fails?



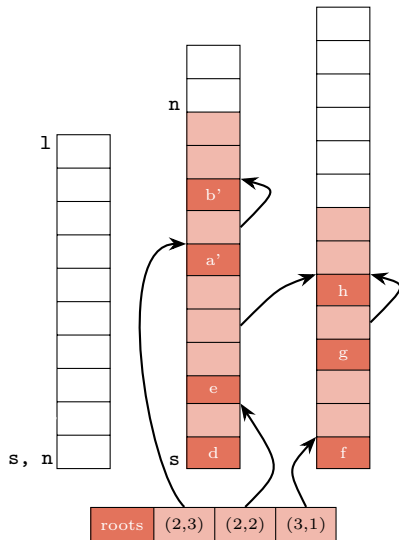
Overview of Operations

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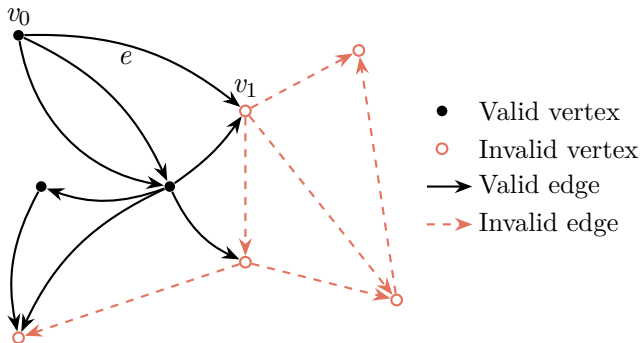
Sources of Complexity

- variable-length objects
- disambiguate int/ptr
- determine v 's gen
- determine gen size
- what if `malloc` fails?
- mutator's max `alloc`?



Instantiating GC_Graph

A PreGraph is a hextuple ($VType$, $EType$, $vvalid$, $evalid$, src , dst)



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GC_PreGraph: VType := nat * nat
EType := VType * nat
src := fst
dst := *unrestricted*
 $\forall v. \text{vvalid}(\gamma, v) \Leftrightarrow \text{graph_has_v}(\gamma, v)$
 $\forall v, out. \text{evalid}(\gamma, (v, out)) \Leftrightarrow$
 $\text{vvalid}(\gamma, v) \wedge \text{In } out \text{ (get_edges}(\gamma, v))$

A LabeledGraph is a quadruple (PreGraph, VL, EL, GL)

GC_Graph: GC_PreGraph as shown

VL := raw_vert_block

EL := unit

GL := list gen_info

Instantiating GC_Graph

A LabeledGraph is a quadruple (PreGraph, VL, EL, GL)

GC_Graph: GC_PreGraph as shown

VL := raw_vert_block

EL := unit

GL := list gen_info

Definition

raw_fld := Z + GC_Ptr.

Record raw_vert_block :=
{ raw_mark: bool;
 copied_vertex: VType;
 raw_flds: list raw_fld;
 (* elided *) }.

Record gen_info :=
{ s_addr: val;
 s_ok: isptr s_addr;
 num_vert: nat;
 (* elided *) }.

forward is robust

```
void forward (value *s, *l, **n, *p) {  
    value * v; value va = *p;  
    if(Is_block(va)) {  
        v = (value*)iop2ptr(va);  
        if(Is_from(s, l, v)) {  
            header_t hd = Hd_val(v);  
            if(hd == 0) {  
                *p = Field(v,0);  
            } else { /* elided */  

```

forward is **robust**
pointer?

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forward is **robust**

pointer? in from space? already forwarded?

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void forward (value *s, *l, **n, *p) {  
  value * v; value va = *p;  
  if(Is_block(va)) {  
    v = (value*)iop2ptr(va);  
    if(Is_from(s, l, v)) {  
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pointer? in from space? already forwarded?
and **versatile**

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forward is **robust**

pointer? in from space? already forwarded?
and **versatile**
called on root set called on heap

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void forward (value *s, *l, **n, *p) {  
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```

$$\left\{ \begin{array}{l} \forall \gamma, from, to, v, n. \text{gc_graph}(\gamma) \wedge \text{compat}(\gamma, from, to) \wedge \\ s = \text{start}(\gamma, from) \wedge l = s + \text{gensz}(\gamma, from) \wedge \\ n = \text{nxtaddr}(to) \wedge p = \text{vaddr}(\gamma, v) + n \end{array} \right\} \stackrel{\text{def}}{=} \phi_1$$

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  }
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```
void forward (value *s, *l, **n, *p) {
```

```
    /* elided */
```

```
    if(hd == 0) {
```

```
        *p = Field(v,0);
```

$$\left\{ \begin{array}{l} \phi_1 \wedge \exists \gamma'. \text{gc_graph}(\gamma') \wedge \gamma' = \text{upd_edge}(\gamma, e, \text{copy}(\gamma, v)) \wedge \\ \text{compat}(\gamma', from, to) \wedge \text{fwd_relation}(\gamma, \gamma', from, to, v, n) \end{array} \right\}$$

```
}
```

```
else {  
  int i; int sz; value *new; sz = size(hd);  
  new = *next+1; *next = new+sz; Hd_val(new) = hd;  
  for(i = 0; i < sz; i++)  
    Field(new, i) = Field(v, i);  
}
```

```
else {  
  int i; int sz; value *new; sz = size(hd);  
  new = *next+1; *next = new+sz; Hd_val(new) = hd;  
  for(i = 0; i < sz; i++)  
    Field(new, i) = Field(v, i);  
  
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```



```
else {  
  int i; int sz; value *new; sz = size(hd);  
  new = *next+1; *next = new+sz; Hd_val(new) = hd;  
  for(i = 0; i < sz; i++)  
    Field(new, i) = Field(v, i);  
  {  
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     $\gamma' = \text{copy\_vertex}(\gamma, to, v, v') \wedge \text{compat}(\gamma', from, to)$   
  }  $\stackrel{\text{def}}{=} \phi_2$   
  Hd_val(v) = 0; Field(v, 0) = p2iop((void *)new);  
  *p = p2iop((void *)new);  
}
```

```

else {
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  for(i = 0; i < sz; i++)
    Field(new, i) = Field(v, i);
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  {
 $\phi_2 \wedge \exists \gamma''. \text{gc\_graph}(\gamma'') \wedge \gamma'' = \text{upd\_edge}(\gamma', e, v') \wedge$ 
 $\text{compat}(\gamma'', from, to) \wedge \text{fwd\_relation}(\gamma, \gamma'', from, to, v, n)$ 
 $\}$ 
}

```

Inductive fwd_relation from to :

forward_t -> LGraph -> LGraph -> Prop :=

```
Inductive fwd_relation from to :  
  forward_t -> LGraph -> LGraph -> Prop :=  
| fr_v_not_in : forall v g,  
  vgen v <> from ->  
  fwd_relation from to (inl (inr v)) g g
```

```
Inductive fwd_relation from to :  
  forward_t -> LGraph -> LGraph -> Prop :=  
| fr_v_not_in : forall v g,  
  vgen v <> from ->  
  fwd_relation from to (inl (inr v)) g g  
| fr_e_to_fwdded : forall e g,  
  vgen (dst g e) = from ->  
  raw_mark (vlabel g (dst g e)) = true ->  
  let new_g := labeledgraph_gen_dst g e  
    (copied_vertex (vlabel g (dst g e))) in  
  fwd_relation from to (inr e) g new_g
```

```
| fr_e_to_not_fwded_Sn : forall e g g',  
  vgen (dst g e) = from ->  
  raw_mark (vlabel g (dst g e)) = false ->  
  let new_g :=  
    labeledgraph_gen_dst (lgraph_copy1v g (dst g e) to)  
      e (copy1v_new_v g to) in  
  fwd_loop from to  
    (make_fields new_g (copy1v_new_v g to)) new_g g' ->  
  fwd_relation from to (inr e) g g'
```

Similar to `forward_relation`, we have

`forward_roots_relation`

`do_scan_relation`

`do_generation_relation`

`garbage_collect_relation`

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`forward_roots_relation`

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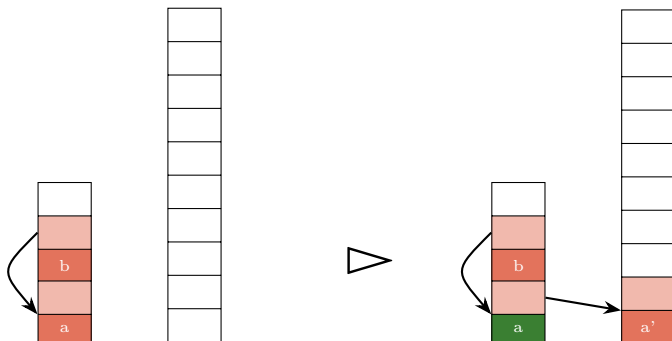
`garbage_collect_relation`

A composition of these gives us our **isomorphism**

Isomorphism

But the journey is far from easy!

A brief look at `semi_iso`:



The general iterative pattern:

$$\frac{}{\gamma \triangleright \gamma}$$

The general iterative pattern:

$$\frac{}{\gamma \triangleright \gamma} \qquad \frac{\gamma \triangleright \gamma_i \qquad \gamma_i \rightsquigarrow \gamma_{i+1}}{\gamma \triangleright \gamma_{i+1}}$$

The general iterative pattern:

$$\frac{}{\gamma \triangleright \gamma}$$

$$\frac{\gamma \triangleright \gamma_i \quad \gamma_i \rightsquigarrow \gamma_{i+1}}{\gamma \triangleright \gamma_{i+1}}$$

$$\gamma_\alpha \triangleright \gamma_\omega$$

A specific example:

```
Lemma semi_iso_refl: forall g from to,  
  sound_gc_graph g -> semi_iso g g from to nil.
```

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Lemma semi_iso_refl: forall g from to,  
  sound_gc_graph g -> semi_iso g g from to nil.
```

```
Lemma fwd_rel_semi_iso:  
  forall from to p g1 g2 g3 roots,  
    semi_iso g1 g2 from to l1 ->  
    forward_relation from to p g2 g3 ->  
    semi_iso g1 g3 from to
```

And eventually,

```
Theorem garbage_collect_iso: forall roots1 roots2 g1 g2,  
  ...  
  garbage_collect_relation roots1 roots2 g1 g2 ->  
  gc_graph_iso g1 roots1 g2 roots2.
```

And eventually,

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Theorem garbage_collect_iso: forall roots1 roots2 g1 g2,  
  ...  
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The graphs are **isomorphic**

up to the vertices reachable from roots

The space between **n** and **l** is **available for alloc**

And eventually,

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Theorem garbage_collect_iso: forall roots1 roots2 g1 g2,  
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Note that we may still not achieve full isomorphism:
the **graph label** changes to accommodate new vertices
and may even grow to accommodate new generations

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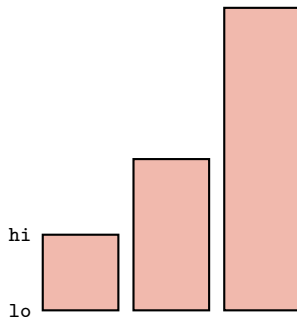
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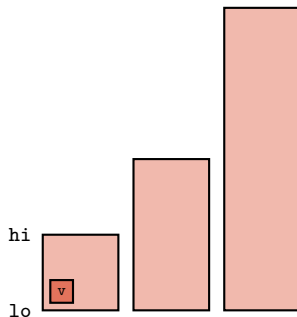
Double-bounded pointer comparisons:

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int Is_from(value * lo, value * hi, value * v) {  
    return (lo <= v && v < hi); }
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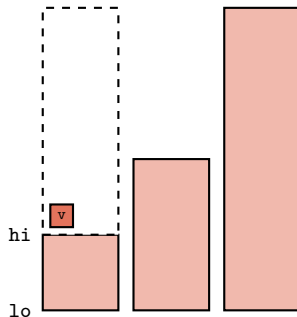
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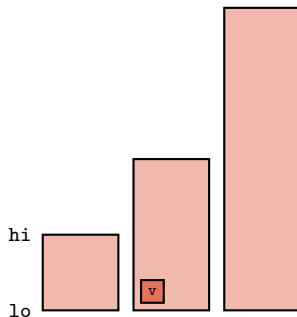
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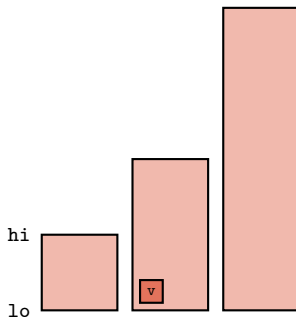
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Resolved using CompCert's `extcall_properties`

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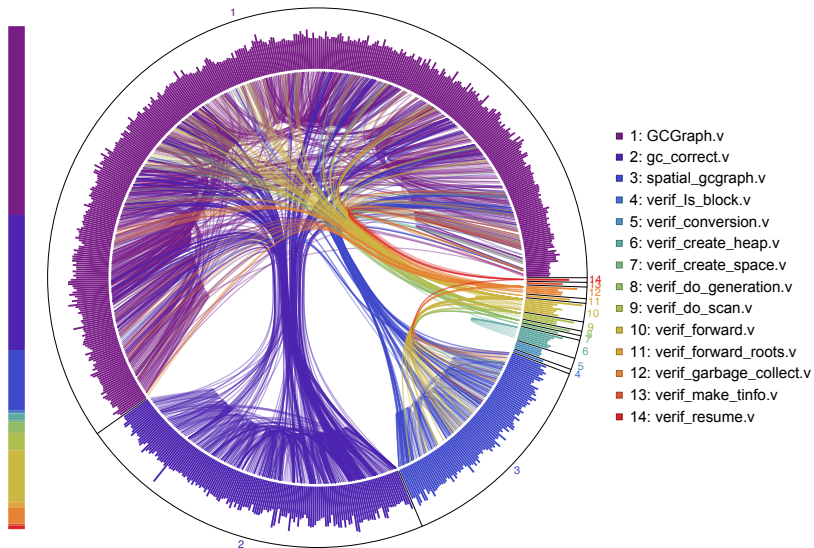
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Discussing char alignment issues with CompCert

Reusability: separation between pure and spatial reasoning



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Further refinements required in C semantics
before we can **specify** and **verify** OCaml's GC?