Probabilistic Analysis of Structure

Authors

Delos-Haoran Liang, Atom-Yatong Wu, Rick-Shoupei Wang

SUMMARY

This report presents a numerical algorithm for a full probabilistic analysis of a container, using Monte Carlo (MC) method. The proposed MC numerical procedure was used to compute the uncertainties in properties of the steel (e.g., the maximum bending moment the structure could sustain), and to derive the statistics of the dynamic response in the presence of uncertainty in the wind loading. The displacement method was utilized to obtain the internal forces of the container. This report also presents the computation of the structural fragility curves of the container under extreme winds.

The results of this report provide some cases about the expected losses associated with the failure of the failure of the container and related preventive measures to reduce the losses.

Keywords: Displacement method, Monte Carlo method, fragility analysis, material science.

1. INTRODUCTION

The purpose of structural calculation is to ensure that the designed structure and structural components can meet the expected safety and usability requirements during construction and work. In China, with the development of engineering technology, the design method of building structures has also begun to shift from the long-term fixed value method to the probabilistic design method. In the research process of probabilistic design, the uncertainty of load and material strength will be considered preferentially, and the probability methods are utilized to determine their values. In fact, this is still not an ideal full probability design method, and the full probability analysis involved in this project paves the way for Chinese students preparing to enter the North American structural design industry.

Notations	
N_{MC}	number of sampling points of Monte Carlo simulations
B	width of the structure
z	building height
M_e	maximum bending moment the structure could sustain
T	pre-selected threshold
M_{max}	maximum bending moment the wind loads act on the structure
q	unit load

2. BACKGROUND

Figure 1. Container

We find this kind of containers in our daily life. In foreign countries, this kind of container is designed in various shapes, but in China, it is mainly used in temporary housing on construction sites.

It is worth studying whether such buildings are safe and stable in daily life in windy weather. Therefore, we chose the most common container on the construction site as the research object for structural analysis and research.

3. STRUCTURAL ANALYSIS *by Atom-Yatong Wu*

We first get the structure loaded with vertical load in **Figure 2**.

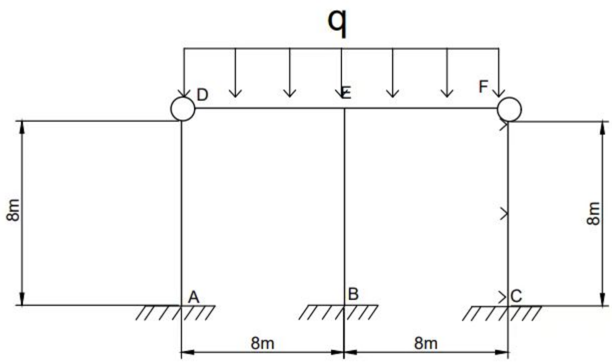
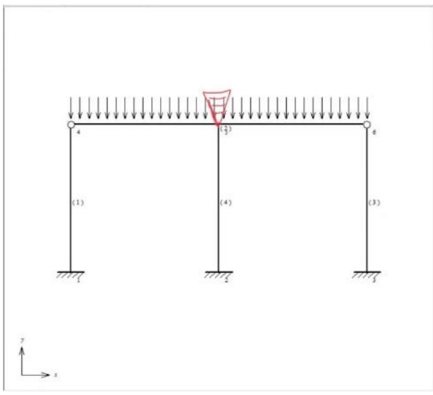


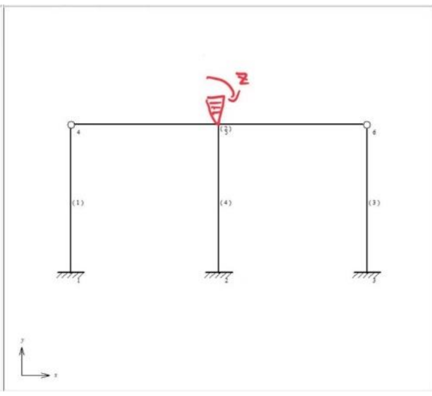
Figure 2. Vertical load

We assumed the element DF is loaded by a uniform load q and the length is L; node A B C E are the rigid joints; node D F are the hinged joints respectively; The structure has 3 superfluous constraints.

Using the **displacement method** to calculate it.

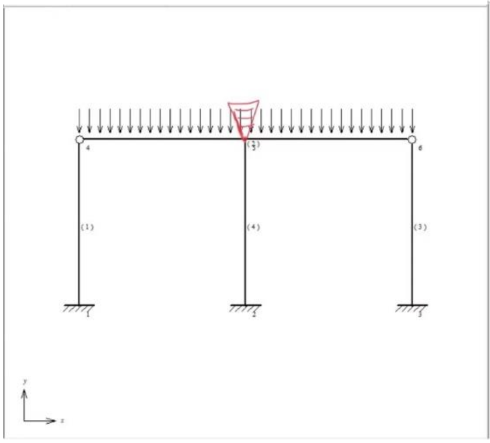
1. Fix the structure and then break it down

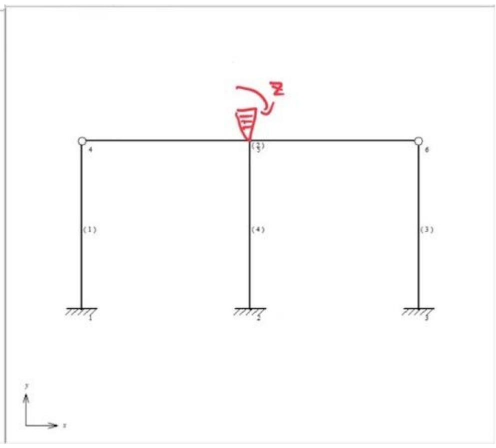




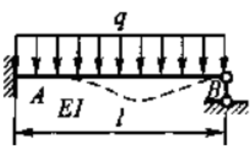
We should assume the angle is 1

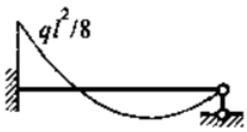
2. Analyze the node E

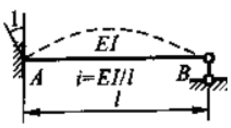


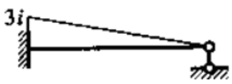


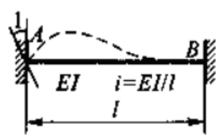
We can look up in the table:

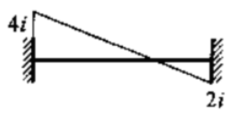




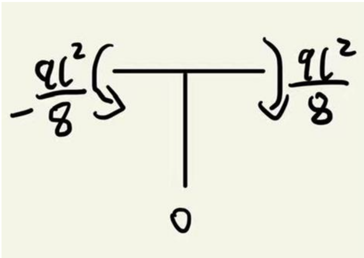


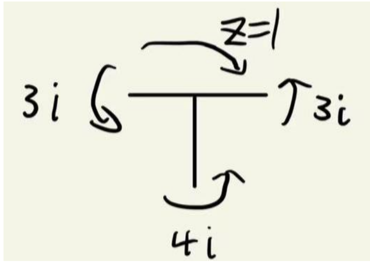






We can get:





3. Build the equation of displacement method

$$\sum R_{ij} \cdot \triangle_j + F_{iP} = 0 \tag{3.1}$$

Then we get:

$$R_{11} = -\frac{ql^2}{8} + \frac{ql^2}{8} = 0 \tag{3.2}$$

$$r_{11} = 3i + 3i + 4i = 10i \tag{3.3}$$

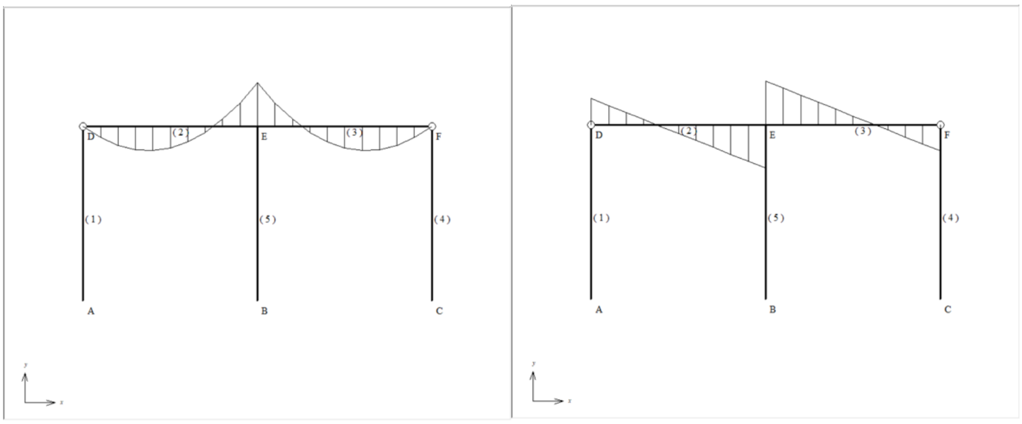
$$R_{11} - r_{11} \cdot \triangle = 0 \tag{3.4}$$

$$\triangle = 0 \tag{3.5}$$

4. Calculate the bending moment of the left side of node E

$$M_c = -\frac{ql^2}{8} + 3i\,\triangle = -fracql^2 8 \tag{3.6}$$

5. Draw the final internal force diagrams



(a) Bending moment diagram (b) Shear force diagram

Figure 3. Internal force diagrams

6. Draw the internal force diagrams when the structure is affected by a left wind

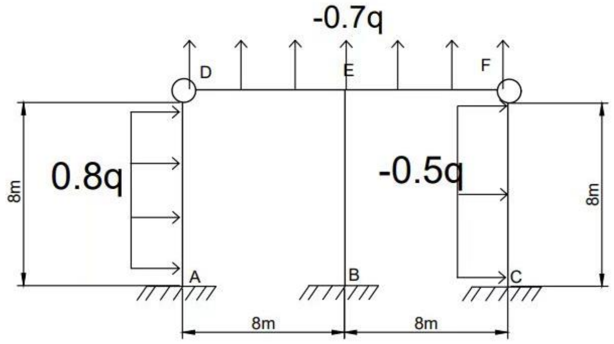
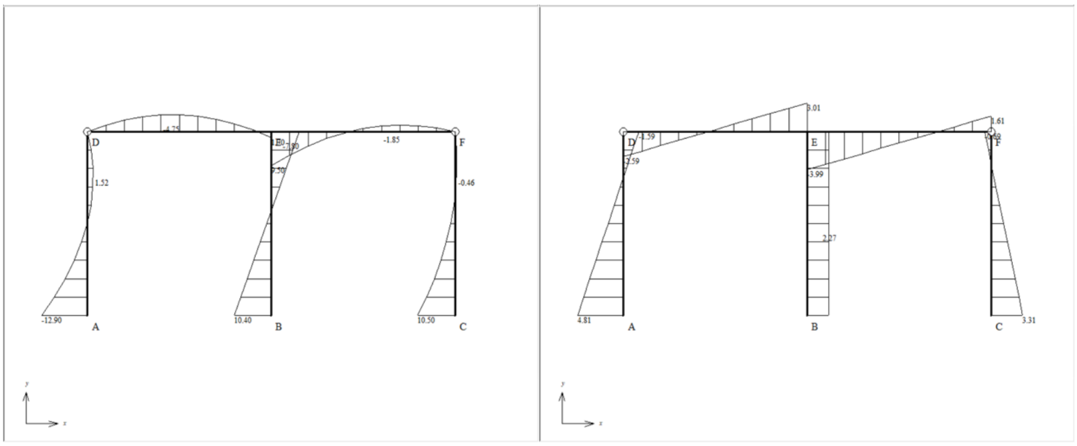


Figure 4. Left wind load

We can get:



(a) Bending moment diagram (b) Shear force diagram

Figure 5. Internal force diagrams

4. PROBABILISTIC ANALYSIS *by Delos-Haoran Liang*

In this section, we combined what we learned from *Probability Concepts in Engineering* and *Numerical analysis*, and we identified the uncertainties both in the capacity of our structure and variability in the load. Then we succeeded in obtaining multiple fragility curves and the probability of the structure through **Monte Carlo method**.

4. 1 Uncertainties in capacity

Firstly, we select **hot rolled ordinary I-beam** of Q235 steel, this kind of I-beam is used for a column with a hinged upper end and a fixed lower end, and the length of the column is 8m. The shape of the I-beam is shown in **Figure 6**, and the standard and the strength design value are shown in **Table 1** and **Table 2** respectively.

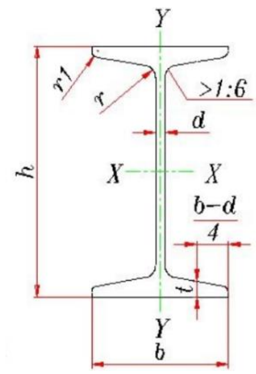


Figure 6. Shape of I-beam

Table 1

Standard of I-beam

Size(mm)					cs-area	x-x axis	
h	b	d	t	R	(cm^2)	W_X(cm^3)	I_X/S_X(cm)
200	100	7.0	11.4	9.0	35.5	237	17.2

Table 2

Strength design value

Steel		Bending stiffness f	Shear stiffness f_v
Type	Diameter(mm)	(N/mm^2)	(N/mm^2)
Q235 steel	$\leq$ 16	215	125

According to our preliminary calculation, the maximum shear force loading on the structure is far smaller than the design value of shear force, so we only took the bending failure into consideration. And we didn't consider the destruction of the boards, thus, we assumed the loading on boards would transmit to the columns on both sides uniformly.

The formula for calculating **standard value of maximum bending moment** is as follows [1]:

$$M_e = \gamma_x W_{nx} f \tag{4.1.1}$$

Where:

γ_x——Sectional Plasticity Development Coefficient

W_{nx}——Section modulus with respect to the x-axis

f——Design value of bending stiffness of steel

M_e obeys **Gamma Distribution** [2], the related parameters are shown in **Table 3**.

Table 3

Parameters of Gamma Distribution

random variable	mean	coefficient of variation [3]
M_e(kN·m)	53.503	0.016

We illustrate the histogram of Monte Carlo simulations results in **Figure 7**.

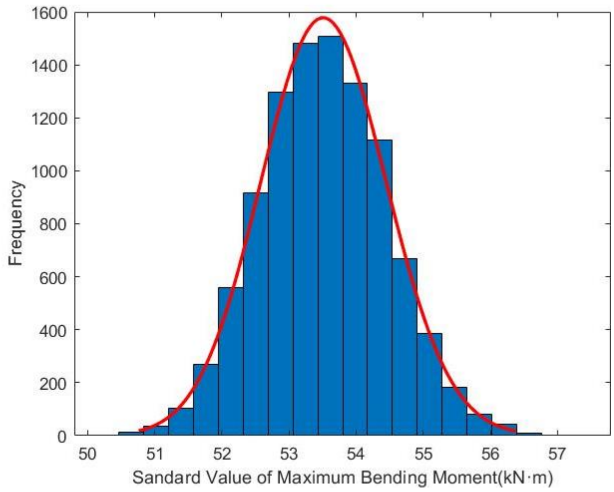


Figure 7. Histogram of frequency when N_{MC} = 10,000

4.2 Uncertainties in loads

According to *Load Code for Building Structure* [4], the statistical samples of snow pressure and wind velocity obey the **Extreme Value Type I Distribution**.

The distribution function of the snow pressure and wind velocity is as follows [4]:

$$F(x) = e^{-e^{-a(x-u)}} \tag{4.2.1}$$

$$\alpha = \frac{1.28255}{\sigma} \tag{4.2.2}$$

$$u = \mu - \frac{0.57722}{\alpha} \tag{4.2.3}$$

Where:

x——a sample of the maximum snow pressure in a year or the maximum wind speed in a year
u——the location parameter of the distribution, i.e. the mode of the distribution
α——the scale parameter of the distribution
σ——standard deviation of sample
μ——mean of sample

To simplify our analysis, we ignore the loads caused by snow pressure and the dead weight of the structure, we will only focus on the wind loads. Furthermore, According to *GB50009-2012*, the wind load is calculated by basic wind pressure w_0 which obeys a relationship with basic wind velocity v_0:

$$w_0 = \frac{1}{2}\rho v_0^2 \tag{4.2.4}$$

Where:

v_0——basic wind velocity
ρ——air density(t/m^3)

GB50009-2012 only provides the data of basic wind pressure, so we assumed that basic wind pressure also obeys **Extreme Value Type I Distribution**, and we set the largest basic wind pressure in Peking during the past 50 years as the mean value [4], the related parameters are shown in **Table 4**.

Table 4

Parameters of Extreme Value Type I Distribution

random variable	mean	coefficient of variation
w_0(kN/m^2)	0.5	0.01

The equivalent uniform load on the structure is shown in **Figure 8**.

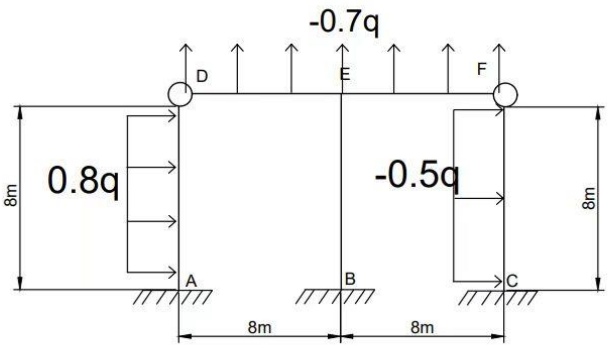


Figure 8. Equivalent uniform load on the structure

The formula for calculating wind load is as follows [4]:

$$w_k = b_z u_s u_z w_0 \tag{4.2.5}$$

Where:

w_k——standard value of wind load(kN/m^2)

b_z——coefficient of wind vibration at height z

u_s——form coefficient of wind load

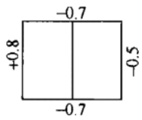
u_z——variation coefficient of wind pressure altitude

w_0——basic wind pressure(kN/m^2)

Given windward width of the structure B = 8m, we obtain the specific values of the structure [4], the values are shown in **Table 5**.

Table 5

Values of coefficient

coefficient	b_z	u_s	u_z
value	2.04		0.65

We illustrate the histogram of Monte Carlo simulations results in **Figure 9**.

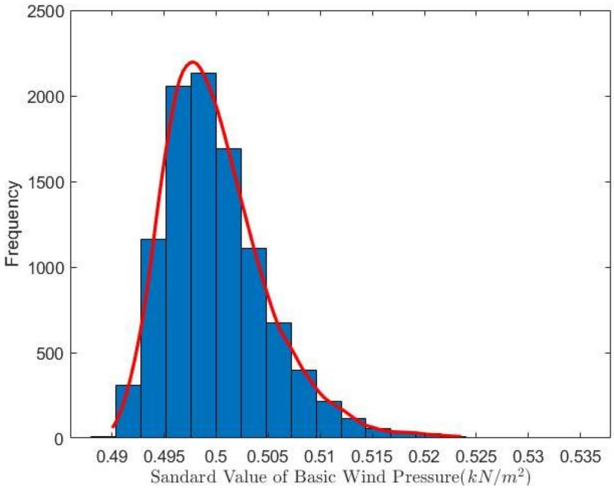


Figure 9. Histogram of frequency when N_{MC} = 10,000

4.3 Fragility analysis

In **Figure 10** the dimensional displacement thresholds used in this study (T1 to T5) were defined as a function of the building width, variable between 0.012%B and 0.028%B (B = 8m).

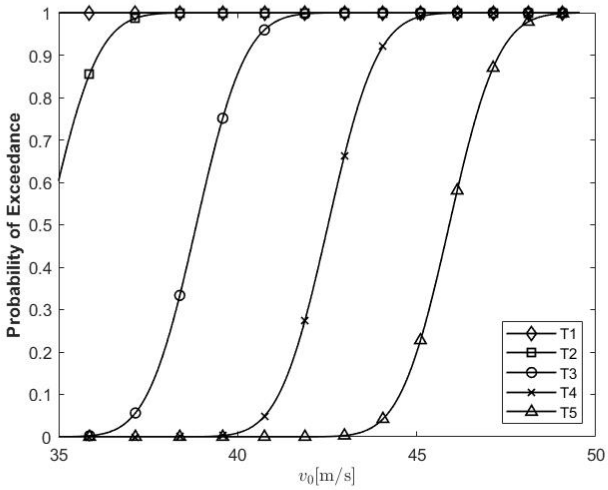


Figure 10. Example of structural fragility curves for uncertain distribution of M_e as a function of the increased velocity.

These thresholds, listed in **Table 6**, were selected as a first attempt at defining appropriate and realistic limit states (e.g., Maximum lateral drift criterion) which were provided with much more physical meaning and can be measured much more easily in specific situations [2].

Table 6

Thresholds for use in the fragility analysis of the structure.

Label	Threshold level (mm)
T1	0.012%B = 0.96
T2	0.016%B = 1.30
T3	0.020%B = 1.60
T4	0.024%B = 1.90
T5	0.028%B = 2.20

4.3.1 Probability of failure

In order to obtain the probability of failure, we utilized **unit load method** to build a connection between the outcomes of structural analysis and the M_{max} that we need in this section. As we can see from **Figure 11**, node A is most likely to fail, and we could extract the important coefficient from the bending moment diagram.

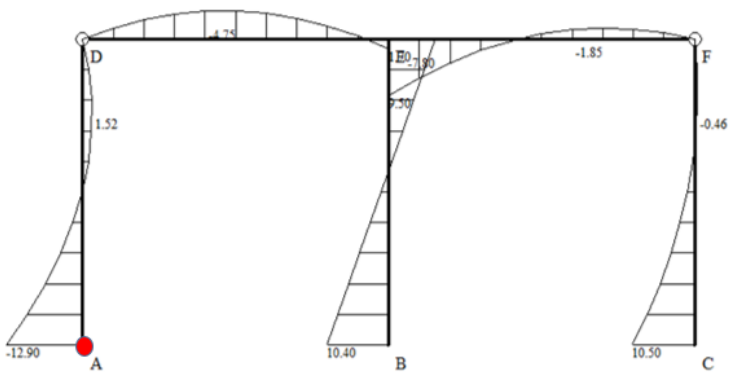


Figure 11. Bending moment diagram (unit q kN·m) ● point of bending failure

Then we build the connection through equations below:

$$M_{max} = 12.9 \cdot q(kN \cdot)m \tag{4.3.1.1}$$

$$q = \frac{1}{2} \cdot w_k \cdot B(kN/m^2) \tag{4.3.1.2}$$

Then we compared the MCS results of M_{max} and M_e through MATLAB matrix operation, at last, we obtained the probability of failure, the flow chart of the calculation is shown in **Figure 12**.

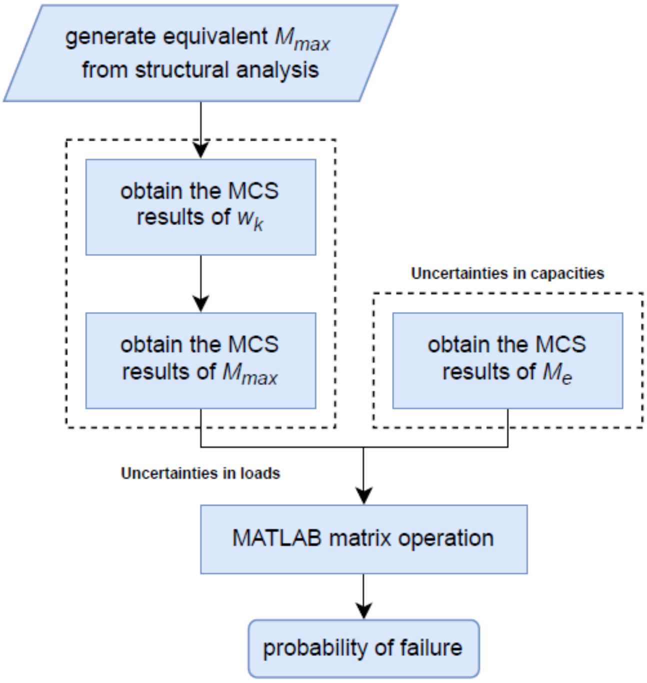
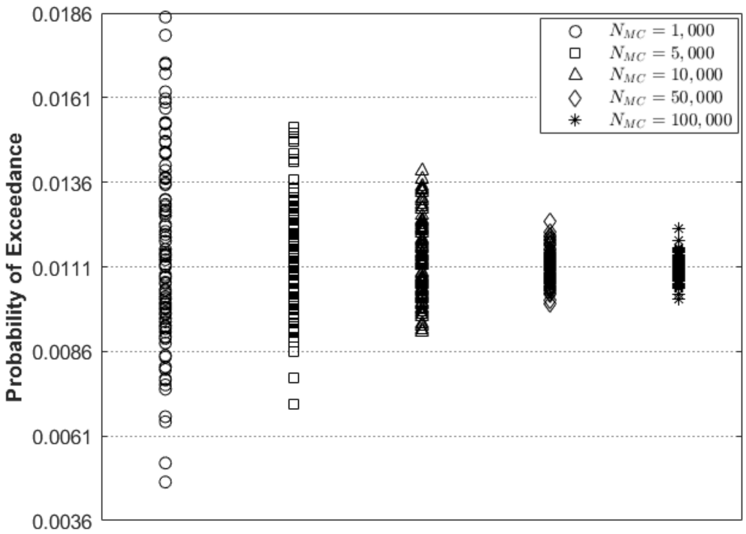


Figure 12. Flow chart of the calculation of the P(Failure)

In **Figure 13(a)**, we obtain the probability of failure, computed by the Monte Carlo method at B = 12m, we extend the width for the calculation due to the reason that when B = 8m, we need to run millions of times of sampling to make the coefficient of variation to be low enough to meet the requirement. As expected, with the increase of the sample sizes, there is a trend of decrement of coefficient of variation (CoV). The maximum absolute error is variable between 10% for N_{MC} = 10,000 and 3% for N_{MC} = 100,000. The CoV, computed for each sample population and different N_{MC}, was utilized as a countable indicator for the choice of the "best" N_{MC} value [2] (**Figure 13(b)**).

a



b

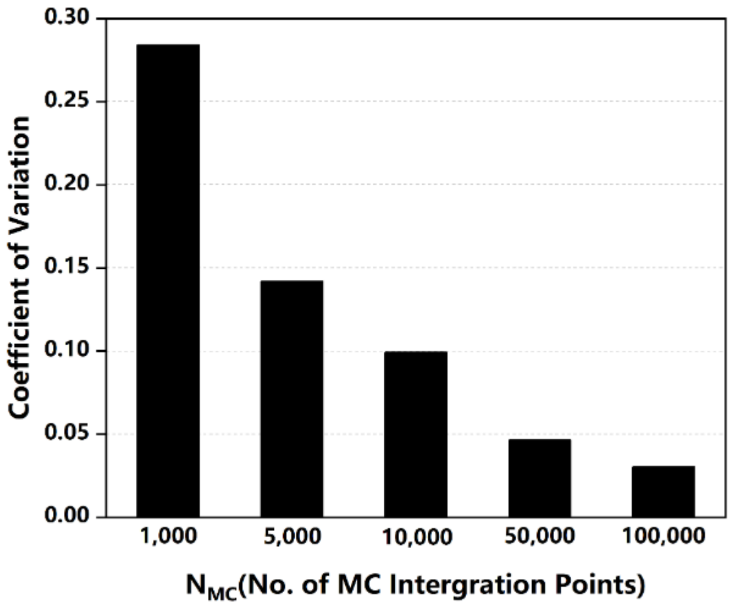


Figure 13. Probability of failure of the structure at B = 12m, computed by MC method – (a) scatter plot of 100 consecutive MC simulations as a function of N_{MC}, μ = 0.0111 when N_{MC} = 100,000; (b) CoV vs. N_{MC}.

Table 7 concludes the influences of N_{MC} on the run time of MC algorithm, all the value were normalized according to the run time when N_{MC} = 100,000 [2].

Table 7
MC validation: run time vs. N_{MC}.

N_{MC}	Average relative run time [a]
1,000	0.0097
5,000	0.0483
10,000	0.0965
50,000	0.4961
100,000	1.0000

[a] Run time relative to N_{MC} = 100,000.

4.3.2 Suggestion for design value

As we can see from (4.3.1.2), without additional support, the unit lode q is proportional to the width B, so with the increase of B, the probability of failure rises respectively, which could be seen in **Figure 14**, the P(Failure) = 1E-5 when B = 11.5m, which is acceptable, but it increases sharply to 0.0111 when we extend the width to 12m.

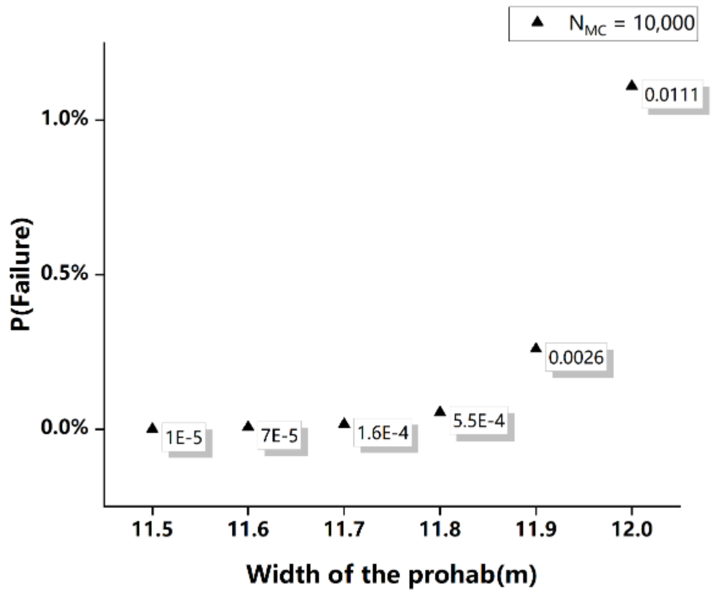


Figure 14. Probability of failure as a function of the increased width of the container when N_{MC} = 10,000.

If the maximum tolerance of the P(Failure) = 0.01%, then the theoretical threshold of the design value of the width will be 11.6m(**Figure 14**).

However, in fact, the realistic design value of the width of the type of the container that we selected is usually between 8m and 10m, due to the unpredictable uncertainties caused by manual manipulation especially when installing, so setting up the design value a bit less than the theoretical threshold can guarantee the safety and stability of the structure to the utmost extent.

5. ADDITIONAL INSIGHT *by Rick-Shoupei Wang*

This part focus on the expected losses in the construction of the structure to give a general way to reduce the losses.

Building loss is an important problem in building construction which could influence the budget of the whole project and the life of the building, and reasonable estimation of loss could effectively prolong the lifespan of building and save energy to achieve sustainability [5]. Expected loss is a part of building loss which is considered before the completion of whole project [6]. pointed that the loss of construction refers to the loss of value caused by the diminishing utility of buildings due to various reasons during the using of buildings. Expected losses could give a guidance to engineers and workers to consider all kinds of factors to avoid damage in the future and make sure the safety of the construction. There are two main parts involved in expected loss, material loss and invisible waste, which involve different factors like energy, economy, environment, society. Material loss mainly involves energy loss (electricity, internal energy, fuel), material loss (concrete, steel), structure damage (earthquake, typhon, deluge). Thus, material loss mainly focuses on those things which could be estimated and calculated and means it is easier to control. For invisible waste, it mainly involves economic loss and space waste. Economic loss refers that there could be some economic exposure in every project which is influenced by many factors like weather, population development, national policy and so on, making it more difficult to be measured. Space waste refers that it could be some space can't be used such as the economic factors. In summary, expected loss is an important part of project design which is influenced by many factors.

5.1 Expected loss in the structure

The structure in this group project is steel structure, which is like simple plant, meaning the expected loss in the structure focus on the steel. For material factors, dimensional error could be a problem in the preparation of material, which could increase the use of steel and cause waste. Corrosion of steel could the most important problem while considering steel structure which is the most common risk in many projects. And for economic factors, changes in material prices and reduced utilization of space could be the expected loss in the structure. To quantify the loss of structure, *Gao* and *Zhang* pointed the calculation method of loss of building [7].

$$Coefficient\ of\ loss = \frac{1 - Residual\ value\ rate\ of\ buildings}{Designed\ service\ life\ of\ building} \times Service\ life\ of\ building$$

$$Loss\ of\ building = Coefficient\ of\ loss \times Replacement\ price\ of\ buildings$$

Both equations give a method to normalize the amount of loss in buildings.

5.2 Measures to reduce the losses

Protection of steel is quite fundamental and necessary in construction and maintenance of containers of steel structure, and we will introduce to methods.

5.2.1 Anti-rust paint

This part only considers the material loss in the structure because the inviable waste is too difficult to be measured as mentioned before. To solve the corrosion of steel, anti-rust paint is the most common method in engineering. *Ye* developed a material called TN-922G specially developed for long-term exposed steel is like our project [8]. The material is composed of inorganic cementitious materials and non-toxic polymer additives, and has the advantages of non-toxic, tasteless, non-combustible, non-volatile, corrosion-resistant, strong sealing, simple operation and no special maintenance. It is mixed with $425^{\#}$ or $525^{\#}$ "ordinary Portland cement" in the weight ratio of $1{:}1.5 - 1{:}2$, and then evenly applied to the surface of the reinforcement to form a rust proof coating.

The performance and technical indexes of the coating material are as follows:

1. Appearance: brown transparent viscous liquid
2. Potential of hydrogen: 7-8
3. Viscosity $(20^{\circ}C) \geq 0.2Pa \cdot S$
4. Breathability$(0.3Mpa, 20\text{min})$
5. Shearing Strength$(14h) \geq 3.3Mpa$

6. Tensile Strength$\geq 1.5Mpa$
7. Water permeability$(0.3Mpa, 30\text{min})$
8. Corrosion Resistance: $Alkali\ proof(40\%\ NaOH, 168h), acid\ proof(36\%H_2SO_4, 168h),$ $Salt\ tolerance(3\%NaCl, 168h)$

The construction procedure is shown in **Figure 15**.

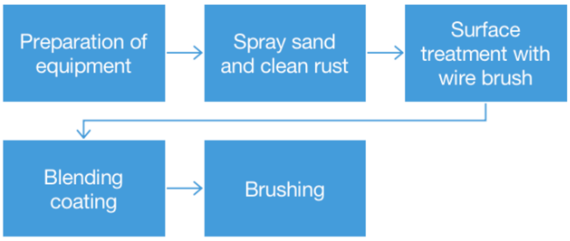


Figure 15. Workflow of the whole construction

5.2.2 Measure to decrease the consumption of steel

Engineering management could be the solutions of the consumption of steel. *Bi and Ye* developed a refined management to reduce the loss of steel [8]. Firstly, build backstage management system which uses computer science to record the use of steel in every time slot and calculate the changes in different part of the project to control the consumption of steel. In this way, the consumption of steel could be controlled on the computer to give more clear charts expressing the changes. Secondly, build using scattered material management system which means the material should be charged by different departments to avoid abuse of power. Finally, build accountability system which is like the first system but should be more detailed.

Conclusion

This report summarizes all the contents aiming to present a full probabilistic analysis of the container. A typical Monte Carlo method was proposed for driving the probability of failure and fragility curves, accounting for uncertainties both in capacities and loads. And the displacement method was utilized to analyze the internal forces of the container, and the results of it were quite helpful for probabilistic analysis. At last, this report provides additional insight concerning the expected losses caused by the failure of the container and the preventive measures to reduce the losses.

Strength and Weakness

Strength

1. Visualization: To the best of our knowledge, we utilize MATLAB and Origin to perform our results, which are generally thought to be precise and intuitive.

2. Simpleness & Effectiveness: We build a simple and effective methodology without considering all the details of the structural and probabilistic analysis.

Weakness

Missing the consideration of the combination of the influences by all possible loads.

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Appendices

Appendix A Core Codes for Calculation of P(Failure)

```
%% function of calculation of P(Failure)
function y = pFs(x,L,no)
% no: number of Monte Carlo simulations (sample size) ; L: width of the structure
mu1 = 53.503;    % E(x) = 53.503kN·m  μ  of standard value of bending moment
cv1 = 0.016;    % CoV = 1.6%
sigma1 = mu1 * cv1;    %  μ = E(x)·CoV
a1 = mu1^2/sigma1;    % parameter a
b = sigma1/mu1;    % parameter b
u1 = unifrnd(0,1,no,round(a1));
X = log(u1);
Y = X';
Z = sum(Y(:,1:no));
A = - (b)*Z;
B = A';    % Gamma Distribution
mu2 = 2.0962*L*(1+0.8902*sqrt(L+50*exp(-L/50)-50));    % E(x) = 0.5 kN/m2,
% ? of standard value of basic wind pressure
cv2 = 0.01;    % CoV = 1.0%
sigma2 = mu2 * cv2;    %  μ = E(x)·CoV
gamma = 0.5772156649;    % Euler's constant  γ  = 0.5772156649
a2 = pi/sqrt(6)*1/sigma2;    % parameter a
u2 = mu2 - gamma/a2;    % parameter u
v = unifrnd(0,1,no,1);
F = u2 - (1/a2).*(log(log(1./v)));    % Gumbel Distribution
m = (B - F);
n = sum(m<=0);
pFs = n/no;    % P(Failure)
y = pFs
```