Homework Set 4

Problem 1. A random variable X has the mean $\mu = 100$ and standard deviation $\sigma = 30$.

(a) Compute the probabilities for the events $\{X > 130\}$ and $\{X > 160\}$ and list them for the following distributions: uniform, gamma, normal, lognormal.

(b) The probability of which event is more sensitive to the choice of the distribution? Try to explain why this is the case.

Problem 2. The seismic fragility of a building, denoted $g(x)$, is defined as the conditional probability of failure of the building for a given peak ground acceleration x, i.e.,

$$g(x) = P(\text{Failure} \mid \text{peak ground acceleration} = x)$$

For a particular class of buildings, the fragility function is given by

$$g(x) = \begin{cases} 0 & x < 0.1 \\ 2.5(x - 0.1) & 0.1 < x < 0.5 \\ 1 & 0.5 < x \end{cases}$$

where x is measured in units of gravity acceleration. Suppose the peak ground acceleration of an earthquake has the exponential distribution with mean 0.05 units of gravity acceleration. determine:

(a) The probability of failure of the building during an earthquake.

(b) If the building is known to have failed, what is the probability density function of the peak ground acceleration of the earthquake that caused the failure.

Note: you may use the integral $\int x \exp(-ax)\,dx = -\left(\frac{1+ax}{a^2}\right)\exp(-ax)$.

Problem 3. Cracks in the weld of a structural member have random lengths A with the PDF

$$f_A(a) = \lambda \exp(-\lambda a) \quad a \geq 0$$

where $\lambda = 10\,mm^{-1}$. An X-ray device is used to detect the welds. The probability that a crack will be detected depends on its length and is given by

$$\begin{aligned} P(\text{crack will be detected} \mid A = a) &= 25a^2 \quad 0 \leq a \leq 0.2\,mm \\ &= 1 \quad\quad\; a \geq 0.2\,mm \end{aligned}$$

(a) Determine the PDF of the length of a crack that has been detected.

(b) Determine the PDF of the length of a crack that has escaped detection.

(c) Plot and compare the above two PDF's together with the PDF of the crack length before detection.

You may use the relation

$$\int x^2 \exp(ax)\,dx = \frac{\exp(ax)}{a^3}\left(a^2x^2 - 2ax + 2\right)$$