2023秋高数下期末试题 (回忆版)

1. Determine whether the following limits exist:

(a) $\lim\limits_{(x,y)\to(1,2)} \frac{\ln(1-x+xy)}{x-1}$

(b) $\lim\limits_{(x,y)\to(0,0)} (x+y)^{x+y+xy}$

2. Let f be the function defined by $f(x,y)=\begin{cases} \frac{2x+1}{x^2+y^2}\sin(x^2+y^2) & \text{if } (x,y)\neq(0,0) \\ 1 & \text{if } (x,y)=(0,0) \end{cases}$

Determine if the function f is continuous at the origin.

3. Let n be the normal unit vector pointing inside the surface $3x^2+y^2+z^2=3$. Compute the directional derivative of the function

$$f(x,y,z)=\frac{\sqrt{x^2+y^2+z^2}}{(y+z+1)^2}$$

at the point $(1,0,0)$ in the direction n.

4. Find the equations of the tangent plane and the normal line for the surface

$$xy+z+2^{xy}=4$$

at the point $(1,1,1)$.

5. Use the Lagrange multipliers to find the minimal and maximal values of the function

$$f(x,y,z)=x^{\frac{5}{2}}+y^{\frac{5}{2}}+z^{\frac{5}{2}}$$

on the sphere $x^2+y^2+z^2=1$

6. Compute the integral $I=\iint_D x^2y^2\,dxdy$, where D is the plane domain bounded by the curves $x=\frac{1}{2}$ and $y^2=2x$.

7. Determine the area of the region bounded by the curves $r=\sin\theta$ and $r=\cos\theta$.

8. Find the volume of the solid bounded by the surfaces S_1 and S_2,

$$S_1:=\{(x,y,z): x^2+y^2+4z^2=9,\ z\geq 0\},$$

$$S_2:=\{(x,y,z): z=\sqrt{x^2+y^2}\}.$$

9. Determine the work done by the vector field

$$F=e^{y+2z}(i+xj+2xk)$$

along the curve of the intersection of the surfaces $x^2+2y^2+3z^2=3$ and $x+y+z=0$ joining the points $A(1,-1,0)$ and $B(-1,1,0)$.

10. Calculate the circulation of the vector field

$$F=yz^2i+2xz^2j+xyz\,k$$

along the curve of intersection of the sphere $x^2+y^2+z^2=1$ and the cone $z=\sqrt{x^2+y^2}$ traversed in the counterclockwise direction around the z-axis when viewed from above.