2018春高数下期中试题2（回忆版）

1. Determine which of the following series converges absolutely, converges or diverges. Use any method, and give reasons for your answers.

(1) $\sum_{n=1}^{\infty} \frac{2^n+4^n}{3^n+4^n}$;

(2) $\sum_{n=1}^{\infty} \frac{1}{n(\ln n)^2}$;

(3) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$;

(4) $\sum_{n=1}^{\infty} \frac{n!(n+1)!(n+2)!}{(3n)!}$;

(5) $\sum_{n=1}^{\infty} (-1)^n(\sqrt{n^2+1}-n)$

2. (1) Find the radius and interval of convergence of the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{\sqrt{n^2+3}}$$

(2) For what values of x does the series converge absolutely, or conditionally?

3. Find the Maclaurin series of the function

$$f(x) = (x+1)e^x.$$

4. Use series to evaluate the limit

$$\lim_{x\to 0} \frac{\ln(1+x^2)}{1-\cos x}$$

5. Find the length of astroid

$$x=\cos^3 t, \quad y=\sin^3 t, \quad 0\le t\le 2\pi.$$

6. Find the area of the region bounded by the circle $r=2\sin\theta$ for $\frac{\pi}{4}\le\theta\le\frac{\pi}{2}$.

7. Find the first four terms of the binomial series for the function.

$$(1+x)^{\frac{1}{2}}.$$

8. Does the following sequence converge? If so, to what value?

$$x_1=1, \quad x_{n+1}=\frac{x_n}{2}+\frac{1}{x_n}.$$