淑芬精讲回忆版

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一、判断

1. 一阶导等于0，x0存在严格局部极小值当且仅当二阶导大于0　**（X）**
2. R^n 任意范数等价
3. x∈(0,1)，x^n不一致收敛到0
4. 等度连续则一致连续
5. a＞0，$\log_a^x$ 不是凸函数
6. 紧空间存在可数稠密子集
7. fn连续+逐点收敛不能推f连续
8. 偏导数存在不能推可微
9. 可逆线性算子是$L(R^n)$ 的开集
10. 还有一个不记得了？？

二、积分

$$\int_0^{\pi} sin^3xdx$$

$$\int_1^2 xlnxdx$$

三、

$$f \text{ 连续可微，} f(1)=0, \int_0^1 f^4(x)dx=1, \text{证明} \int_0^1 f^{'}(x)^2dx\int_0^1 x^2f^6(x)dx > \frac{1}{16}$$

解：

用Cauchy不等式即

$$\left(\int_a^b f(x)g(x)dx\right)^2 \leq \int_a^b f^2(x)dx \cdot \int_a^b g^2(x)dx.$$

放缩，接着用分部积分证明不等式

四、给出 f(x, y) 和 v 和x_0，求 $f'(x_0)(v)$　**要用矩阵，转为梯度算然后8分没了**

五、一致收敛和一致Cauchy等价证明，改为了sup$|f_n - f_m|$，本质一样

7.8 Theorem *The sequence of functions $\{f_n\}$, defined on E, converges uniformly on E if and only if for every $\varepsilon > 0$ there exists an integer N such that $m \geq N$, $n \geq N$, $x \in E$ implies*

$$|f_n(x) - f_m(x)| \leq \varepsilon. \tag{13}$$

Proof Suppose $\{f_n\}$ converges uniformly on E, and let f be the limit function. Then there is an integer N such that $n \geq N$, $x \in E$ implies

$$|f_n(x) - f(x)| \leq \frac{\varepsilon}{2},$$

so that

$$|f_n(x) - f_m(x)| \leq |f_n(x) - f(x)| + |f(x) - f_m(x)| \leq \varepsilon$$

if $n \geq N$, $m \geq N$, $x \in E$. Conversely, suppose the Cauchy condition holds. Then, the sequence $\{f_n(x)\}$ converges, for every x, to a limit which we may call $f(x)$. Thus the sequence $\{f_n\}$ converges on E, to f. We have to prove that the convergence is uniform.

Let $\varepsilon > 0$ be given, and choose N such that (13) holds. Fix n, and let $m \to \infty$ in (13). Since $f_m(x) \to f(x)$ as $m \to \infty$, this gives

$$|f_n(x) - f(x)| \leq \varepsilon \tag{14}$$

for every $n \geq N$ and every $x \in E$, which completes the proof.

六、证明，题干条件改为了f连续可微，集合凸有界闭，删去范数有界

9.19 Theorem *Suppose* $\mathbf{f}$ *maps a convex open set* $E \subset R^n$ *into* R^m, $\mathbf{f}$ *is differentiable in* E, *and there is a real number* M *such that*

$$\|\mathbf{f}'(\mathbf{x})\| \leq M$$

for every $\mathbf{x} \in E$. *Then*

$$|\mathbf{f}(\mathbf{b}) - \mathbf{f}(\mathbf{a})| \leq M|\mathbf{b} - \mathbf{a}| \qquad \textit{for all } \mathbf{a} \in E, \mathbf{b} \in E.$$

Proof Fix $\mathbf{a} \in E$, $\mathbf{b} \in E$. Define

$$\gamma(t) = (1 - t)\mathbf{a} + t\mathbf{b}$$

for all $t \in R^1$ such that $\gamma(t) \in E$. Since E is convex, $\gamma(t) \in E$ if $0 \leq t \leq 1$. Put

$$\mathbf{g}(t) = \mathbf{f}(\gamma(t)).$$

Then

$$\mathbf{g}'(t) = \mathbf{f}'(\gamma(t))\gamma'(t) = \mathbf{f}'(\gamma(t))(\mathbf{b} - \mathbf{a}),$$

so that

$$|\mathbf{g}'(t)| \leq \|\mathbf{f}'(\gamma(t))\| \, |\mathbf{b} - \mathbf{a}| \leq M|\mathbf{b} - \mathbf{a}|$$

for all $t \in [0, 1]$. By Theorem 5.19,

$$|\mathbf{g}(1) - \mathbf{g}(0)| \leq M|\mathbf{b} - \mathbf{a}|.$$

But $\mathbf{g}(0) = \mathbf{f}(\mathbf{a})$ and $\mathbf{g}(1) = \mathbf{f}(\mathbf{b})$. This completes the proof.

七、不动点定理证明 题干改为了d(Tx, Ty) < d(x, y)，

反证法，用课件方法证明然后11分没了，可自行查百度

9.23 Theorem *If X is a complete metric space, and if φ is a contraction of X into X, then there exists one and only one $x \in X$ such that $\varphi(x) = x$.*

Proof Pick $x_0 \in X$ arbitrarily, and define $\{x_n\}$ recursively, by setting

(44) $$x_{n+1} = \varphi(x_n) \qquad (n = 0, 1, 2, \ldots).$$

Choose $c < 1$ so that (43) holds. For $n \geq 1$ we then have

$$d(x_{n+1}, x_n) = d(\varphi(x_n), \varphi(x_{n-1})) \leq c\, d(x_n, x_{n-1}).$$

Hence induction gives

(45) $$d(x_{n+1}, x_n) \leq c^n\, d(x_1, x_0) \qquad (n = 0, 1, 2, \ldots).$$

If $n < m$, it follows that

$$\begin{aligned} d(x_n, x_m) &\leq \sum_{i=n+1}^{m} d(x_i, x_{i-1}) \\ &\leq (c^n + c^{n+1} + \cdots + c^{m-1})\, d(x_1, x_0) \\ &\leq [(1-c)^{-1}\, d(x_1, x_0)]c^n. \end{aligned}$$

Thus $\{x_n\}$ is a Cauchy sequence. Since X is complete, $\lim x_n = x$ for some $x \in X$.

Since φ is a contraction, φ is continuous (in fact, uniformly continuous) on X. Hence

$$\varphi(x) = \lim_{n \to \infty} \varphi(x_n) = \lim_{n \to \infty} x_{n+1} = x.$$