2018春高数下期中试题1（回忆版）

1. Determine whether the following statements are true or false? No justification is necessary.

(a) If both $f_x(x,y)$ and $f_y(x,y)$ exist at (x_0,y_0), then $f(x,y)$ is continuous at (x_0,y_0).

(b) Let $f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ (1)

(c) For the $f(x,y)$ as in (1), both $f_x(0,0)$ and $f_y(0,0)$ exist.

(d) Nonzero vectors u and v are parallel if and only if $u\times v=0$.

(e) The surface $y^2-x^2=z$ is a hyperbolic paraboloid.

(f) If $f(x,y)$ and its partial derivatives f_x, f_y, f_{xy}, and f_{yx} are defined throughout an open region containing a point (a,b), then $f_{xy}(a,b)=f_{yx}(a,b)$.

2. Suppose that the function $f(x,y)$ is differentiable, and $f(0,0)=1$, $f_x(0,0)=2$, $f_y(0,0)=3$. Then $f(x,y)\approx$ ______ when both x and y are small (using the standard linear approximation at $(0,0)$).

3. Find the distance from the point $(1,1,5)$ to the line

$$L:\ x=1+t,\ y=3-t,\ z=2t.$$

4. Find the length of the curve

$$r(t)=(\sqrt{2}t)i+(\sqrt{2}t)j+(1-t^2)k$$

from $(0,0,1)$ to $(\sqrt{2},\sqrt{2},0)$.

5. Find the normal vector and the curvature for the helix.

$$r(t)=(a\cos t)i+(a\sin t)j+(bt)k,\quad a,b\geqslant 0,\ a^2+b^2\neq 0.$$

6. Find $\lim\limits_{(x,y)\to(0,0)}\frac{x^2y}{x^4+y^2}$, if it exists; otherwise give the reason why the limit does not exist.

7. Find $\frac{\partial w}{\partial v}$ when $u=-1$, $v=2$, if $w=xy+\ln z$, $x=\frac{v^2}{u}$, $y=u+v$, $z=\cos u$.

8. Find the critial points of the function $f(x,y)=x^4+y^4+4xy$, and use the second derivative test to classify each point as one where a saddle, local maximum or local minimum occurs.

9. Find the point on the surface $z^2=xy+4$ closest to the origin.

10. Use Taylor's formula for $f(x,y)=xe^y$ at the origin to find quadratic and cubic approximations of f near the origin.