2019.

一、

1~3 F T F

二、

1~5 B D D C D.

三、(1) $\frac{1}{2}$ (2) $(0,1)$ (3) $\ln(\sec 1)$

四、(1) $\lim_{n\to\infty}\sqrt[n]{|a_n|} = (x-1)^2 < 1$

$x \in (0,2)$

① $x=0$ $a_n = \frac{(-1)^{n+1}}{\sqrt{n+9012}\ln n}$.

$|a_n|$ 单减.

且 $\frac{|a_n|}{\frac{1}{n\ln n}} = \infty$ $\sum \frac{1}{n\ln n}$ 发散

∴ 条件收敛.

② $x=2$ $a_n = \frac{(-1)^n}{\sqrt{n+9012}\ln n}$

同上

(2) 绝对收敛 $(0,2)$

条件收敛 $x=0$、$x=2$

五、(1) $\int_0^{2\pi}\int_0^{\frac{\sqrt{2}}{2}}\int_r^{\sqrt{1-r^2}} (r\cos\theta+z)\,r\,dz\,dr\,d\theta$

(2) $\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^1 (\rho\sin\varphi\cos\theta+\rho\cos\varphi)\rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$

六：$V = 8xyz$

$g(x,y,z) = \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} = 1$

$\nabla\vec{V} = \lambda\nabla\vec{g}$

$$\begin{cases} 8yz = \lambda\frac{2x}{a^2} \\ 8xz = \lambda\frac{2y}{b^2} \\ 8xy = \lambda\frac{2z}{c^2}\end{cases}$$

① $\lambda=0$ 不成立

② $\lambda\neq 0$

$\frac{x^2}{a^2} = \frac{y^2}{b^2} = \frac{z^2}{c^2} \Rightarrow x = \frac{a}{\sqrt{3}}$ $y=\frac{b}{\sqrt{3}}$ $z=\frac{c}{\sqrt{3}}$

∴ $V = \frac{8abc}{3\sqrt{3}}$

七：$y=x^2$ $x=1$

$\vec{r}(t) = (t, t^2)$

$\vec{v}(t) = (1, 2t)$

$|\vec{v}(t)| = \sqrt{4t^2+1}$

$\vec{T} = \left(\frac{1}{\sqrt{4t^2+1}}, \frac{2t}{\sqrt{4t^2+1}}\right)$

$\frac{d\vec{T}}{dt} = \left(-\frac{4t}{(1+4t^2)^{\frac{3}{2}}}, \frac{2}{(1+4t^2)^{\frac{3}{2}}}\right)$

$K(1) = \frac{2}{5\sqrt{5}}$ $r = \frac{5\sqrt{5}}{2}$

$(1,1) + \frac{5\sqrt{5}}{2}\cdot\frac{1}{\sqrt{5}}(-2,1) = (-4, \frac{7}{2})$

∴ $(x+4)^2+(y-\frac{7}{2})^2 = \frac{125}{4}$

八 $M = \int_0^2 \int_0^{\sqrt{x}} \int_0^{4-x^2} xy \, dz\,dy\,dx = \frac{32}{15}$

$M_{xy} = \frac{256}{105}$

$M_{yz} = \frac{8}{3}$

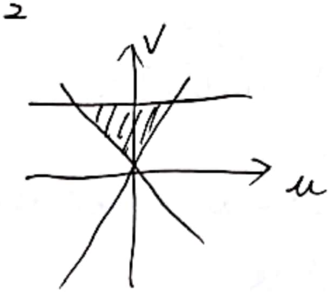
$M_{xz} = \frac{256\sqrt{2}}{231}$

$\therefore \bar{x} = \frac{M_{yz}}{M} = \frac{5}{4} \quad \bar{y} = \frac{M_{xz}}{M} = \frac{40\sqrt{2}}{77} \quad \bar{z} = \frac{M_{xy}}{M} = \frac{8}{7}$.

九 $u = y - x$ $\quad$ $J = \left\| \begin{matrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{matrix} \right\| = \frac{1}{2}$

$v = y + x$

$$\left.\begin{matrix} x=0 & v=u \\ y=0 & u=-v \\ x+y=2 & v=2 \end{matrix}\right\} \Rightarrow$$



$I = \int_0^2 \int_{-v}^{v} e^{\frac{u}{v}} \cdot \frac{1}{2} du\,dv = e - e^{-1}$

十、(1) $\frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}$; $\frac{\partial P}{\partial x} = \frac{\partial M}{\partial z}$; $\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$

$\therefore$ exact.

(2) $\frac{\partial f}{\partial x} = e^x \ln y$

$f = e^x \ln y + g(y, z)$

$\frac{\partial f}{\partial z} = y \cos z \quad \therefore f = e^x \ln y + y \sin z + h(y)$

$\frac{\partial f}{\partial y} = \frac{e^x}{y} + \sin z + \frac{\partial h}{\partial z} \quad \therefore \frac{\partial h}{\partial z} = C$

$\therefore f = e^x \ln y + y \sin z + C$

$f(1,3,\pi) - f(1,1,1) = e \ln 3 - \sin 1$

十一、$\nabla \cdot (4x\vec{j}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 4x & 0 \end{vmatrix} = 4\vec{k}$

$\therefore \iint_S \nabla \times (4x\vec{j}) \cdot \vec{n}\, d\sigma = \iint_S 4\vec{k} \cdot \vec{k}\, d\sigma = 4\iint_S d\sigma = 64\pi$

十二、$\nabla \cdot \vec{F} = 6 + 4y$ $\quad$ stocks' 定理.

$\iiint_V (6+4y) dV = \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} (6+4y)\, dz\,dy\,dx$

$= \int_0^{\frac{\pi}{2}} \int_0^1 \int_0^r (6 + 4r\sin\theta) r\, dz\,dr\,d\theta$

$= \pi + 1$

十三、 $\cos a_n - a_n = \cos b_n$

$\cos a_n - \cos b_n = a_n > 0$

$\Rightarrow 0 < a_n < b_n$

$\sum_{n=1}^{\infty} b_n$ 收敛 $\Rightarrow \lim_{n\to\infty} b_n = 0$

$0 \leq \lim_{n\to\infty} a_n \leq b_n$ $\therefore \lim_{n\to\infty} a_n = 0$.