Photogrammetric Computer Vision

Exercise 2

Winter semester 24/25

(Course materials for internal use only!)

Computer Vision in Engineering – Prof. Dr. Rodehorst
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Agenda

Topics

Assignment 1.	Points and lines in the plane, first steps in MATLAB / Octave
Assignment 2.	**Projective transformation (Homography)**
Assignment 3.	Camera calibration using direct linear transformation (DLT)
Assignment 4.	Orientation of an image pair
Assignment 5.	Projective and direct Euclidean reconstruction
Assignment 6.	Stereo image matching
Final Project	- will be announced later -

Agenda

	Start date		Deadline
Assignment 1.	~~21.10.24~~	~~–~~	~~03.11.24~~
Assignment 2.	**04.11.24**	**–**	**17.11.24**
Assignment 3.	18.11.24	–	01.12.24
Assignment 4.	02.12.24	–	15.12.24
Assignment 5.	16.12.24	–	12.01.25
Assignment 6.	13.01.25	–	26.01.25
Final Project.	27.01.25	–	16.03.25

Assignment 1 – sample solution

Part 1

1. You would like to compute the connecting line between two 2D points. What happens, if the two points are identical?

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ w \end{pmatrix}, \quad \mathbf{l} = \mathbf{x} \times \mathbf{x} = \begin{pmatrix} yw - wy \\ wx - xw \\ xy - yx \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad |\mathbf{l}| = 0$$

Not defined!

2. Where does the general line x cos φ + y sin φ = d intersect the line $(0, 0, 1)^T$ given in homogeneous coordinates? How can this point be interpreted?

$$\mathbf{l}_1 = \begin{pmatrix} \cos\phi \\ \sin\phi \\ -d \end{pmatrix}, \quad \mathbf{l}_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{x} = \mathbf{l}_1 \times \mathbf{l}_2 = \begin{pmatrix} \sin\phi \\ -\cos\phi \\ 0 \end{pmatrix}$$

Section at infinity in the direction l_1

3. Show that the horizon is a straight line by showing that three points on the horizon are always collinear.

$$\det\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 0 & 0 & 0 \end{pmatrix} = x_1 \cdot y_2 \cdot 0 - x_3 \cdot y_2 \cdot 0 + x_2 \cdot y_3 \cdot 0 - x_1 \cdot y_3 \cdot 0 + x_3 \cdot y_1 \cdot 0 - x_2 \cdot y_1 \cdot 0 = 0$$

Part 2

All objects in MATLAB are matrices. A matrix is created with `[1,2;3,4]`, where semicolons separate the rows. For a matrix multiplication use * and for a matrix `A` is `A'` the transpose. For the solution of this exercise the commands `cross`, `sin`, `cos`, `pi` and `inv` can be helpful.

1. The two points $\mathbf{x} = (2, 3)^T$ and $\mathbf{y} = (-4, 5)^T$ are given.
   a. Determine the connecting line $\mathbf{l}$ between the two points.
   b. *Move* $\mathbf{x}$ and $\mathbf{y}$ in the direction $\mathbf{t} = (6, -7)^T$,
      rotate afterwards using the angle $\varphi = 15°$ and finally
      scale with factor $\lambda = 8$.
   c. Accomplish the same operations with the line $\mathbf{l}$.
2. Examine whether the transformed points $\mathbf{x}'$ and $\mathbf{y}'$ are on the transformed line $\mathbf{l}'$.

Part 2

$$\mathbf{l}^\mathrm{T}\mathbf{x} = 0$$

$$\boxed{\mathbf{l}^\mathrm{T}\mathbf{H}^{-1}}\boxed{\mathbf{H}\mathbf{x}} = 0$$

$$\mathbf{l}^\mathrm{T}\mathbf{H}^{-1} = \mathbf{l}'^\mathrm{T} \qquad \mathbf{H}\mathbf{x} = \mathbf{x}'$$

$$\mathbf{H}^{-\mathrm{T}}\mathbf{l} = \mathbf{l}'$$

$$\mathbf{l}'^\mathrm{T}\mathbf{x}' = 0$$

```
function Exercise1                         % Planar similarity transformation
%        =========
x = [ 2; 3; 1];                            % Points x and y in homogeneous coordinates
y = [-4; 5; 1];
l = cross(x, y)                            % Joining line l using cross product

H = Scale(8) * Rot(15) * Trans(6,-7);      % Transformation concatenation
                          % 1. Translation, 2. Rotation, 3. Global scaling
x2 = H * x                                 % Apply transformation to points x and y
y2 = H * y
l2 = inv(H') * l;                          % Apply transformation to line l

x2' * l2                                   % Incidence test: scalar product 0?
y2' * l2

function T = Trans(x,y)
%        ==============
T = [ 1  0  x ;                            % Translation matrix
      0  1  y ;
      0  0  1 ];

function R = Rot(a)
%        ==========
phi = a * pi / 180;                        % Degree -> radiant
R = [ cos(phi) -sin(phi)  0 ;              % Rotation matrix
      sin(phi)  cos(phi)  0 ;
         0         0      1 ];

function S = Scale(s)
%        ============
S = [ s  0  0 ;                            % Global scaling matrix
      0  s  0 ;
      0  0  1 ];
```

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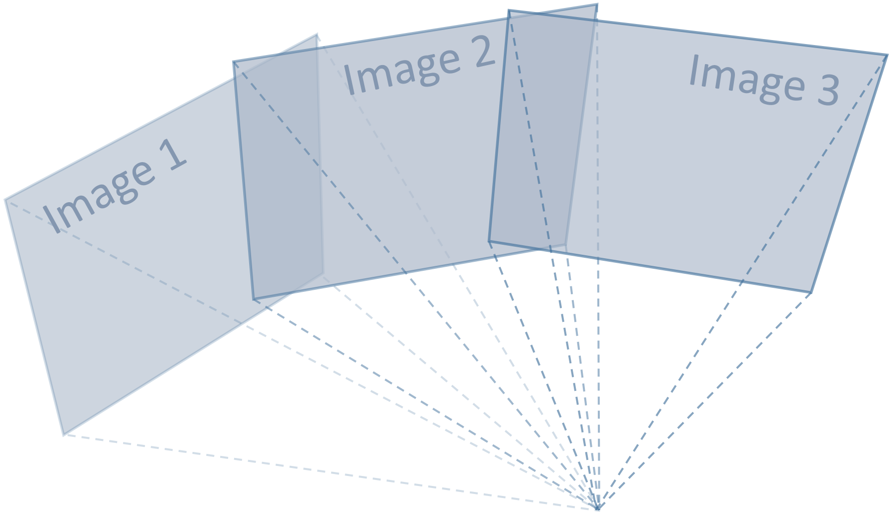
Assignment 2

Assignment 2: *Projective Transformation*

panoramic image mosaic

multiple overlapping images



Assignment 2: *Projective Transformation*

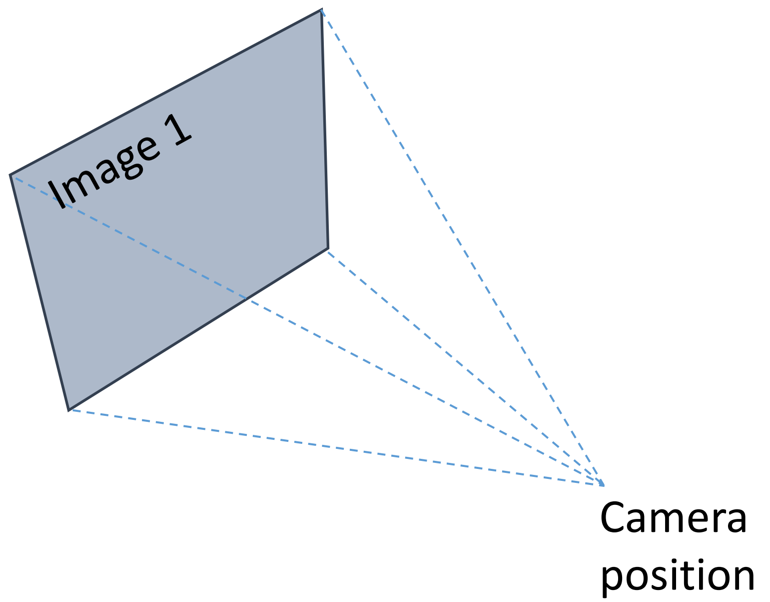
1) Image acquisition
   - 3 images
   - at least **30%** overlap

2) Correspondence analysis
   - **interactive** point selection

3) Homography computation
   - **H_{12}** (first to second image)
   - **H_{32}** (third image to intermediate mosaic)

4) Projective rectification
   - auxiliary program ***geokor***

5) Visualization



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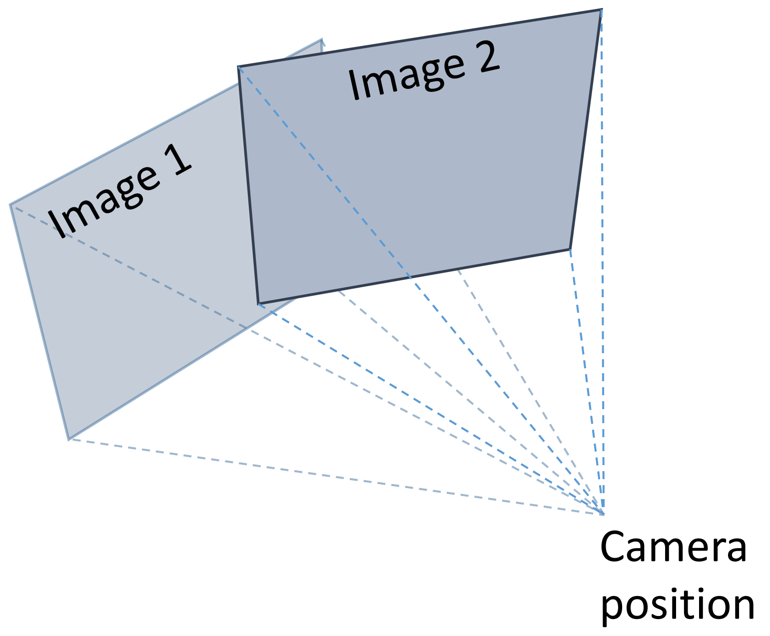

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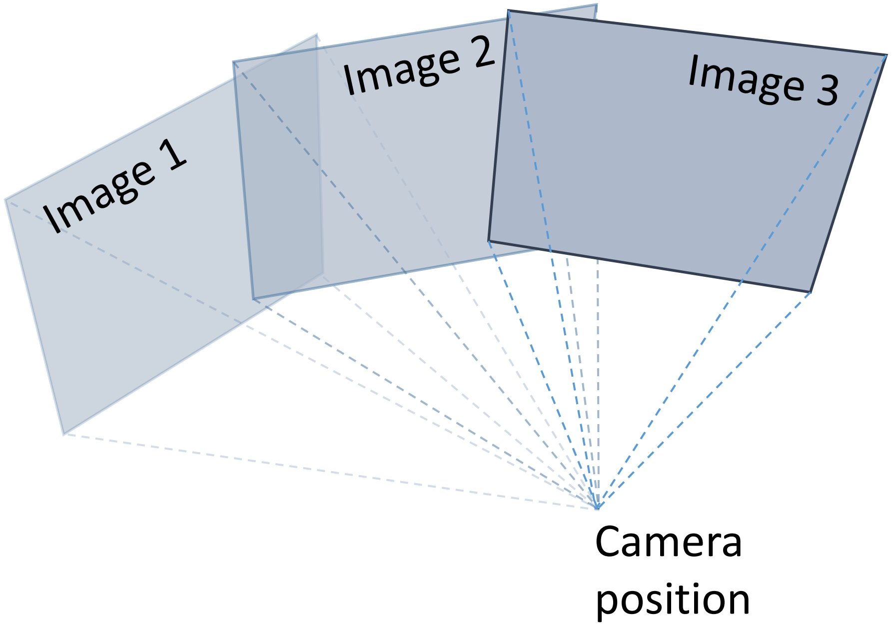

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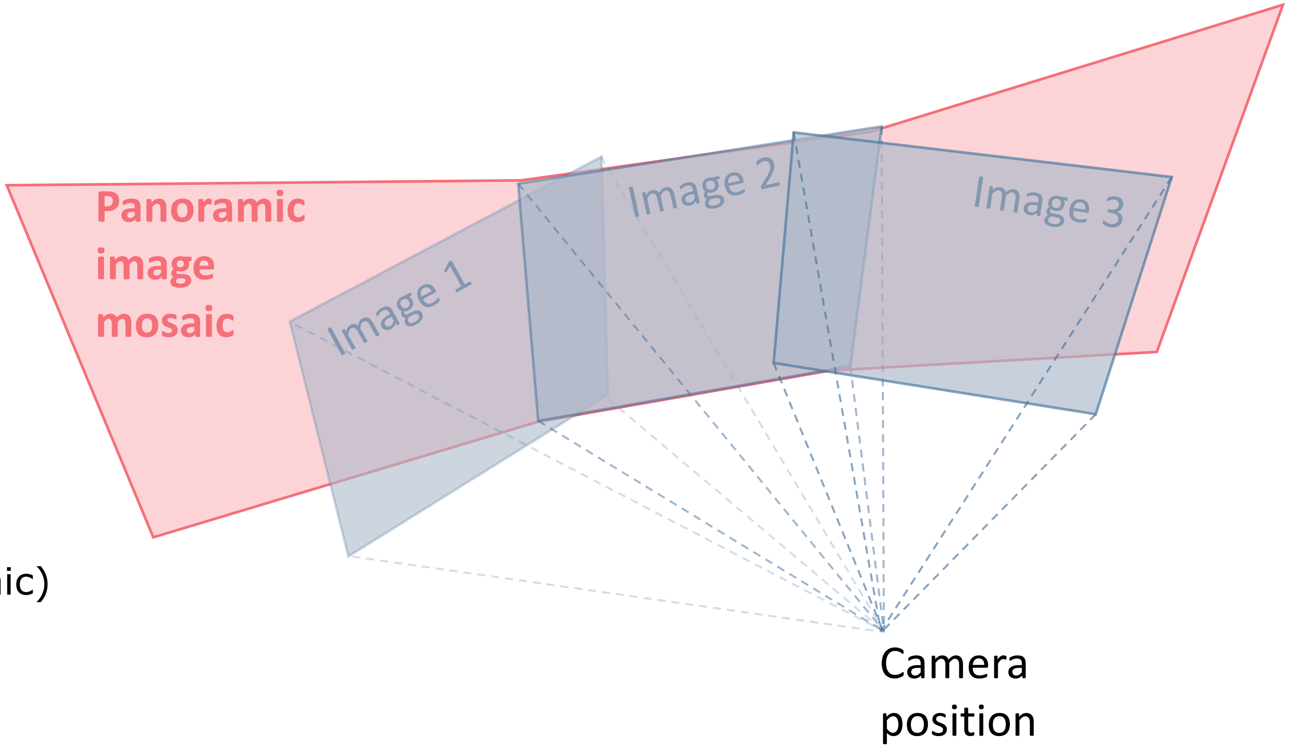

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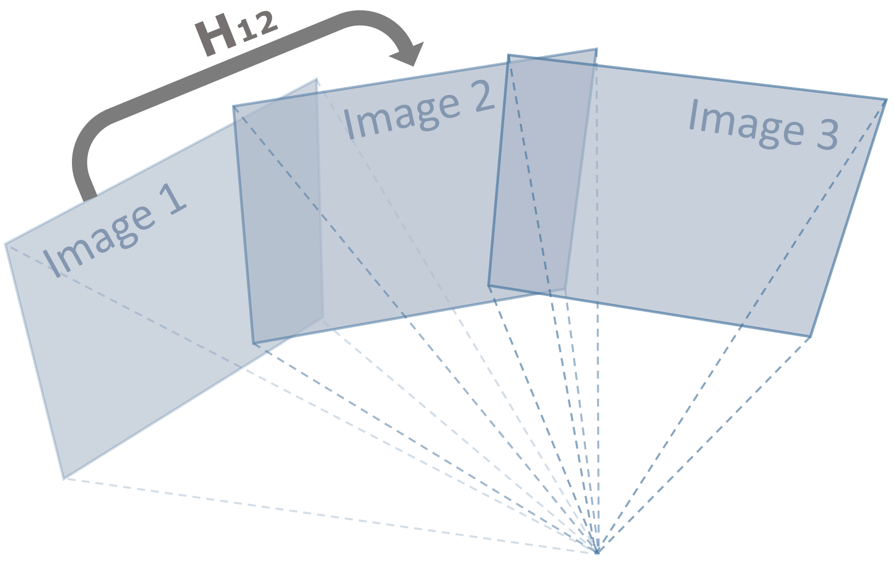

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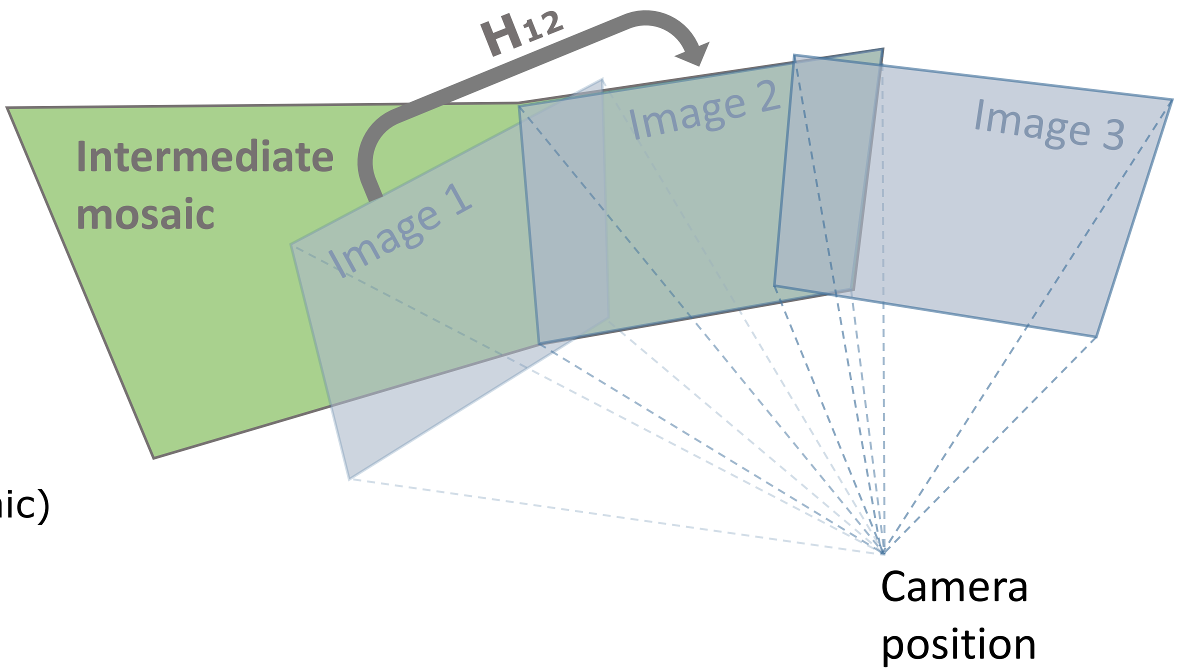

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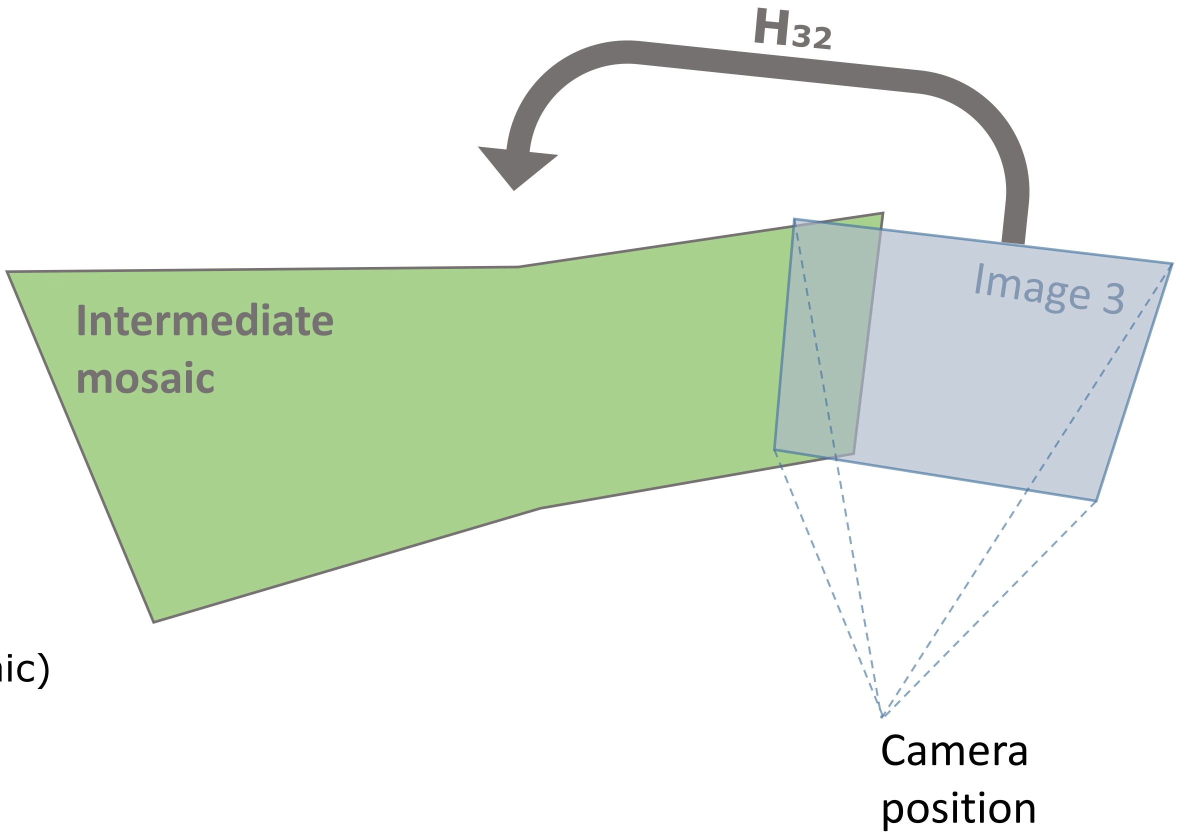

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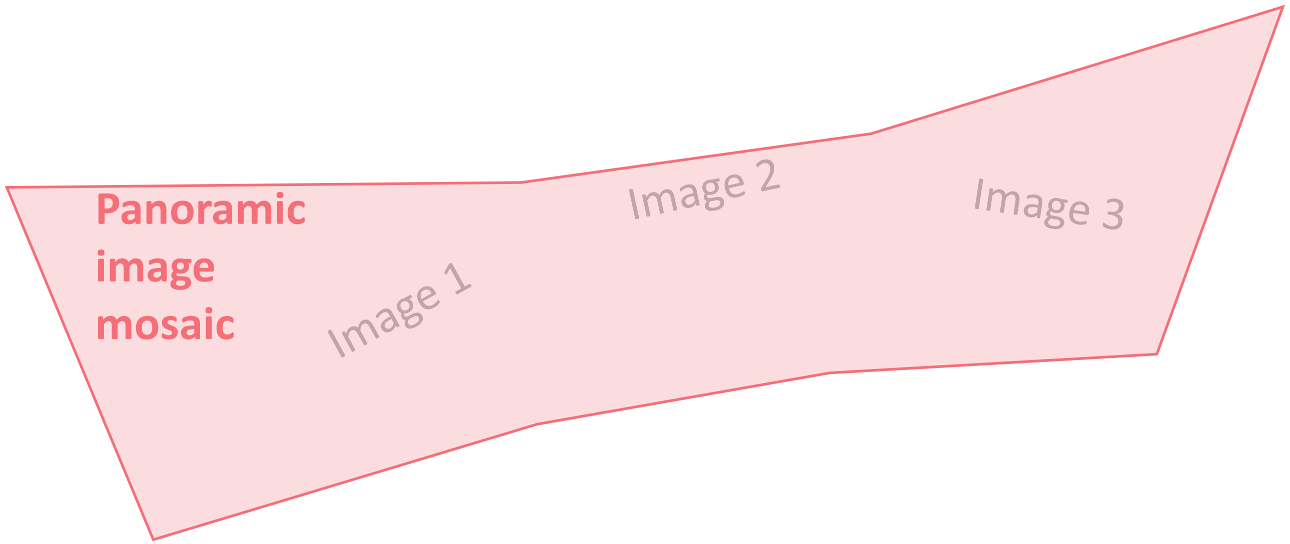

Assignment 2: *Projective Transformation*

Hints:

- If you do not have a tripod to stabilize the camera, choosing to photograph distant planar object is a good way to approximate the static camera projection center.
- Make sure to take pictures of objects with sufficient distinct features. You should be able to easily find and select the corresponding feature points between two overlapping images.
- Work with homogenous coordinates. You can simply add 1 as a 3rd component to the 2-component image coordinates of the selected feature points.
- Apply conditioning to the coordinates of the selected feature points. (See lecture 4 for more details on conditioning.)
- Remember to reverse the conditioning before obtaining the final solution for **H**.
- If in doubt, you can always test your implementation using the numerical example from lecture 4 as an input.

Assignment 2: *Example Result*

Image Source: *student submission; WS 19/20*