Bauhaus-Universität Weimar

Fakultät Medien

Photogrammetric Computer Vision WiSe2024

Assignment 1

Group 30

Muhammad Farzad Ali(126559)

Cesar Fernando Gamba Tiusaba(127138)

Chun Ting Lin(127130)

Parya Miraki(127187)

I. Points and Lines in the Plane

1. You would like to compute the connecting line between two 2D points. What happens if the two points are identical?

   The line will be undefined, because if we want to connect 2 points with coordinates (X_1,Y_1) and (X_2,Y_2), the line equation between these point will be

   $$y = \frac{y_2 - y_1}{x_2 - x_1} * x + b$$

   If both points have the same coordinates the slope of the line will be 0/0 that is not possible

2. Where does the general line x cos φ + y sin φ= d intersect the line $(0, 0, 1)^T$ given in homogeneous coordinates? How can this point be interpreted?

   We do the cross product between the general line and the line $(0,0,1)^T$

   $$\begin{matrix} \cos\varphi \\ \sin\varphi \\ -D \end{matrix} \times \begin{matrix} 0 \\ 1 \\ 1 \end{matrix} = \begin{matrix} \sin\varphi \\ -\cos\varphi \\ 0 \end{matrix}$$

   This intersection point ($\sin\varphi$, $-\cos\varphi$,0) lies on the line at infinity (since the third coordinate is zero), which indicates that the two lines intersect at a point on the horizon. Geometrically, this point can be interpreted as the direction vector of the original line in Euclidean space, representing the "vanishing point" of parallel lines with the same direction as L.

3. Show that the horizon is a straight line by showing that three points on the horizon are always collinear. (Hint: use projective geometry.)

   We have three points, with the next coordinates

   $$\begin{matrix} (a_1 & b_1 & c_1)^T \\ (a_2 & b_2 & c_2)^T \\ (a_3 & b_3 & c_3)^T \end{matrix}$$

Where C_1, $C_{2,}$ $C_{3,}$ are 0 meaning they all lie on the line at infinity.

Following the duality principle that says that

$$\det(l_1 \quad l_2 \quad l_3) = 0 \leftrightarrow \det(x_1 \quad x_2 \quad x_3) = 0$$
$$det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_2 \\ 0 & 0 & 0 \end{pmatrix} = 0$$

Since the determinant is zero, P1, P2, and P3 are indeed collinear, meaning that any three points on the horizon lie on a single straight line. Therefore, the horizon is itself a straight line.

II. First Steps in MATLAB

2.1 the MATLAB file is attached to the zip file as" *assignment_1_2.1_Task*".

```
% Given points
x = [2; 3; 1]; % Adding 1 for homogeneous coordinates
y = [-4; 5; 1]; % Adding 1 for homogeneous coordinates

% Compute line l as the cross product of x and y
l = cross(x, y);
disp('Connecting line l between points x and y:');
disp(l);
% Translation vector
t = [6; -7];
% Rotation angle in radians
phi = 15 * (pi / 180);
% Scaling factor
lambda = 8;
% Translation: move x and y by vector t
x_trans = [x(1) + t(1); x(2) + t(2); 1];
y_trans = [y(1) + t(1); y(2) + t(2); 1];
% Rotation matrix
R = [cos(phi), -sin(phi); sin(phi), cos(phi)];
```

```
% Rotate the translated points
x_rot = R * x_trans(1:2);
y_rot = R * y_trans(1:2);
% Scale the rotated points
x_prime = lambda * x_rot;
y_prime = lambda * y_rot;
disp('Transformed point x_prime:');
disp(x_prime);
disp('Transformed point y_prime:');
disp(y_prime);

% Homogeneous transformation matrix for translation
T = [1, 0, t(1); 0, 1, t(2); 0, 0, 1];

% Homogeneous transformation matrix for rotation
R_hom = [cos(phi), -sin(phi), 0; sin(phi), cos(phi), 0; 0, 0, 1];

% Homogeneous transformation matrix for scaling
S = [lambda, 0, 0; 0, lambda, 0; 0, 0, 1];

% Combined transformation matrix
transformation = S * R_hom * T;

% Apply transformation to line l
l_prime = transformation' * l;
disp('Transformed line l_prime:');
disp(l_prime);
```

```
% Convert x_prime and y_prime back to homogeneous coordinates
x_prime_hom = [x_prime; 1];
y_prime_hom = [y_prime; 1];

% Check if x' and y' lie on the line l'
is_x_on_l = abs(l_prime' * x_prime_hom) < 1e-10;
is_y_on_l = abs(l_prime' * y_prime_hom) < 1e-10;

if is_x_on_l
disp('Transformed point x_prime lies on transformed line l_prime');
else
disp('Transformed point x_prime does not lie on transformed line l_prime');
end

if is_y_on_l
disp('Transformed point y_prime lies on transformed line l_prime');
else
disp('Transformed point y_prime does not lie on transformed line l_prime');
end
```

2.2 If x′ and y′ do not lie on l′, it could be due to:

1. Numerical Inaccuracies: Small errors from rotation and scaling transformations.

2. Inconsistent Transformations: Slight mismatches in how transformations are applied to the points versus the line.

3. Order of Transformations: If the order (translation, rotation, scaling) differs between the points and line, it can lead to misalignment.