Photogrammetric Computer Vision

Exercise 3

Winter semester 24/25

(Course materials for internal use only!)

Computer Vision in Engineering – Prof. Dr. Rodehorst
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Agenda

Topics

Assignment 1.	Points and lines in the plane, first steps in MATLAB / Octave
Assignment 2.	Projective transformation (Homography)
Assignment 3.	**Camera calibration using direct linear transformation (DLT)**
Assignment 4.	Orientation of an image pair
Assignment 5.	Projective and direct Euclidean reconstruction
Assignment 6.	Stereo image matching
Final Project	- will be announced later -

Agenda

	Start date		Deadline
Assignment 1.	~~21.10.24~~	~~–~~	~~03.11.24~~
Assignment 2.	~~04.11.24~~	~~–~~	~~17.11.24~~
Assignment 3.	**18.11.24**	**–**	**01.12.24**
Assignment 4.	02.12.24	–	15.12.24
Assignment 5.	16.12.24	–	12.01.25
Assignment 6.	13.01.25	–	26.01.25
Final Project.	27.01.25	–	16.03.25

Assignment 2 – sample solution

Sample code 1/2

```
% Image mosaicking using general projective 2D transformations (homographies)

function exercise2
%        =========
f = imread('image1.ppm');                              % Read three images f, g, h
g = imread('image2.ppm');
h = imread('image3.ppm');                        % Adjust image f to image g and then
i = mosaic_image(h, mosaic_image(f, g));          % image h to the combined image
imshow(i);                                                  % Show mosaic result
```

```
function i = mosaic_image(f, g)                              % Combine two images
%            ==================
imshow(f); x1 = get_points;    % Show image f and g and get 4 homologous points
imshow(g); x2 = get_points;                    % in image f and then in image g
H = homography2(x1, x2);                                  % Compute homography H
i = geokor(H, f, g);            % Rectify image f using H and combine it with g
```

```
function p = get_points          % Measure homogeneous coordinates interactively
%           ==========                                        | x1, x2,     xn |
p = []; but = 1;                                      % p = | y1, y2, ... yn |
while but == 1                                        %     | 1 , 1 ,     1  |
    [x, y, but] = ginput(1);
    if but == 1
        p = [p [x y 1]'];
        hold on; plot(x, y, 'r+'); hold off;
    end
end
```

```
function H = homography2(x1, x2)              % Planar projective transformation
%            ====================
T1 = condition2(x1); c1 = T1 * x1;                    % Image point conditioning
T2 = condition2(x2); c2 = T2 * x2;
A = design_homo2(c1, c2);                                % Build design matrix
h = solve_dlt(A);                                % Linear least squares solution
H = inv(T2) * reshape(h, 3, 3)' * T1;     % Reshape row-wise and deconditioning
```

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Sample code 1/2

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```

Sample code 2/2

```
function H = homography2(x1, x2)              % Planar projective transformation
%            ====================
T1 = condition2(x1); c1 = T1 * x1;                    % Image point conditioning
T2 = condition2(x2); c2 = T2 * x2;
A = design_homo2(c1, c2);                                % Build design matrix
h = solve_dlt(A);                                % Linear least squares solution
H = inv(T2) * reshape(h, 3, 3)' * T1;     % Reshape row-wise and deconditioning
```

```
function T = condition2(x)                % Conditioning matrix for image points
%            =============
tx = mean(x(1,:));             ty = mean(x(2,:));             % Translation tx, ty
sx = mean(abs(x(1,:) - tx)); sy = mean(abs(x(2,:) - ty));      % Scaling sx, sy
T = [1/sx 0    -tx/sx;
     0    1/sy -ty/sy;
     0    0     1];
```

```
function A = design_homo2(x1, x2)            % Design matrix for 2D homography
%            ====================
A = [];
for i = 1 : size(x1, 2)
    A = [ A; -x2(3, i)*x1(:, i)'  0  0  0  x2(1, i)*x1(:, i)' ;
             0  0  0  -x2(3, i)*x1(:, i)'  x2(2, i)*x1(:, i)' ];
end
```

```
function x = solve_dlt(A)    % Direct linear transformation, solver for A*x = 0
%            ============
[U, D, V] = svd(A);                              % Singular value decomposition
x = V(:, end); % Last column is singular vector to the smallest singular value
```

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Sample code 2/2

```
function H = homography2(x1, x2)          % Planar projective transformation
%                 ====================
T1 = condition2(x1); c1 = T1 * x1;                % Image point conditioning
T2 = condition2(x2); c2 = T2 * x2;
A = design_homo2(c1, c2);                                % Build design matrix
h = solve_dlt(A);                                % Linear least squares solution
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```

column-wise: reshape(p, 3, 3)

≠

row-wise: reshape(p, 3, 3)‘

```
>> p = [1:9]

p =

     1     2     3     4     5     6     7     8     9

>> p_mat = reshape(p, 3, 3)

p_mat =

     1     4     7
     2     5     8
     3     6     9

>> p_mat2 = reshape(p, 3, 3)'

p_mat2 =

     1     2     3
     4     5     6
     7     8     9
```

Sample code 2/2

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end
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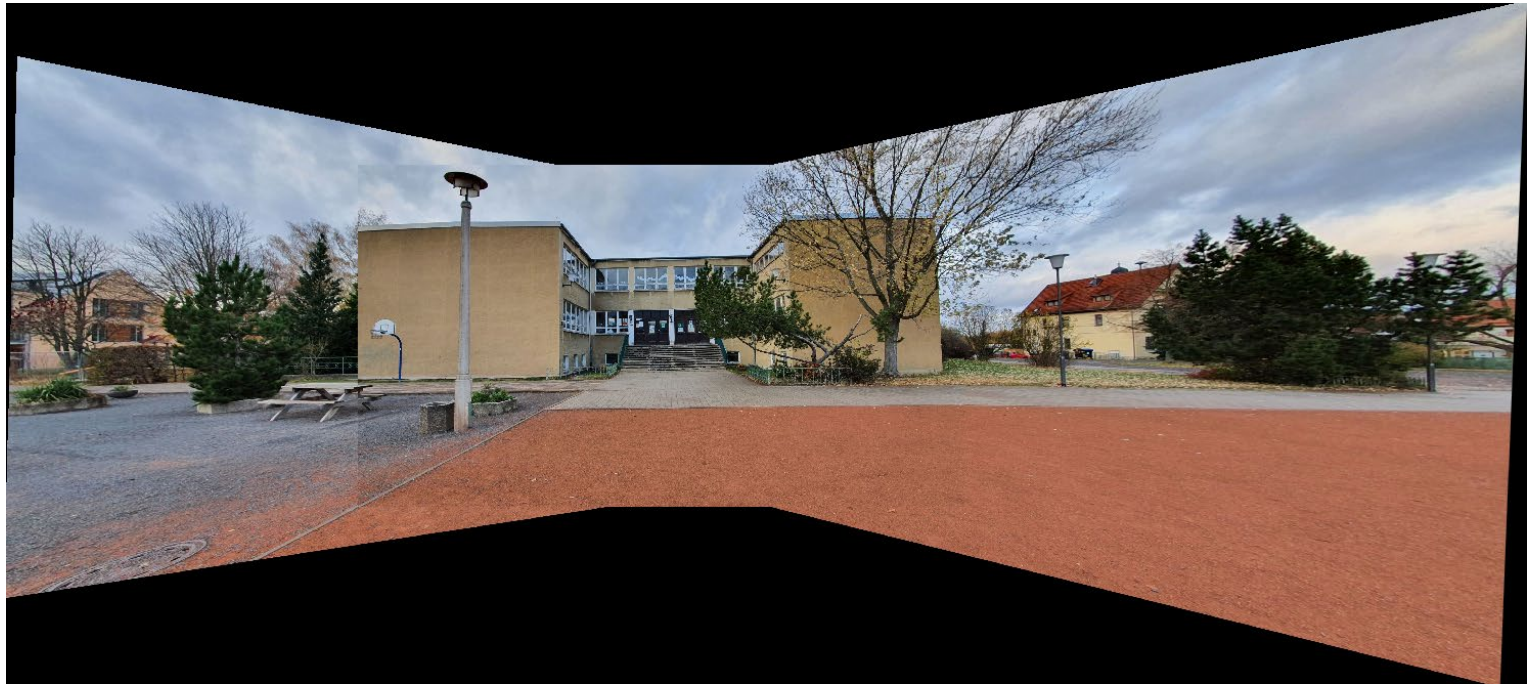
Assignment 2 – example results



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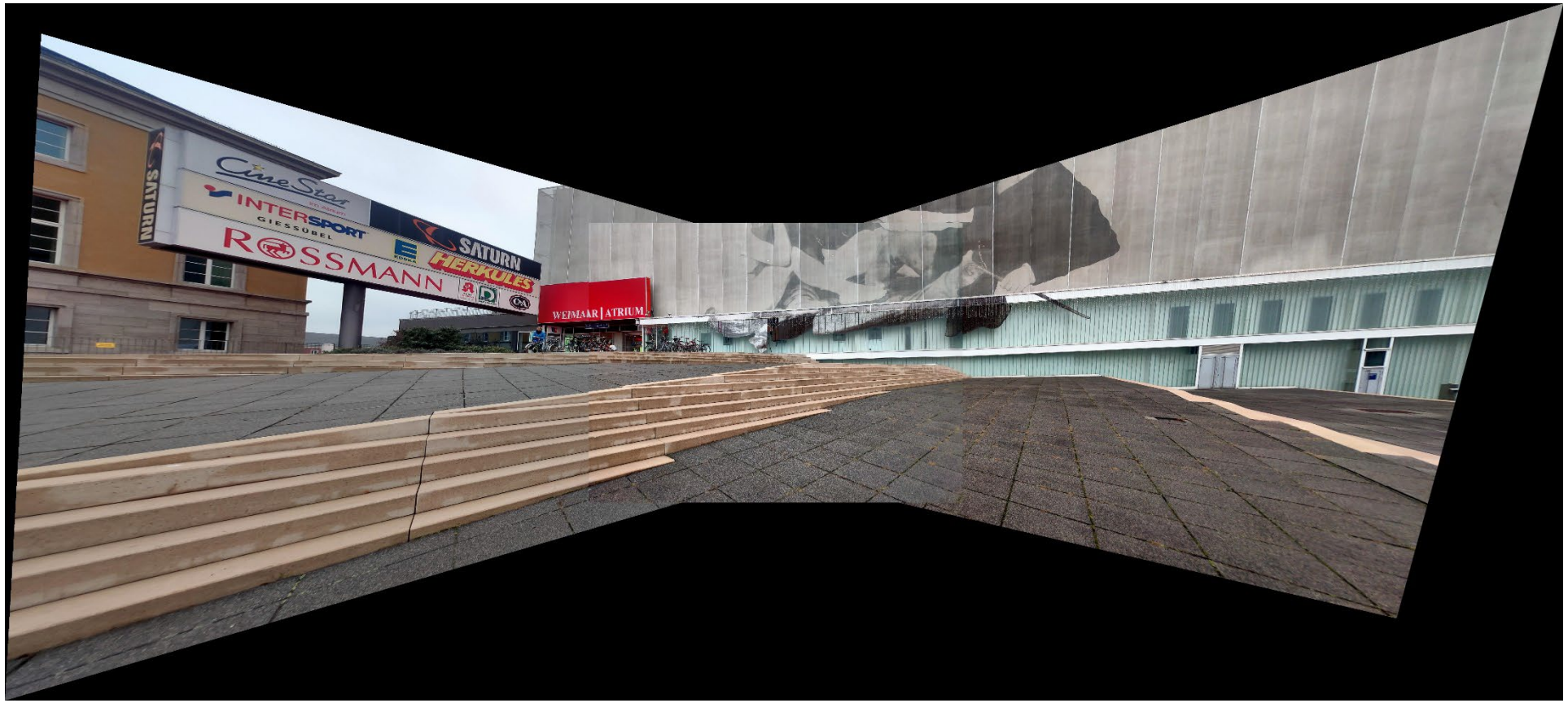

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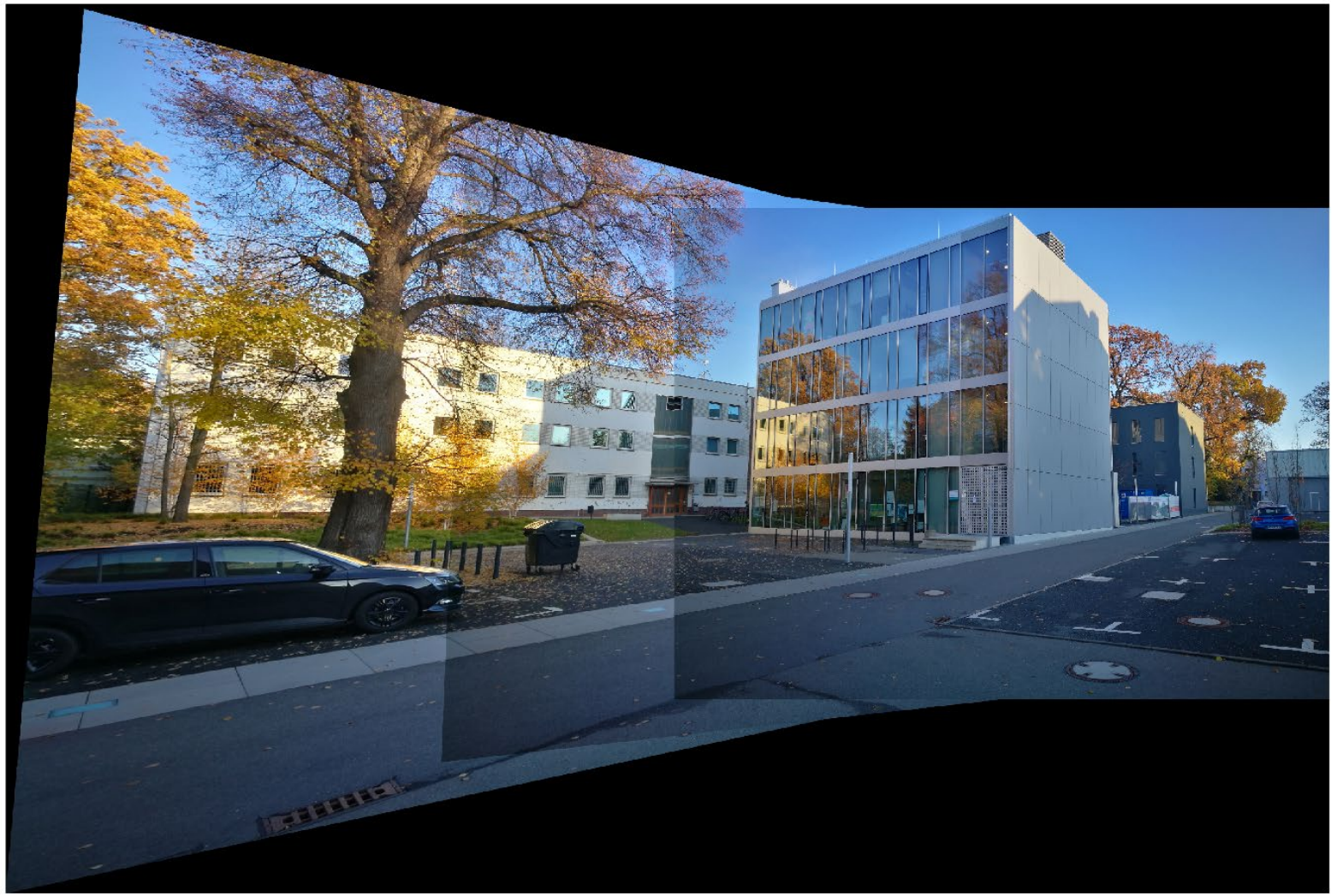

Assignment 3: *Camera calibration using DLT*

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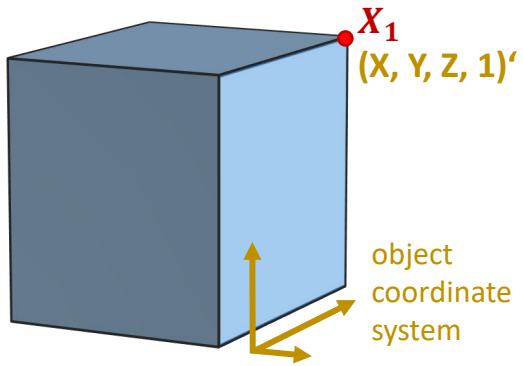


calibration object

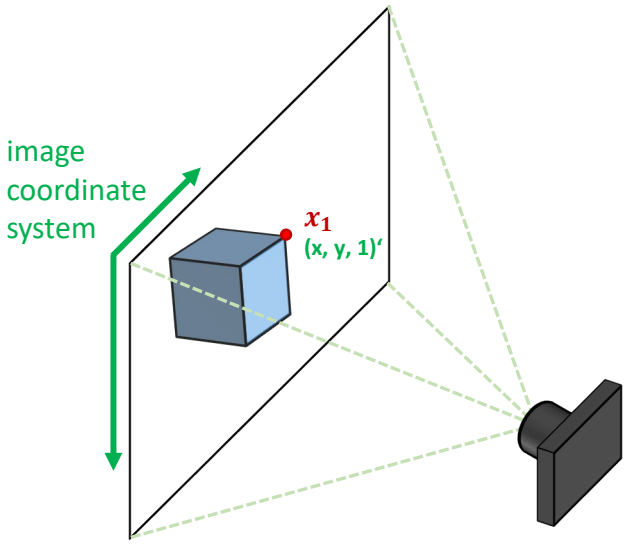


image of the
calibration object

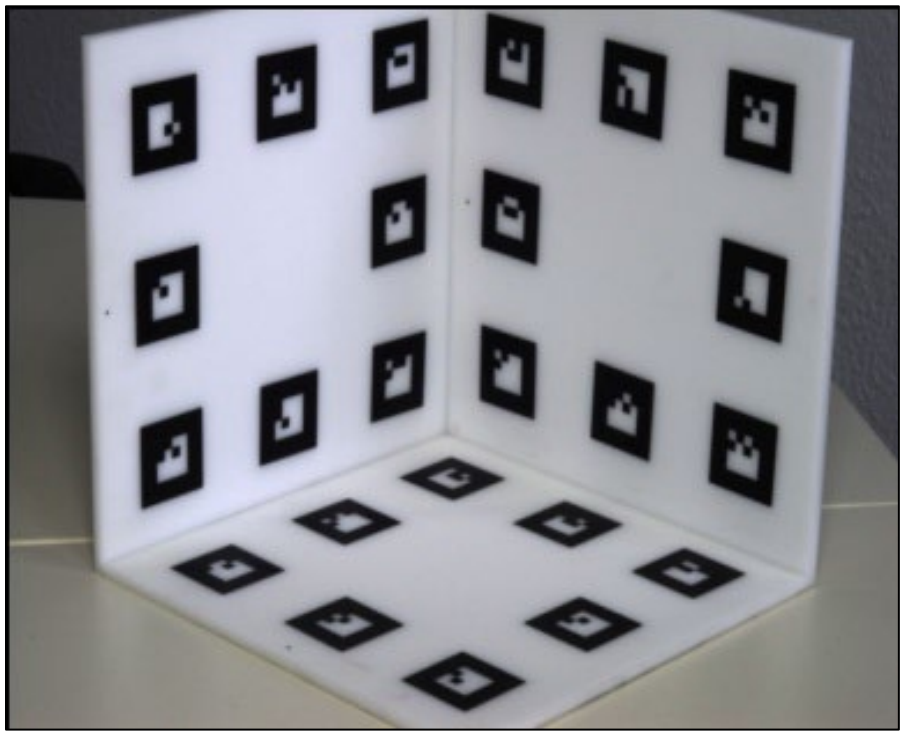
$$x_1 = PX_1$$
$$x_2 = PX_2$$
$$\dots$$
$$x_6 = PX_6$$

$$x_i = \underbrace{PX_i}_{K\ \&\ R, C}$$

Assignment 3: *Camera calibration using DLT*

1) Image acquisition

- one picture of an appropriate calibration object
- brief description of the chosen calibration object
- technical information about the used camera



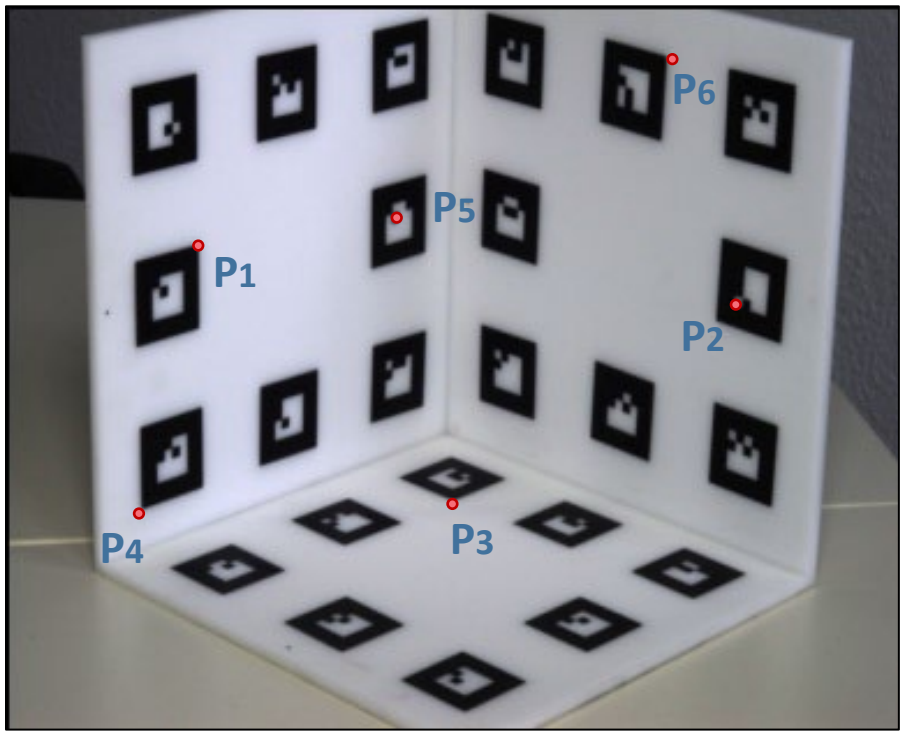
Assignment 3: *Camera calibration using DLT*

1) Image acquisition

- one picture of an appropriate calibration object
- brief description of the chosen calibration object
- technical information about the used camera

2) Control point measurements

- object coordinates of at least 6 control points
- axes of the object coordinate system
- measurements precision



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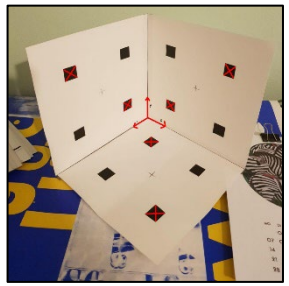
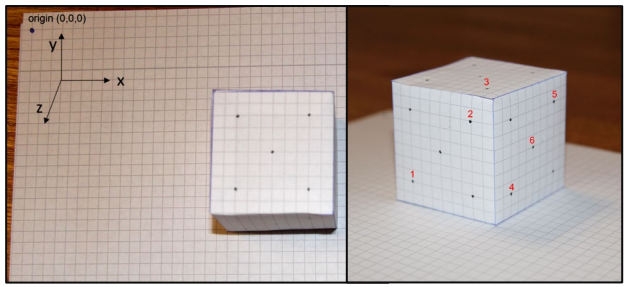
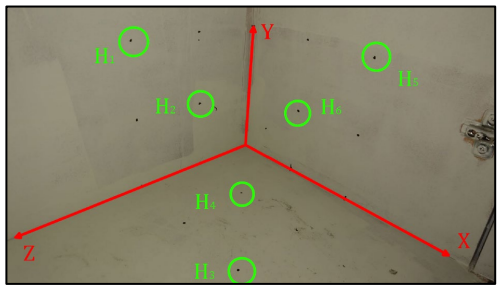
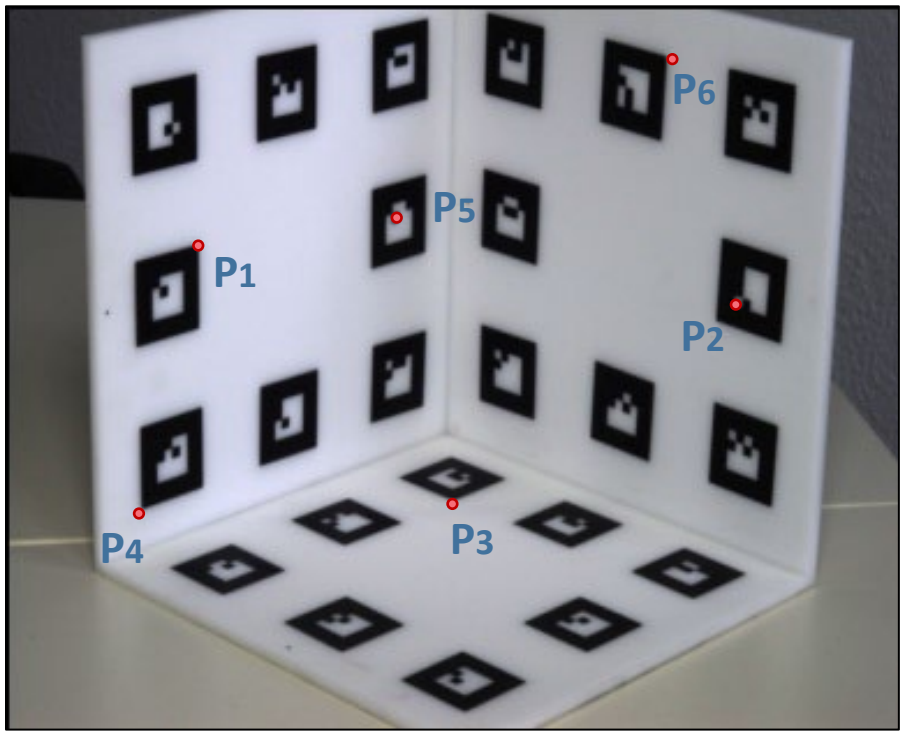






examples of annotated calibration objects from previous students' submissions



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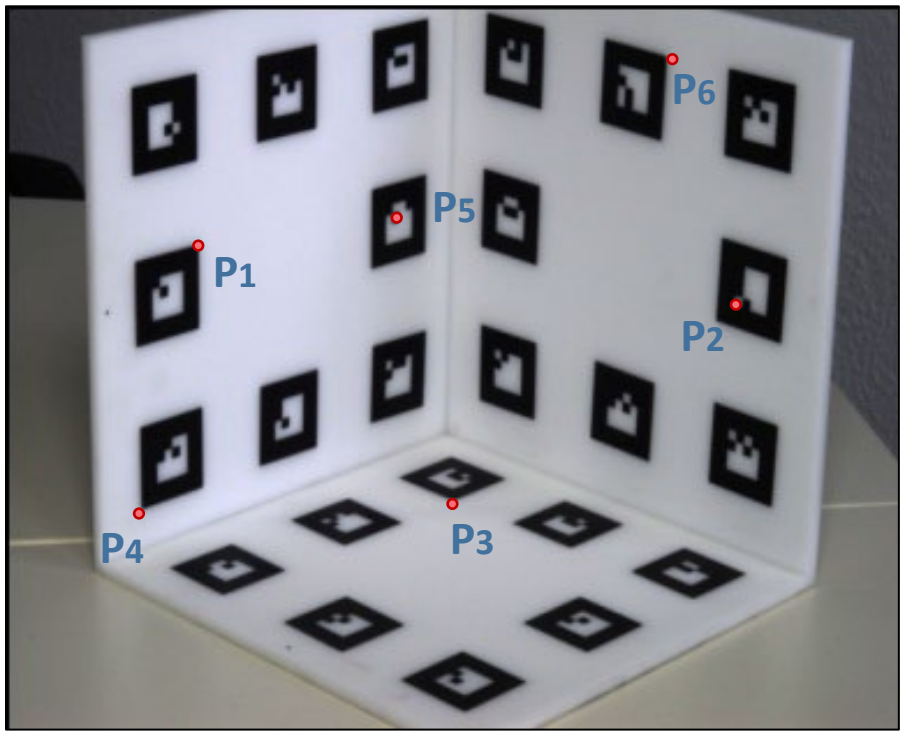
3) Computation of the projection matrix

- spatial resection using DLT with help of SVD



Assignment 3: *Camera calibration using DLT*

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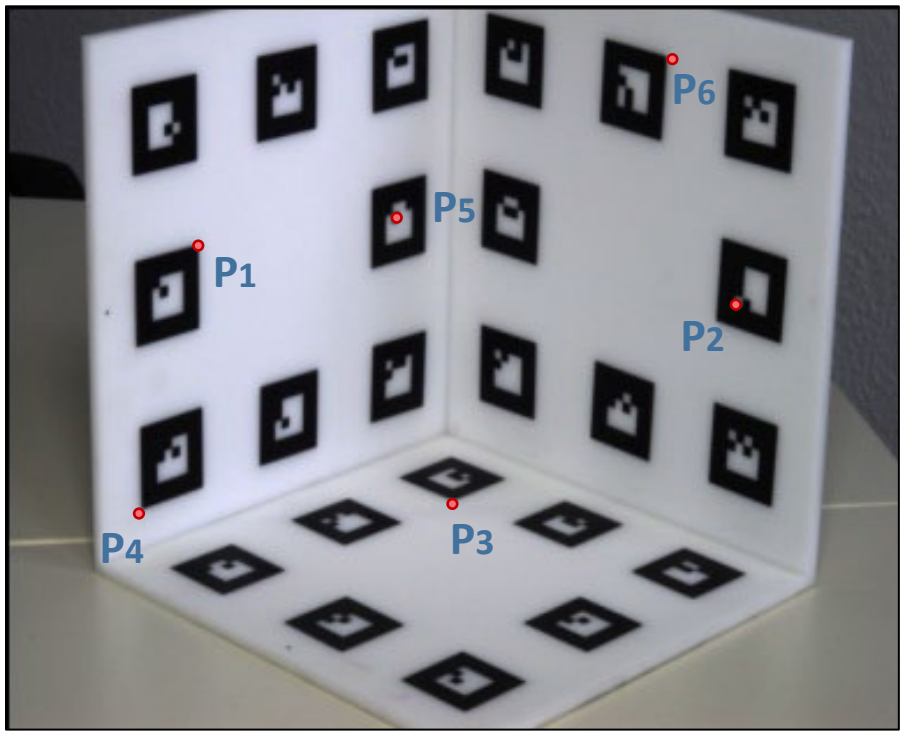
2) Control point measurements
   - object coordinates of at least 6 control points
   - axes of the object coordinate system
   - measurements precision

3) Computation of the projection matrix
   - spatial resection using DLT with help of SVD

4) Interpretation of the projection matrix
   - factorization of projection matrix (RQD)
   - meaning of extracted parameters
   - quality assessment



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