Álgebra Matricial – Tarea 03

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1) Obtenga la descomposición LU/PLU de las siguientes matrices:

a) $\begin{pmatrix} 2 & 3 & 1 & 4 \\ 1 & -1 & 6 & 4 \\ 0 & -1 & 5 & 8 \\ 1 & 2 & -1 & 4 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 3 & -1 & 6 \\ 4 & 7 & 2 & 1 \\ -2 & 5 & -2 & 0 \\ 0 & -4 & 5 & 2 \end{pmatrix}$

Para (a)

$$\begin{pmatrix} 2 & 3 & 1 & 4 \\ 1 & -1 & 6 & 4 \\ 0 & -1 & 5 & 8 \\ 1 & 2 & -1 & 4 \end{pmatrix} \xrightarrow{F_4 \leftarrow -\frac{1}{2}F_1 + F_4} \begin{pmatrix} 2 & 3 & 1 & 4 \\ 1 & -1 & 6 & 4 \\ 0 & -1 & 5 & 8 \\ 0 & 1/2 & -3/2 & 2 \end{pmatrix} \Bigg| \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{cccc} -1 & -\frac{3}{2} & -\frac{1}{2} & -2 \\ 1 & 2 & -1 & 4 \\ \hline 0 & \frac{1}{2} & -\frac{3}{2} & 2 \end{array}$$

$$\xrightarrow{F_2 \leftarrow -\frac{1}{2}F_1 + F_2} \begin{pmatrix} 2 & 3 & 1 & 4 \\ 0 & -5/2 & 11/2 & 2 \\ 0 & -1 & 5 & 8 \\ 0 & 1/2 & -3/2 & 2 \end{pmatrix} \Bigg| \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1/2 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{cccc} -1 & -3/2 & -1/2 & -2 \\ 1 & -1 & 6 & 4 \\ \hline 0 & -5/2 & 11/2 & 2 \end{array}$$

$$\xrightarrow{F_3 \leftarrow -\frac{2}{5}F_2 + F_3} \begin{pmatrix} 2 & 3 & 1 & 4 \\ 0 & -5/2 & 11/2 & 2 \\ 0 & 0 & 14/5 & 36/5 \\ 0 & 1/2 & -3/2 & 2 \end{pmatrix} \Bigg| \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 0 & -2/5 & 1 & 0 \\ -1/2 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{cccc} 0 & 1 & -\frac{11}{5} & -\frac{4}{5} \\ 0 & -1 & 5 & 8 \\ \hline 0 & 0 & \frac{14}{5} & \frac{36}{5} \end{array}$$

$$\xrightarrow{F_4 \leftarrow +\frac{1}{5}F_2 + F_4} \begin{pmatrix} 2 & 3 & 1 & 4 \\ 0 & -5/2 & 11/2 & 2 \\ 0 & 0 & 14/5 & 36/5 \\ 0 & 0 & 13/5 & 12/5 \end{pmatrix} \Bigg| \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 0 & -2/5 & 1 & 0 \\ -1/2 & 1/5 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{cccc} 0 & 1/2 & -3/2 & 2 \\ 0 & -1/2 & -\frac{11}{10} & \frac{2}{5} \\ \hline 0 & 0 & \frac{13}{5} & \frac{12}{5} \end{array}$$

$F_4 \leftarrow -\frac{13}{35} f_3 + f_4$

$$\longrightarrow$$

$$\begin{array}{cccc} 0 & 0 & \frac{-13}{5} & \frac{468}{175} \\ 0 & 0 & \frac{13}{5} & \frac{12}{5} \\ \hline 0 & 0 & 0 & \end{array}$$

$$\left(\begin{array}{cccc} 2 & 3 & 1 & 4 \\ 0 & -5/2 & 11/2 & 2 \\ 0 & 0 & 14/2 & 36/5 \\ 0 & 0 & 0 & \frac{24}{7} \end{array}\right) \Bigg| \left(\begin{array}{cccc} 1 & 0 & 0 & 0 \\ -1/2 & 1 & 0 & 0 \\ 0 & -2/5 & 1 & 0 \\ -1/2 & 1/5 & -1/7 & 1 \end{array}\right)$$

$$\therefore\ U = \begin{pmatrix} 2 & 3 & 1 & 4 \\ 0 & \frac{-5}{2} & \frac{11}{2} & 2 \\ 0 & 0 & \frac{14}{5} & \frac{36}{5} \\ 0 & 0 & 0 & \frac{24}{7} \end{pmatrix} \qquad L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 2/5 & 1 & 0 \\ 1/2 & -1/5 & -1/7 & 1 \end{pmatrix}$$

$$\Rightarrow \underbrace{\begin{pmatrix} 2 & 3 & 1 & 4 \\ 1 & -1 & 6 & 4 \\ 0 & -1 & 5 & 8 \\ 1 & 2 & -1 & 4 \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 2/5 & 1 & 0 \\ 1/2 & -1/5 & -1/7 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 2 & 3 & 1 & 4 \\ 0 & -5/2 & 11/2 & 2 \\ 0 & 0 & 14/5 & 36/5 \\ 0 & 0 & 0 & 24/7 \end{pmatrix}}_{U}$$

Para (b)

$$\begin{pmatrix} 2 & 3 & -1 & 6 \\ 4 & 7 & 2 & 1 \\ -2 & 5 & -2 & 0 \\ 0 & -4 & 5 & 2 \end{pmatrix} \xrightarrow{f_3 \leftarrow f_1 + f_3} \begin{pmatrix} 2 & 3 & -1 & 6 \\ 4 & 7 & 2 & 1 \\ 0 & 8 & -3 & 6 \\ 0 & -4 & 5 & 2 \end{pmatrix} \Bigg| \Bigg| \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{cccc} 2 & 3 & -1 & 6 \\ -2 & 5 & -2 & 0 \\ \hline 0 & 8 & -3 & 6 \end{array}$$

$$\xrightarrow{f_2 \leftarrow -2f_1 + f_2} \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 8 & -3 & 6 \\ 0 & -4 & 5 & 2 \end{pmatrix} \Bigg| \Bigg| \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{cccc} -4 & -6 & 2 & -12 \\ 4 & 7 & 2 & 1 \\ \hline 0 & 1 & 4 & -11 \end{array}$$

$$\xrightarrow{f_3 \leftarrow -8f_2 + f_3} \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 0 & -35 & 94 \\ 0 & -4 & 5 & 2 \end{pmatrix} \Bigg| \Bigg| \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -8 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{cccc} 0 & -8 & -32 & 88 \\ 0 & 8 & -3 & 6 \\ \hline 0 & 0 & -35 & 94 \end{array}$$

$$\xrightarrow{f_4 \leftarrow +4f_2 + f_4} \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 0 & -35 & 94 \\ 0 & 0 & 21 & -42 \end{pmatrix} \Bigg| \Bigg| \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -8 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{cccc} 0 & 4 & 16 & -44 \\ 0 & -4 & 5 & 2 \\ \hline 0 & 0 & 21 & -42 \end{array}$$

$$\xrightarrow{f_4 \leftarrow +\frac{21}{35} f_3 + f_4} \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 0 & -35 & 94 \\ 0 & 0 & 0 & \frac{72}{5} \end{pmatrix} \Bigg| \Bigg| \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -8 & 1 & 0 \\ 0 & 4 & \frac{21}{35} & 1 \end{pmatrix}$$

$$\begin{array}{cccc} 0 & 0 & -21 & \frac{285}{5} \\ 0 & 0 & 21 & -42 \\ \hline 0 & 0 & 0 & \frac{72}{5} \end{array}$$

$$\left\{ \begin{array}{l} U = \begin{pmatrix} 2 & 3 & -1 & 6 \\ 0 & 1 & 4 & -11 \\ 0 & 0 & -35 & 94 \\ 0 & 0 & 0 & 72/5 \end{pmatrix} \\ \\ L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ +2 & 1 & 0 & 0 \\ -1 & +8 & 1 & 0 \\ 0 & -4 & -\frac{3}{5} & 1 \end{pmatrix} \end{array} \right.$$

2- Calcule la descomposición Cholesky de las siguientes matrices:

a) $\begin{pmatrix} 4 & 10 & 8 \\ 10 & 26 & 26 \\ 8 & 26 & 61 \end{pmatrix}$ b) $\begin{pmatrix} 25 & 15 & -5 \\ 15 & 18 & 0 \\ -5 & 0 & 11 \end{pmatrix}$

Para (a)

Sabemos que para la descomposición Cholesky necesitamos encontrar una matriz T tq $TT^t = A$ y $T = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix}$

$\Rightarrow l_{11} = \sqrt{a_{11}} = \sqrt{4} = 2$

$l_{21} = \frac{a_{21}}{l_{11}} = \frac{10}{2} = 5$

$l_{31} = \frac{a_{31}}{l_{11}} = \frac{8}{2} = 4$

$l_{22} = \sqrt{a_{22} - l_{21}^2} = \sqrt{26 - 5^2} = \sqrt{26-25} = 1$

$l_{32} = \frac{a_{32} - l_{21} l_{31}}{l_{22}} = \frac{26 - 5(4)}{1} = 6$

$l_{33} = \sqrt{a_{33} - l_{31}^2 - l_{32}^2} = \sqrt{61 - 4^2 - 6^2} = \sqrt{9} = 3$

$\therefore T = \begin{pmatrix} 2 & 0 & 0 \\ 5 & 1 & 0 \\ 4 & 6 & 3 \end{pmatrix}$ y $T^t = \begin{pmatrix} 2 & 5 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 3 \end{pmatrix}$

$\Rightarrow TT^t = \begin{pmatrix} 2 & 0 & 0 \\ 5 & 1 & 0 \\ 4 & 6 & 3 \end{pmatrix} \begin{pmatrix} 2 & 5 & 4 \\ 0 & 1 & 6 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 10 & 8 \\ 10 & 26 & 26 \\ 8 & 26 & 61 \end{pmatrix}$

Para (b)

$$\underbrace{\begin{pmatrix} 25 & 15 & -5 \\ 15 & 18 & 0 \\ -5 & 0 & 11 \end{pmatrix}}_{A}, \quad \text{quiero } T = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \text{ tq } T \cdot T^t = A$$

$\Rightarrow$

$$l_{11} = \sqrt{a_{11}} = \sqrt{25} = 5 \qquad l_{22} = \sqrt{a_{22} - l_{21}^2} = \sqrt{18-9} = \sqrt{9} = 3$$

$$l_{21} = \frac{a_{21}}{l_{11}} = \frac{15}{5} = 3 \qquad l_{32} = \frac{a_{32} - l_{21} l_{31}}{l_{22}} = \frac{0 - 3 \cdot (-1)}{3} = \frac{3}{3} = 1$$

$$l_{31} = \frac{a_{31}}{l_{11}} = \frac{-5}{5} = -1 \qquad l_{33} = \sqrt{a_{33} - l_{31}^2 - l_{32}^2} = \sqrt{11 - (-1)^2 - (1)^2}$$

$$= \sqrt{11 - 1 - 1} = \sqrt{9} = 3$$

$$\therefore \; T = \begin{pmatrix} 5 & 0 & 0 \\ 3 & 3 & 0 \\ -1 & 1 & 3 \end{pmatrix} \quad y \quad T^t = \begin{pmatrix} 5 & 3 & -1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\Rightarrow TT^t = \begin{pmatrix} 5 & 0 & 0 \\ 3 & 3 & 0 \\ -1 & 1 & 3 \end{pmatrix} \begin{pmatrix} 5 & 3 & -1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 25 & 15 & -5 \\ 15 & 18 & 0 \\ -5 & 0 & 11 \end{pmatrix}$$