Syllabus

MA Math Camp 2021

Department of Economics, Columbia University

Last Updated : August 15, 2021

Course Information

- Instructor : César Barilla
- Email : cesar.barilla@columbia.edu
- Graders : Utkarsh Kumar and Akanksha Vardani

- Course Website : `https://cesarbarilla.github.io/Columbia-MA-Math-Camp-2021`
- Dates : Monday Aug. 16 - Thur Sep. 2
- Time : 9:30am-12:00pm
- Place : Schermerhorn 614 and on Zoom (link and recordings will be shared via email)
- Office Hours : Wednesday 1pm-2pm, on Zoom

- Problem Sets : TBA
- Exam : TBA

Course Description

The course will cover the mathematical tools and concepts required for the first year sequence of the Master's in Economics. The main goal of the course is to prepare for first year classes by reviewing or introducing fundamental concepts in various domains of mathematics – analysis, linear algebra, calculus, probability, optimization. A strong emphasis will be put on proof-writing skills and proper mathematical rigor, as well as problem-solving and application of the tools. Students are expected to have taken courses in elementary analysis and unidimensional calculus, as well as have some familiarity with concepts in probability and linear algebra.

The class will be taught in a hybrid format from Monday August 16th to Thursday September 2nd. Lectures will be held in person (Schermerhorn 614) every weekday from 9.30am to 12pm EST ; they

will simultaneously be available on Zoom as well as recorded for asynchroneous attendance. If possible, students are strongly encouraged to attend the lectures in real time.

The course is largely self-contained. Lecture notes will be posted on the website ; teaching itself will mostly take place on the blackboard but additional notes or slides might be provided. Some additional notes and textbook references are provided below.

Problem sets will be assigned weekly. They are important practice and will be graded for feedback, although no grade will be given for the class. Problem sets will have to be submitted online (modalities to be specified) and will have to be typed – LaTeXis very strongly encouraged as it is an extremely valuable skill that students should acquire as soon as possible. There will be a final exam – the date and modality of the exam will be announced later.

Course Material

Course Outline and Lecture Notes

Here is a tentative course outline :

1. Preliminaries : Mathematical Logic, Sets, Functions, Numbers
    a) Introduction to Mathematical Logic
    b) Sets
    c) Relations
    d) Functions
    e) Numbers
    f) Countability and Cardinality
2. Real Analysis
    a) Metric Spaces
    b) Basic Topology
    c) Sequences and Convergence
    d) Compactness
    e) Cauchy Sequences and Completeness
    f) Continuity of Functions
3. Linear Algebra
    a) Vectors and Vector Spaces
    b) Matrices
    c) Systems of Linear Equations
    d) Eigenvalues, Eigenvectors, and Diagonalization
    e) Quadratic Forms
4. Multivariate Calculus

a) Derivatives
b) Mean Value Theorem
c) Higher order derivatives and Taylor Expansions
d) Log-Linearization
e) Implicit and Inverse Function Theorems
f) (Riemanian) Integration

5. Convexity
   a) Convex Sets, Separation Theorem, Fixed Point Theorems
   b) Convex and Concave Functions
   c) Quasi-convex and Quasi-concave functions
6. Optimization
   a) General Setup
   b) Result on the set of Maximizers
   c) Kuhn-Tucker Theorem
   d) Optimization on R^n
   e) A brief introduction to dynamic programming
7. Probability (if time permits)
8. Correspondences (if time permits)

The outline will be updated shortly with more details.
Lectures notes will be posted on the website :

`https://github.com/CesarBarilla/Columbia-MA-Math-Camp-2021/#course-outline-and-lecture-notes`

Problem Sets

Problem sets will be posted on the website and due each Monday ; the modality of submission is yet to be specified. Problem sets will have to be typed and LaTeXis strongly encouraged.

References and Textbooks

Lecture notes from last year's math camp are available on the website along with other useful documents.

Below is a list of useful references and textbooks sorted by theme. Within each theme, references are listed in (approximately) increasing complexity. References marked with a **(!)** are more advanced and are included either for future references or very motivated students.

- General references
  - Knut Sydsaeter, Peter Hammond, Arne Strom and Andrés Carvajal. "Esssential mathematics for economic analysis.", 5th Edition, (2016), Pearson.

  - Knut Sydsaeter, Peter Hammond, Atle Seierstad and and Arne Strom. "Further mathematics for economic analysis.", 2nd Edition, (2008), Pearson.

- Analysis
  - Walter Rudin. "Principles of Mathematical Analysis" (1976), International Series in Pure & Applied Mathematics, McGraw-Hill.
  - Ok, Efe A. "Real Analysis with Economic Applications" (2007).
  - **(!)** Walter Rudin, Real and Complex Analysis, Third Edition (1987), McGraw-Hill.

- Linear Algebra
  - Treil, Sergei. "Linear Algebra Done Wrong." (2014) (available online at `http://www.math.brown.edu/treil/papers/LADW/LADW-2014-09)`
  - Lang, Serge. "Linear Algebra", Third Edition (2004), Springer Undergraduate Texts in Mathematics.

- Optimization
  - Rangarajan K. Sundaram, "A First Course in Optimization Theory" (1996), Cambridge University Press.

- Probability and Measure Theory
  - Of, Efe A. "Measure and Probability Theory with Economic Applications". Available online at `https://sites.google.com/a/nyu.edu/efeok/books`
  - Rick Durrett, "Probability Theory and Examples", Version 5 (2019), available online at `https://services.math.duke.edu/~rtd/PTE/pte.html`
  - Patrick Billingsley, "Probability and Measure", Third Edition (1994), Wiley Series in Probability and Mathematical Statistics.

- Dynamic Programming
  - Klaus Walde, "Applied Intertemporal Optimization" (2020), available online for free at this link on the author's website.
  - Nancy L. Stokey, Robert E. Lucas Jr, and Edward C. Prescott. "Recursive Methods in Economic Dynamics" (1989), Harvard University Press.