TÉCNICAS DE MINERÍA DE DATOS

Regresión Lineal

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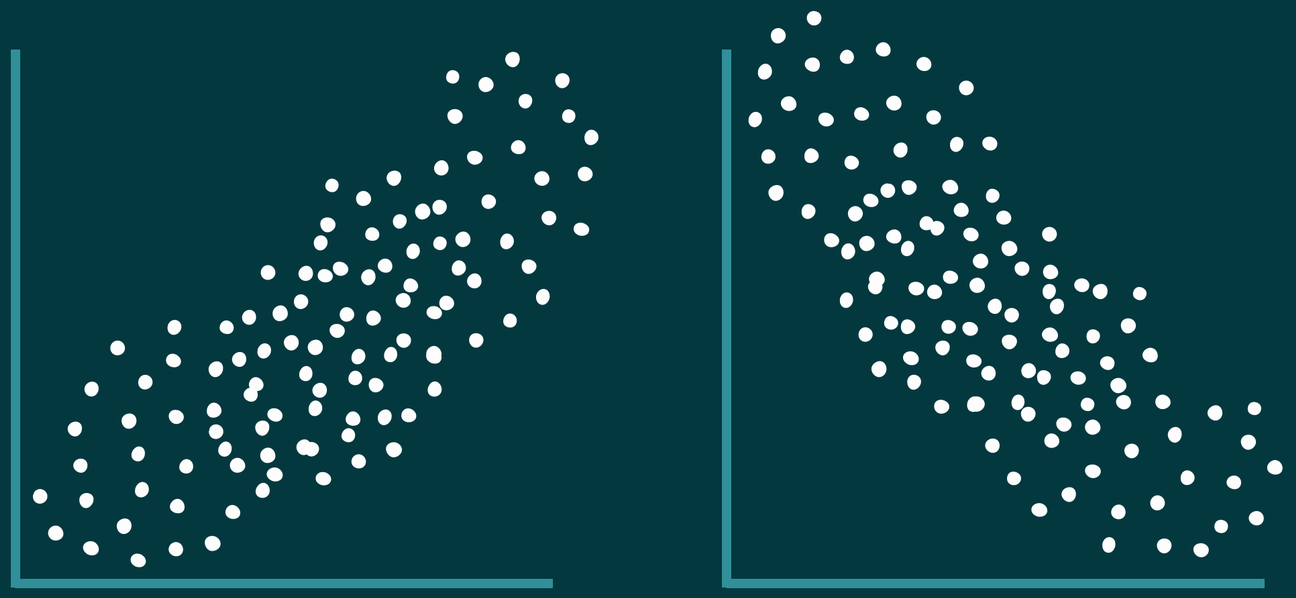
REGRESIÓN LINEAL SIMPLE



$$r = \sqrt{1 - \frac{\sum(Y - \hat{Y})^2}{\sum(Y - \bar{Y})^2}}$$

SSE = $\sum(Y - \hat{Y})^2$

Variación de Y alrededor de la recta.

SST = $\sum(Y - \bar{Y})^2$

Variación de Y alrededor de la media

$$y = \beta 0 + \beta 1x + \varepsilon$$

LA FÓRMULA DE MINIMOS CUADRADOS QUEDA:

$$S(\beta_0, \beta_1) = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

SE DERIVA:

$$\frac{\partial S}{\partial \beta_0} = -2 \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0$$

$$\frac{\partial S}{\partial \beta_1} = -2 \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)\, x_i = 0$$

REGRESIÓN LINEAL SIMPLE

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} x_i y_i - \frac{(\sum_{i=1}^{n} x_i)(\sum_{i=1}^{n} y_i)}{n}}{\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}$$

En la ecuación general de la recta de regresión, B1 es la pendiente y B0 el valor de la variable dependiente Y para la que X = 0.

REGRESIÓN LINEAL MÚLTIPLE

Un modelo de regresión donde interviene más de una variable regresora, se llama modelo de regresión múltiple, el cual maneja la siguiente ecuación:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon$$

LA FÓRMULA DE MÍNIMOS CUADRADOS QUEDA:

$$S(\beta_0, \beta_1, \dots, \beta_k) = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \beta_0 - \sum_{j=1}^{k} \beta_j x_{ij})^2$$

SE DERIVA:

$$\frac{\delta s}{\delta \beta_0} = -2 \sum_{i=1}^{n} \left(y_1 - \widehat{\beta_0} - \sum_{j=1}^{k} \widehat{\beta_1}\, x_{ij} \right) = 0$$

$$\frac{\delta s}{\delta \beta_j} = -2 \sum_{i=1}^{n} \left(y_1 - \widehat{\beta_0} - \sum_{j=1}^{k} \widehat{\beta_1}\, x_{ij} \right) x_{ij} = 0$$

ECUACIÓN DE MÍNIMOS CUADRADOS:

$$n\widehat{\beta_0} + \widehat{\beta_1}\sum_{i=1}^{n} x_{i1} + \widehat{\beta_2}\sum_{i=1}^{n} x_{i2} + \cdots + \widehat{\beta_k}\sum_{i=1}^{n} x_{ik} = \sum_{i=1}^{n} y_i$$

$$\widehat{\beta_0}\sum_{i=1}^{n} x_{i1} + \widehat{\beta_1}\sum_{i=1}^{n} {x_{i1}}^2 + \widehat{\beta_2}\sum_{i=1}^{n} x_{i1}x_{i2} + \cdots + \widehat{\beta_k}\sum_{i=1}^{n} x_{i1}x_{ik} = \sum_{i=1}^{n} x_{i1}y_i$$

$$\vdots$$

$$\widehat{\beta_0}\sum_{i=1}^{n} x_{ik} + \widehat{\beta_1}\sum_{i=1}^{n} x_{ik}x_{i1} + \widehat{\beta_2}\sum_{i=1}^{n} x_{ik}x_{i2} + \cdots + \widehat{\beta_k}\sum_{i=1}^{n} {x_{ik}}^2 = \sum_{i=1}^{n} x_{ik}y_i$$

ECUACIÓN MATRICIAL DEL MODELO:

$$y = x\beta + \varepsilon$$

LOS VALORES ESTÁN DADOS POR:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \quad \mathrm{x} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{12} & x_{22} & \cdots & x_{2k} \\ & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_n \end{bmatrix} \quad e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$$

ECUACIÓN DE MÍNIMOS CUADRADOS:

$$x'x\hat{\beta} = x'y$$

ESTIMADOR DE MÍNIMOS CUADRADOS:

$$\hat{\beta} = (x'x)^{-1}x'y$$

MULTICOLINEALIDAD

La multicolinealidad es la relación de dependencia lineal fuerte entre más de dos variables explicativas en una regresión múltiple.

¿COMO MEDIRLA?

Obtenemos el factor de inflación de varianza para el j-ésimo coeficiente de la regresión.

$$VIF_j = \frac{1}{1 - R_j^2}$$

Los factores VIF > 10 presentan problemas de multicolinealidad.

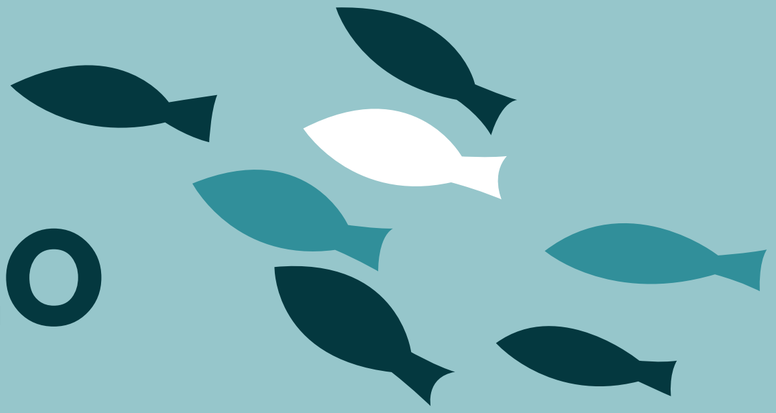
PRUEBA DE SIGNIFICANCIA

H0: B1 = 0, la regresión no es significativa

H1: B1 =! 0, la regresión es significativa

Si la regresión es significativa entonces el modelo si se ajusta al comportamiento de los datos.

EJEMPLO



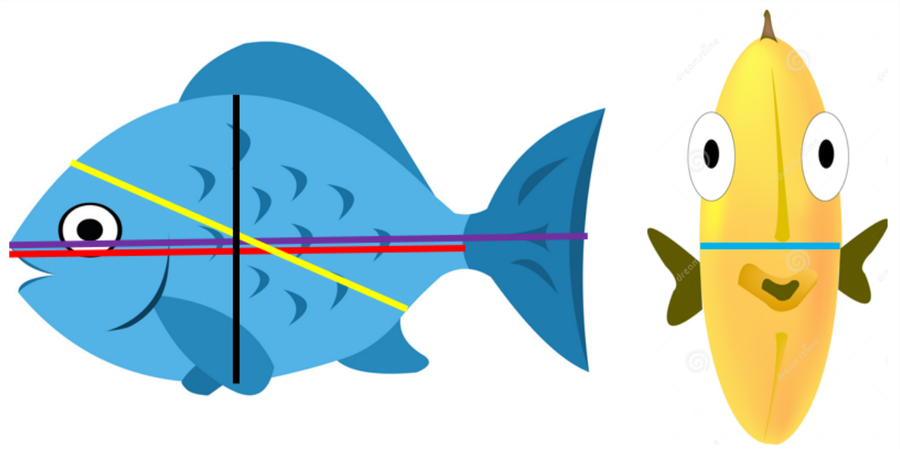
FISH MARKET DATASET

Predecir el peso de los peces de la especie "Perch".

IMPORTAR LIBRERIAS Y DATOS

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt # Visualización
import seaborn as sns #Visualización
from statsmodels.stats.anova import anova_lm
from statsmodels.formula.api import ols
df = pd.read_csv("Fish.csv")
```

DATASET



```
print(df.head())
print(df.shape)
```

```
  Species  Weight  Length1  Length2  Length3   Height   Width
0   Bream   242.0     23.2     25.4     30.0  11.5200  4.0200
1   Bream   290.0     24.0     26.3     31.2  12.4800  4.3056
2   Bream   340.0     23.9     26.5     31.1  12.3778  4.6961
3   Bream   363.0     26.3     29.0     33.5  12.7300  4.4555
4   Bream   430.0     26.5     29.0     34.0  12.4440  5.1340
(159, 7)
```

BÚSQUEDA DE DATOS NULOS

```
df.isnull().sum()
```

```
Species    0
Weight     0
Length1    0
Length2    0
Length3    0
Height     0
Width      0
dtype: int64
```

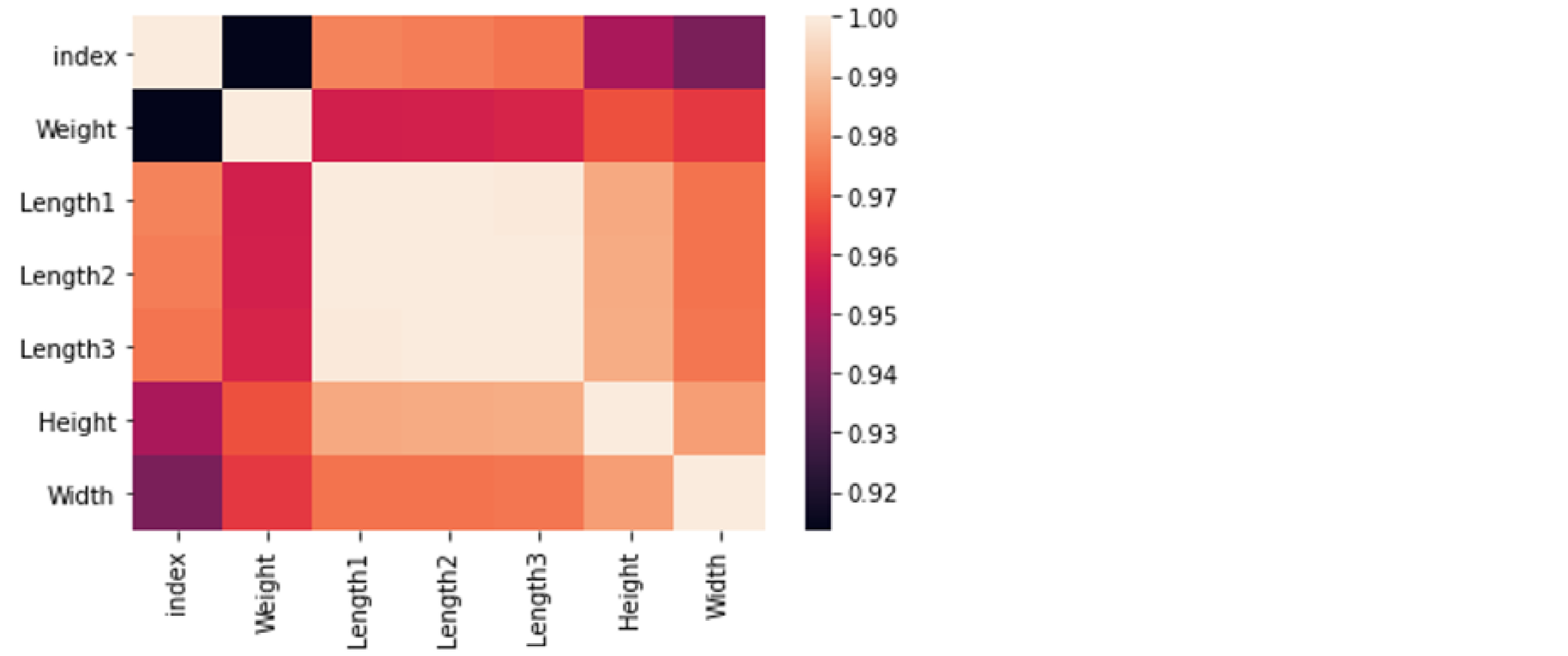
EXTRACCIÓN DE DATOS "PERCH"

```
perch=df[df['Species']=='Perch']
perch=perch.drop("Species",axis=1)
perch=perch.reset_index()
perch=perch.drop("index",axis=1)
```

CORRELOGRAMA

```
corr = perch.corr(method='pearson')
sns.heatmap(corr, xticklabels = corr.columns.values, yticklabels=corr.columns.values)
```

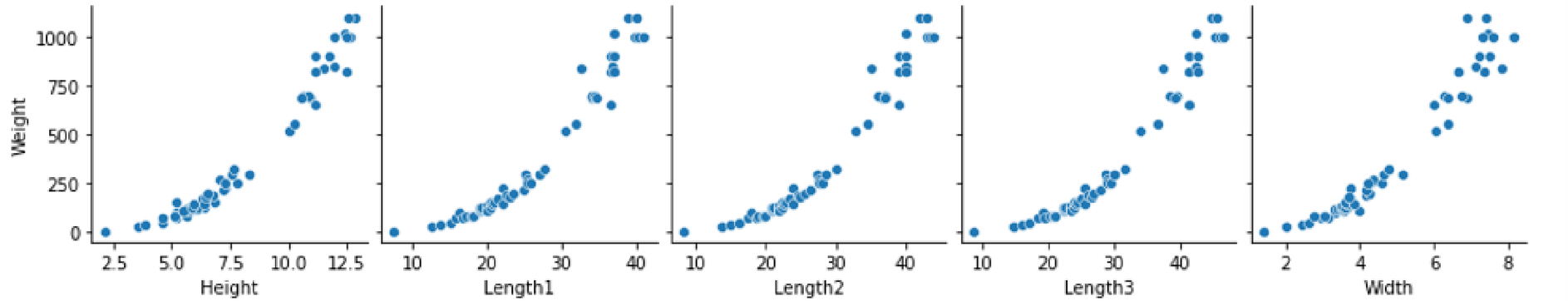
```
<matplotlib.axes._subplots.AxesSubplot at 0x7fcd6edb2950>
```



DISPERSIÓN WEIGHT VS OTRAS VARIABLES

```
sns.pairplot(perch, x_vars=['Height','Length1', 'Length2', 'Length3', 'Width'], y_vars='Weight')
```

```
<seaborn.axisgrid.PairGrid at 0x7fcd6ec65410>
```



DECLARACIÓN DE VARIABLES Y DEFINICIONES

```
from sklearn.feature_selection import RFE
from sklearn.linear_model import LinearRegression
import statsmodels.api as sm
from statsmodels.stats.outliers_influence import variance_inflation_factor
```

```
# Fit the model
y = perch["Weight"]
x1 = perch["Length1"]
x2 = perch["Length2"]
x3 = perch["Length3"]
x4 = perch["Height"]
x5 = perch["Width"]
d = {"x1": x1, "x2": x2, "x3": x3,"x4":x4,"x5":x5}
X=pd.DataFrame(d)
```

```
def build_model(X,y):
    X = sm.add_constant(X) #Adding the constant
    lm = sm.OLS(y,X).fit() # fitting the model
    print(lm.summary()) # model summary
    return X

def checkVIF(X):
    vif = pd.DataFrame()
    vif['Features'] = X.columns
    vif['VIF'] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1])]
    vif['VIF'] = round(vif['VIF'], 2)
    vif = vif.sort_values(by = "VIF", ascending = False)
    return(vif)
```

MODELO

```
[141] model1=build_model(X,y)
```

OLS Regression Results

Dep. Variable:	Weight	R-squared:	0.943
Model:	OLS	Adj. R-squared:	0.937
Method:	Least Squares	F-statistic:	165.2
Date:	Wed, 08 Sep 2021	Prob (F-statistic):	7.60e-30
Time:	19:48:50	Log-Likelihood:	-326.45
No. Observations:	56	AIC:	664.9
Df Residuals:	50	BIC:	677.0
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-556.5865	60.663	-9.175	0.000	-678.431	-434.741
x1	-3.1302	57.880	-0.054	0.957	-119.386	113.126
x2	-38.5019	88.552	-0.435	0.666	-216.364	139.361
x3	42.9174	60.088	0.714	0.478	-77.773	163.608
x4	65.6555	29.998	2.189	0.033	5.402	125.909
x5	64.9023	36.765	1.765	0.084	-8.943	138.748

Omnibus:	21.244	Durbin-Watson:	0.500
Prob(Omnibus):	0.000	Jarque-Bera (JB):	29.351
Skew:	1.415	Prob(JB):	4.23e-07
Kurtosis:	5.137	Cond. No.	495.

Ecuación del modelo:

$$\hat{y} = -556.58 - 3.13x_1 - 38.5x_2$$

$$+42.91x_3 + 65.65x_4 + 64.9x_5$$

```
checkVIF(model1)
```

	Variables	VIF
0	const	27.16
1	x1	1780.08
2	x2	4626.41
3	x3	2376.77
4	x4	54.04
5	x5	30.86

VIF

Como todos los VIFs son altos, hay problemas de multicolinealidad.

REGRESIÓN SIMPLE

```
d1 = {"x4":x4}
X_1=pd.DataFrame(d1)
mod2=build_model(X_1,y)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 Weight   R-squared:                       0.938
Model:                            OLS   Adj. R-squared:                  0.937
Method:                 Least Squares   F-statistic:                     815.2
Date:                Wed, 08 Sep 2021   Prob (F-statistic):           2.92e-34
Time:                        20:18:55   Log-Likelihood:                -328.82
No. Observations:                  56   AIC:                             661.6
Df Residuals:                      54   BIC:                             665.7
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       -537.3275     34.260    -15.684      0.000    -606.015    -468.640
x4           116.9654      4.096     28.553      0.000     108.752     125.178
==============================================================================
Omnibus:                       11.275   Durbin-Watson:                   0.678
Prob(Omnibus):                  0.004   Jarque-Bera (JB):               11.319
Skew:                           0.954   Prob(JB):                      0.00349
Kurtosis:                       4.099   Cond. No.                         24.8
==============================================================================
```

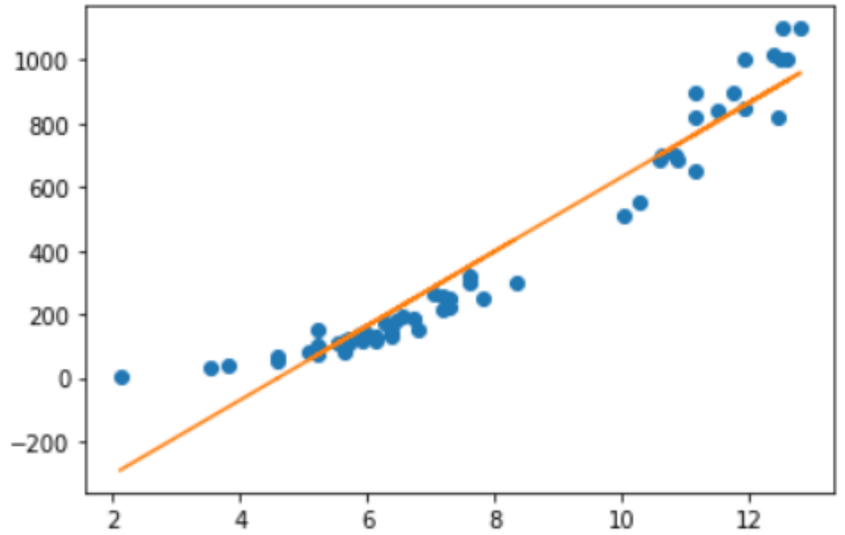
Ecuación del modelo:

$$\hat{y} = -537.32 + 116.96x_4$$

GRÁFICA DEL MODELO

```
plt.plot(x4, y, 'o')
m, b = np.polyfit(x4, y, 1)
plt.plot(x4, m*x4 + b)
```

```
[<matplotlib.lines.Line2D at 0x7fcd656fc410>]
```



REGRESIÓN POLINOMIAL

```
[153] x4_2=perch["Height"]**2
      d2 = {"x4":x4, "x4_2":x4_2}
      X_2=pd.DataFrame(d2)
      mod3=build_model(X_2,y)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 Weight   R-squared:                       0.981
Model:                            OLS   Adj. R-squared:                  0.981
Method:                 Least Squares   F-statistic:                     1395.
Date:                Wed, 08 Sep 2021   Prob (F-statistic):           1.47e-46
Time:                        21:16:28   Log-Likelihood:                -295.11
No. Observations:                  56   AIC:                             596.2
Df Residuals:                      53   BIC:                             602.3
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         92.4902     59.731      1.548      0.127     -27.314     212.295
x4           -54.1540     15.557     -3.481      0.001     -85.357     -22.952
x4_2          10.2295      0.920     11.118      0.000       8.384      12.075
==============================================================================
Omnibus:                       13.550   Durbin-Watson:                   1.563
Prob(Omnibus):                  0.001   Jarque-Bera (JB):               34.089
Skew:                          -0.510   Prob(JB):                     3.96e-08
Kurtosis:                       6.684   Cond. No.                         815.
==============================================================================
```

Ecuación del modelo:

$$\hat{y} = 92.49 - 54.15x_4 + 10.22{x_4}^2$$

REGRESIÓN POLINOMIAL

```
[153] x4_2=perch["Height"]**2
      d2 = {"x4":x4, "x4_2":x4_2}
      X_2=pd.DataFrame(d2)
      mod3=build_model(X_2,y)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 Weight   R-squared:                       0.981
Model:                            OLS   Adj. R-squared:                  0.981
Method:                 Least Squares   F-statistic:                     1395.
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No. Observations:                  56   AIC:                             596.2
Df Residuals:                      53   BIC:                             602.3
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         92.4902     59.731      1.548      0.127     -27.314     212.295
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x4_2          10.2295      0.920     11.118      0.000       8.384      12.075
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Skew:                          -0.510   Prob(JB):                     3.96e-08
Kurtosis:                       6.684   Cond. No.                         815.
==============================================================================
```

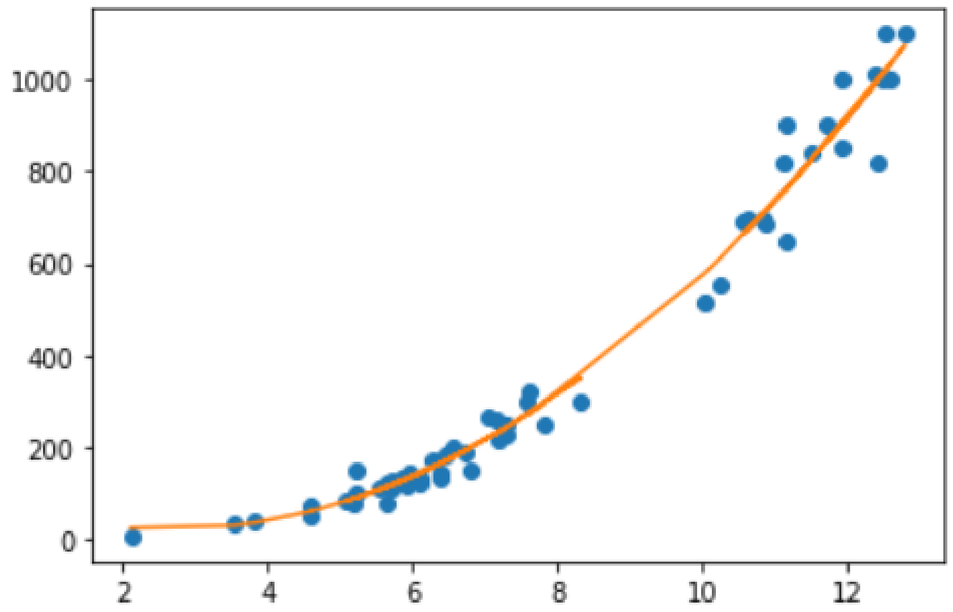
Ecuación del modelo:

$$\hat{y} = 92.49 - 54.15x_4 + 10.22{x_4}^2$$

GRÁFICA DEL MODELO

```
plt.plot(x4, y, 'o')
a,b,c = np.polyfit(x4, y, 2)
plt.plot(x4, a*x4*x4 +b*x4 + c)
```

```
[<matplotlib.lines.Line2D at 0x7fcd657d4a90>]
```



CÓMO HACER UNA ESTIMACIÓN

Haciendo uso de la ecuación del modelo, se sustituye la x.

Estimar el precio de una percha con altura de 9.5 cm

$$\hat{y} = 92.49 - 54.15x_4 + 10.22{x_4}^2$$

$$\hat{y} = 92.49 - 54.15(9.5) + 10.22(9.5)^2$$

$$\hat{y} = 501.2379\ g$$

```
X_2=sm.add_constant(X_2)
y_pred = sm.OLS(y,X_2).fit().predict([1,9.5,9.5**2])
y_pred
```

```
array([501.23791418])
```

PREGUNTAS

- ¿EN LA REGRESIÓN LINEAL SIMPLE, QUÉ DETERMINA EL COEFICIENTE B1?
- ¿QUÉ NOMBRE SE LE ASIGNA A LA(S) VARIABLE(S) INDEPENDIENTE(S) (X'S) EN LOS MODELOS DE REGRESIÓN?
- ¿CUÁNTOS COEFICIENTES B HAY EN UN MODELO DE REGRESIÓN CON TRES VARIABLES PREDICTORAS?
- ¿QUÉ SIGNIFICA UN VIF ALTO?
- ¿ES LA RELACIÓN DE DEPENDENCIA LINEAL FUERTE ENTRE MÁS DE DOS VARIABLES EN REGRESIÓN MULTIPLE?

RESPUESTAS

- DETERMINA LA PENDIENTEDE LA RECTA EN EL MODELO
- VARIABLE(S) PREDICTORA(S)
- CUATRO, B0, B1, B2 Y B3
- SIGNIFICA QUE UNA DE LAS VARIABLES REGRESORAS ESTÁ RELACIONADA LINEALMENTE CON LAS DEMÁS
- MULTICOLINEALIDAD

BIBLIOGRAFÍA

- *Apuntes Métodos Estadísticos* con MET. Alejandra Cerda. Sem Febrero-Junio 2021
- Goyal, S. (2019). *Car Price Prediction (Linear Regression - RFE)*. Obtenido de Kaggle: https://www.kaggle.com/goyalshalini93/car-price-prediction-linear-regression-rfe
- *How to plot a linear regression line on a scatter plot in Python*. (s.f.). Obtenido de Kite: https://www.kite.com/python/answers/how-to-plot-a-linear-regression-line-on-a-scatter-plot-in-python
- *pandas.DataFrame.reset_index*. (s.f.). Obtenido de pandas: https://pandas.pydata.org/pandas-docs/dev/reference/api/pandas.DataFrame.reset_index.html
- Pyae, A. (2019, junio 13). *Fish Market*. Retrieved from Kaggle: https://www.kaggle.com/aungpyaeap/fish-market