Integral Definida - Cambio de variable

Bit Planet

Solución

$$\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx = \int_1^5 \frac{x}{(2x-1)^{\frac{1}{2}}}\,dx$$

$$= \int_1^5 x\,(2x-1)^{-\frac{1}{2}}\,dx$$

$$* \ \frac{d}{dx}x^n = nx^{n-1}$$

$$* \ \frac{d}{dx}c = \emptyset$$

- $u = 2x-1$

$du = 2dx$

$\frac{du}{2} = dx$

$$* \ x = \frac{u+1}{2}$$

$$= \int_1^5 x\,(u)^{-\frac{1}{2}}\,\frac{du}{2} = \int_1^5 \left(\frac{u+1}{2}\right)(u)^{-\frac{1}{2}}\,\frac{du}{2}$$

$$= \frac{1}{4}\int_1^5 u^{\frac{1}{2}} + u^{-\frac{1}{2}}\,du$$

$$* \int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$= \frac{1}{4}\ \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Bigg|_1^5 = \frac{1}{4}\ \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{u^{\frac{1}{2}}}{\frac{1}{2}}$$

$$= \frac{1}{4}\ \frac{2u^{\frac{3}{2}}}{3} + 2u^{\frac{1}{2}}\Bigg|_{1=x_1}^{5=x_2}$$

$*u = 2x-1$

$u_2 = 2(5)-1$

$u_2 = 9$

$u_1 = 2(1)-1$

$u_1 = 1$

$$= \frac{1}{4}\ \frac{2u^{\frac{3}{2}}}{3} + 2u^{\frac{1}{2}}\Bigg|_1^9$$

$$= \frac{1}{4}\left[\frac{2}{3}\left(9^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) + 2\left(9^{\frac{1}{2}} - 1^{\frac{1}{2}}\right)\right]$$

$$= \frac{1}{4}\left[\frac{2}{3}(27-1) + 2(3-1)\right]$$

$$= \frac{1}{4}\left[\frac{2}{3}(26) + 2(2)\right] = \frac{1}{4}\left[\frac{52}{3} + 4\right]$$

$$= \frac{1}{4}\left[\frac{64}{3}\right] = \frac{16}{3}$$