Integral - Impropia -> Límite Infinito

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$$\int_{-\infty}^{\infty} \frac{e^x}{e^x+1}\,dx$$

*Cuando ambos extremos de integración son infinito.

- $$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{a \to -\infty} \int_a^0 f(x)\,dx + \lim_{b \to \infty} \int_0^b f(x)\,dx$$

$$= \lim_{a \to -\infty} \int_a^0 \frac{e^x}{e^x+1}\,dx + \lim_{b \to \infty} \int_0^b \frac{e^x}{e^x+1}\,dx$$

Cambio de variable

$u = e^x + 1$

$du = e^x\,dx$

$dx = \dfrac{du}{e^x}$

$$= \lim_{a \to -\infty} \int_a^0 \frac{\cancel{e^x}}{u} \frac{du}{\cancel{e^x}} + \lim_{b \to \infty} \int_0^b \frac{\cancel{e^x}}{u} \frac{du}{\cancel{e^x}}$$

$$= \lim_{a \to -\infty} \int_a^0 \frac{1}{u}\,du + \lim_{b \to \infty} \int_0^b \frac{1}{u}\,du$$

$*\int \frac{1}{u}\,du = \ln|u| + c$

$$= \lim_{a \to -\infty} \Big[\ln|u|\Big]\Big|_a^0 + \lim_{b \to \infty} \Big[\ln|u|\Big]\Big|_0^b$$

$$= \lim_{a \to -\infty} \Big[\ln|e^x+1|\Big]\Big|_a^0 + \lim_{b \to \infty} \Big[\ln|e^x+1|\Big]\Big|_0^b$$

$$= \lim_{a \to -\infty} \Big[\ln|e^0+1| - \ln|e^a+1|\Big] + \lim_{b \to \infty} \Big[\ln|e^b+1| - \ln|e^0+1|\Big]$$

$$= \left[\ln|e^{0}+1| - \ln|e^{-\infty}+1|\right] + \left[\ln|e^{\infty}+1| - \ln|e^{0}+1|\right]$$

$$= \left[\ln|1+1| - \ln|0+1|\right] + \left[\ln|\infty+1| - \ln|1+1|\right]$$

$$= \left[\ln|2| - \ln|1|\right] + \left[\ln|\infty| - \ln|2|\right]$$

$$= \ln|2| - 0 + \infty = \infty \quad \therefore \text{Diverge}$$