Integral - Impropia -> Limite Infinito

ByteplanetZ

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$$

*Cuando ambos extremos de integracion son infinito.

- $$\int_{-\infty}^{\infty} f(x)\,dx = \lim_{a\to-\infty} \int_{a}^{0} f(x)\,dx + \lim_{b\to\infty} \int_{0}^{b} f(x)\,dx$$

$$= \lim_{a\to-\infty} \int_{a}^{0} \frac{1}{1+x^2}\,dx + \lim_{b\to\infty} \int_{0}^{b} \frac{1}{1+x^2}\,dx$$

(a, u)

$$= \lim_{a\to-\infty} \left[\frac{1}{1}\arctan\left(\frac{x}{1}\right)\right]\Big|_{a}^{0} \quad * \int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$$

$$+ \lim_{b\to\infty} \left[\frac{1}{1}\arctan\left(\frac{x}{1}\right)\right]\Big|_{0}^{b}$$

$$= [\arctan(x)]\Big|_{-\infty}^{0} + [\arctan(x)]\Big|_{0}^{\infty}$$

$$= [\arctan(0) - \arctan(-\infty)] + [\arctan(\infty) - \arctan(0)]$$

$$= \left[0 - \left(-\frac{\pi}{2}\right)\right] + \left[\frac{\pi}{2} - 0\right]$$

$$= +\frac{\pi}{2} + \frac{\pi}{2} = +\pi$$