Integral - Impropia -> Límite Infinito

BytePlanet

$$\int_1^{\infty} \frac{3}{\sqrt[3]{x}}\,dx$$

*Cuando el extremo superior de integración es infinito.

- $\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^{b} f(x)\,dx$

$$= \lim_{b\to\infty} \int_1^{b} \frac{3}{\sqrt[3]{x}}\,dx$$

* $\sqrt{x} = x^{\frac{1}{2}}$
* $\frac{1}{x} = x^{-1}$

$$= \lim_{b\to\infty} \int_1^{b} \frac{3}{x^{\frac{1}{3}}}\,dx$$

* $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

$$= \lim_{b\to\infty} \int_1^{b} 3\,x^{-\frac{1}{3}}\,dx$$

$$= 3\left[\lim_{b\to\infty} \frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}\right]\Bigg|_1^{b} = 3\left[\lim_{b\to\infty} \frac{x^{-\frac{1}{3}+\frac{3}{3}}}{-\frac{1}{3}+\frac{3}{3}}\right]\Bigg|_1^{b}$$

$$= 3\left[\frac{x^{\frac{2}{3}}}{\frac{2}{3}}\right]\Bigg|_1^{\infty} = 3\left[\frac{3}{2}\,x^{\frac{2}{3}}\right]\Bigg|_1^{\infty}$$

$$= \frac{9}{2}\left[x^{\frac{2}{3}}\right]\Bigg|_1^{\infty} = \frac{9}{2}\left[\infty^{\frac{2}{3}} - 1^{\frac{2}{3}}\right]$$

$$\frac{9}{2}\left[\infty - 1^{\frac{2}{3}}\right] = \frac{9}{2}\left[\infty\right] = \underline{\infty}$$

∴ Diverge