1. Números Reales

(a) **Constantes**

(1) $\pi \approx 3.141592653589\ldots$

(2) $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx 2.718281828459\ldots$

(b) **Leyes de los Exponentes y Radicales**

Sean $x, y \in \mathbb{R}$ y $m, n \in \mathbb{Z}$. Se supone el radicando positivo cuando el índice del radical es par.

(3) $x^m x^n = x^{m+n}$

(4) $(x^m)^n = x^{mn}$

(5) $\frac{x^m}{x^n} = x^{m-n};\ x \neq 0$

(6) $(xy)^n = x^n y^n$

(7) $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n};\ y \neq 0$

(8) $x^0 = 1;\ x \neq 0$

(9) $x^{-n} = \frac{1}{x^n};\ x \neq 0$

(10) $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n;\ x \neq 0, y \neq 0$

(11) $x^{\frac{1}{n}} = \sqrt[n]{x}$

(12) $x^{\frac{m}{n}} = \sqrt[n]{x^m}$

(13) $x^{\frac{m}{n}} = (\sqrt[n]{x})^m$

(14) $\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y}$

(15) $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}};\ y \neq 0$

(16) $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}$

(c) **Propiedades de los Logaritmos**

$$\boxed{\log_a(y) = x \Leftrightarrow a^x = y}$$

Sean $a > 0$ con $a \neq 1$ y $x, y > 0$.

(17) $\log_a(xy) = \log_a(x) + \log_a(y)$

(18) $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

(19) $\log_a(x^n) = n\log_a(x)$

(20) $\log_a\left(\frac{1}{x}\right) = -\log_a(x)$

(21) $a^{\log_a x} = x;\ x > 0$

(22) $\log_a(a^x) = x;\ x \in \mathbb{R}$

(23) $\log_a(1) = 0$

(24) $a^{\log_a(a)} = a$

(25) $\log(x) = \log_{10}(x)$

(26) $\ln(x) = \log_e(x)$

(27) $\log_a(x) = \frac{\log_b(x)}{\log_b(a)} = \frac{\ln(x)}{\ln(a)};\ b > 0,\ b \neq 1$

(Fórmula de Cambio de Base)

2. Álgebra Básica

(a) **Productos Notables**

(28) $x(y+z) = xy + xz$

(29) $(x+y)(x-y) = x^2 - y^2$

(30) $(x \pm y)^2 = x^2 \pm 2xy + y^2$

(31) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(32) $(ax+b)(cx+d) = acx^2 + (ad+bc)x + bd$

(33) $(x \pm y)^3 = x^3 \pm 3x^2y + 3xy^2 \pm y^3$

(34) $(x+y)^n = \sum_{i=0}^{n}\binom{n}{r}x^{n-i}y^i;\ n \in \mathbb{N}$

(Teorema del Binomio de Newton)

(35) $\binom{n}{r} = {}_nC_r = \frac{n!}{(n-r)!r!};\ n \geq r$

(Coeficiente Binomial)

(36) $\prod_{i=1}^{n} i = n!$

(b) **Factores Notables**

(37) $x^2 - y^2 = (x-y)(x+y)$

(38) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

(39) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

(40) $x^2 \pm 2xy + y^2 = (x \pm y)^2$

3. Sumas Especiales

(41) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2+n}{2}$

(42) $\sum_{i=1}^{n} i^2 = \frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}(2n^3+3n^2+n)$

(43) $\sum_{i=1}^{n} i^3 = \frac{1}{4}n^2(n+1)^2 = \frac{1}{4}(n^4+2n^3+n^2)$

(44) $\sum_{i=1}^{n} i^4 = \frac{1}{30}(6n^5+15n^4+10n^3-n)$

(45) $\sum_{i=1}^{n}(2i-1) = n^2$

4. Ecuaciones Algebraicas

(a) **Ecuación Cuadrática**

$$\boxed{ax^2 + bx + c = 0;\ a, b, c \in \mathbb{R},\ a \neq 0}$$

(46) $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

(47) $\Delta = b^2 - 4ac$ *(Discriminante)*

- Si $\Delta > 0 \rightarrow$ Raíces Reales Diferentes
- Si $\Delta = 0 \rightarrow$ Raíces Reales Iguales
- Si $\Delta < 0 \rightarrow$ Raíces Complejas

(b) **Ecuación Cúbica**

$$\boxed{ax^3 + bx^2 + cx + d = 0;\ a, b, c, d \in \mathbb{R},\ a \neq 0}$$

(48) $Q = \frac{3b-a^2}{9}$

(49) $R = \frac{9ab-27c-2a^3}{54}$

(50) $S = \sqrt[3]{R + \sqrt{Q^3+R^2}}$

(51) $T = \sqrt[3]{R - \sqrt{Q^3+R^2}}$

(52) $\Delta = Q^3 + R^2$ *(Discriminante)*

- Si $\Delta > 0 \rightarrow$ una raíz real y dos complejas conjugadas
- Si $\Delta = 0 \rightarrow$ raíces reales y por lo menos dos son iguales
- Si $\Delta < 0 \rightarrow$ raíces reales y diferentes

(53) $x_1 = S + T - \dfrac{a}{3}$

(54) $x_2 = -\left(\dfrac{S+T}{2} + \dfrac{a}{3}\right) + \left(\dfrac{(S-T)\sqrt{3}}{2}\right) i$

(55) $x_3 = -\left(\dfrac{S+T}{2} + \dfrac{a}{3}\right) - \left(\dfrac{(S-T)\sqrt{3}}{2}\right) i$

(c) **Fórmula de Euler**

(56) $e^{\alpha \pm i\beta} = e^{\alpha}(\cos\beta \pm i \sin\beta);\ \alpha, \beta \in \mathbb{R}$

5. Trigonometría

(a) Definiciones

(57) $\sin\theta = \dfrac{\mathrm{CO}}{\mathrm{HIP}}$	(60) $\cot\theta = \dfrac{\mathrm{CA}}{\mathrm{CO}}$
(58) $\cos\theta = \dfrac{\mathrm{CA}}{\mathrm{HIP}}$	(61) $\sec\theta = \dfrac{\mathrm{HIP}}{\mathrm{CA}}$
(59) $\tan\theta = \dfrac{\mathrm{CO}}{\mathrm{CA}}$	(62) $\csc\theta = \dfrac{\mathrm{HIP}}{\mathrm{CO}}$

(b) Relaciones entre Grados y Radianes

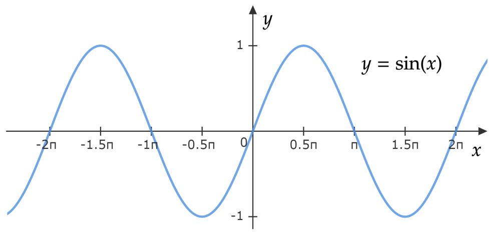
(63) 1 radián $= \left(\frac{180}{\pi}\right)^{\circ} \approx 57.2958^{\circ}$

(64) $1^{\circ} = \frac{\pi}{180}$ radianes ≈ 0.1745 radianes

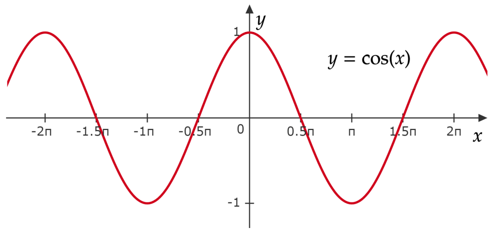
(c) Valores Exactos

Ángulo θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0^{\circ} = 0$	0	1	0
$30^{\circ} = \frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$45^{\circ} = \frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60^{\circ} = \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90^{\circ} = \frac{\pi}{2}$	1	0	$\pm\infty$

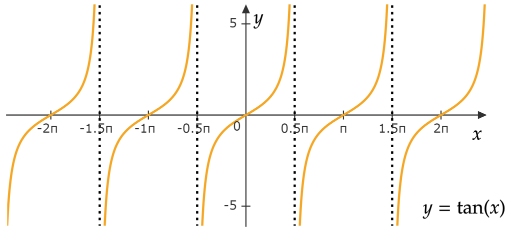
(d) Gráficas



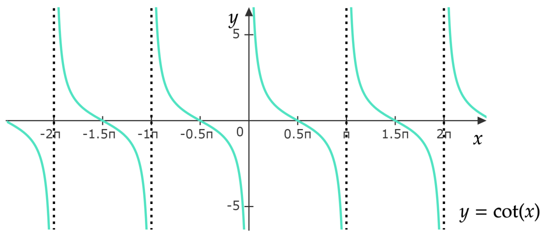
i.



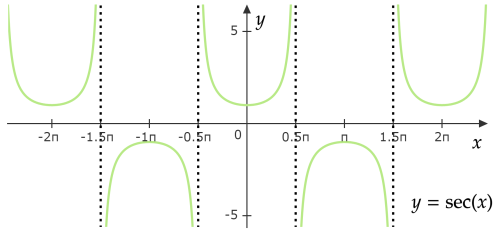
ii.



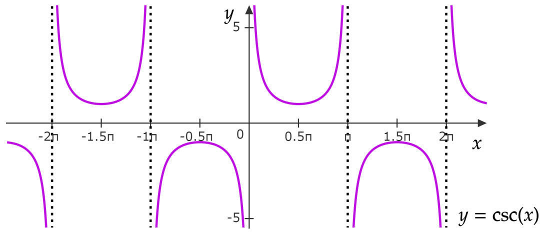
iii.



iv.



v.



vi.

(e) Identidades Básicas

(65) $\sin^2\theta + \cos^2\theta = 1$

(66) $1 + \tan^2\theta = \sec^2\theta$

(67) $1 + \cot^2\theta = \csc^2\theta$

(68) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

(69) $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

(70) $\cot\theta = \dfrac{1}{\tan\theta}$

(71) $\sec\theta = \dfrac{1}{\cos\theta}$

(72) $\csc\theta = \dfrac{1}{\sin\theta}$

(73) $\sin(-\theta) = -\sin\theta$

(74) $\cos(-\theta) = \cos\theta$

(75) $\tan(-\theta) = -\tan\theta$

(76) $\sin\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta$

(77) $\cos\left(\dfrac{\pi}{2} - \theta\right) = \sin\theta$

(78) $\tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta$

(f) **Identidades de Ángulo Doble**

(79) $\sin 2\theta = 2\sin\theta\cos\theta$

(80) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

(81) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

(g) **Identidades de Ángulo Mitad**

(82) $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

(83) $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

(h) **Identidades de Adición y Sustracción**

(84) $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

(85) $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

(86) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

(87) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

(88) $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

(89) $\tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

(i) **Productos de Senos y Cosenos**

(90) $\sin m\theta\sin n\theta = \frac{1}{2}\left[\cos(m-n)\theta - \cos(m+n)\theta\right]$

(91) $\sin m\theta\cos n\theta = \frac{1}{2}\left[\sin(m-n)\theta + \sin(m+n)\theta\right]$

(92) $\cos m\theta\cos n\theta = \frac{1}{2}\left[\cos(m-n)\theta + \cos(m+n)\theta\right]$

(j) **Potencias de Senos y Cosenos**

(93) $\sin^3\theta = \dfrac{3}{4}\sin\theta - \dfrac{1}{4}\sin(3\theta)$

(94) $\cos^3\theta = \dfrac{3}{4}\cos\theta + \dfrac{1}{4}\cos(3\theta)$

(95) $\sin^4\theta = \dfrac{3}{8} - \dfrac{1}{2}\cos(2\theta) + \dfrac{1}{8}\cos(4\theta)$

(96) $\cos^4\theta = \dfrac{3}{8} + \dfrac{1}{2}\cos(2\theta) + \dfrac{1}{8}\cos(4\theta)$

(97) $\sin^5\theta = \dfrac{5}{8}\sin\theta - \dfrac{5}{16}\sin(3\theta) + \dfrac{1}{16}\sin(5\theta)$

(98) $\cos^5\theta = \dfrac{5}{8}\cos\theta + \dfrac{5}{16}\cos(3\theta) + \dfrac{1}{16}\cos(5\theta)$

(k) **Leyes de los Triángulos**

Leyes válidas para cualquier triángulo plano ABC de lados a, b, c y ángulos A, B, C.

(99) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ *(Ley de senos)*

(100) $c^2 = a^2 + b^2 - 2ab\cos C$ *(Ley de cosenos)*

(101) $\dfrac{a+b}{a-b} = \dfrac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}$ *(Ley de tangentes)*

6. Fórmulas de Derivación

u, v, w son funciones de x; F función de u; a, b, c, n constantes con restricciones si así se indica; $\mathrm{e} = 2.71828\ldots$ es la base natural de los logaritmos; $\ln u$ es el logaritmo natural de u (logaritmo base e), donde se supone que $u > 0$ y que todos los ángulos se dan en radianes.

Definición de Derivada

$$\boxed{f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}}$$

(a) **Fórmulas Generales**

(102) $\dfrac{\mathrm{d}}{\mathrm{d}x}(c) = 0;\ c\in\mathbb{R}$

(103) $\dfrac{\mathrm{d}}{\mathrm{d}x}(cx) = c$

(104) $\dfrac{\mathrm{d}}{\mathrm{d}x}(cx^n) = ncx^{n-1}$

(105) $\dfrac{\mathrm{d}}{\mathrm{d}x}[u \pm v \pm w \pm \cdots] = \dfrac{\mathrm{d}u}{\mathrm{d}x} \pm \dfrac{\mathrm{d}v}{\mathrm{d}x} \pm \dfrac{\mathrm{d}w}{\mathrm{d}x} \pm \cdots$

(106) $\dfrac{\mathrm{d}}{\mathrm{d}x}(cu) = c\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(107) $\dfrac{\mathrm{d}}{\mathrm{d}x}(u\cdot v) = u\dfrac{\mathrm{d}v}{\mathrm{d}x} + v\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(108) $\dfrac{\mathrm{d}}{\mathrm{d}x}(uvw) = uv\dfrac{\mathrm{d}w}{\mathrm{d}x} + uw\dfrac{\mathrm{d}v}{\mathrm{d}x} + vw\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(109) $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

(110) $\dfrac{\mathrm{d}}{\mathrm{d}x}(u^n) = nu^{n-1}\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(111) $\dfrac{\mathrm{d}F}{\mathrm{d}x} = \dfrac{\mathrm{d}F}{\mathrm{d}u}\dfrac{\mathrm{d}u}{\mathrm{d}x}$ *(Regla de la Cadena)*

(112) $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{\frac{\mathrm{d}x}{\mathrm{d}u}}$

(113) $\dfrac{\mathrm{d}^n u}{\mathrm{d}x^n} = \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\mathrm{d}^{n-1}u}{\mathrm{d}x^{n-1}}\right)$

(114) $\dfrac{\mathrm{d}}{\mathrm{d}x}(uv)^n = \displaystyle\sum_{i=0}^{n}\binom{n}{i}\dfrac{\mathrm{d}^i u}{\mathrm{d}x^i}\cdot\dfrac{\mathrm{d}^{n-i}v}{\mathrm{d}x^{n-i}}$

(Regla de Leibniz)

(b) **Funciones Exponenciales y Logarítmicas**

(115) $\dfrac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^u = \mathrm{e}^u\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(116) $\dfrac{\mathrm{d}}{\mathrm{d}x}a^u = a^u\ln a\dfrac{\mathrm{d}u}{\mathrm{d}x};\ a > 0$

(117) $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln|u| = \dfrac{1}{u}\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(118) $\dfrac{\mathrm{d}}{\mathrm{d}x}\log_a u = \dfrac{1}{u\ln a}\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(119) $\dfrac{\mathrm{d}}{\mathrm{d}x}u^v = vu^{v-1}\dfrac{\mathrm{d}u}{\mathrm{d}x} + u^v\ln u\dfrac{\mathrm{d}v}{\mathrm{d}x}$

(c) **Funciones Trigonométricas**

(120) $\dfrac{\mathrm{d}}{\mathrm{d}x}\sin u = \cos u\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(121) $\dfrac{\mathrm{d}}{\mathrm{d}x}\cos u = -\sin u\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(122) $\dfrac{\mathrm{d}}{\mathrm{d}x}\tan u = \sec^2 u\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(123) $\dfrac{\mathrm{d}}{\mathrm{d}x}\cot u = -\csc^2 u\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(124) $\dfrac{\mathrm{d}}{\mathrm{d}x}\sec u = \sec u\tan u\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(125) $\dfrac{\mathrm{d}}{\mathrm{d}x}\csc u = -\csc u\cot u\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(d) **Funciones Trigonométricas Inversas**

(126) $\dfrac{\mathrm{d}}{\mathrm{d}x}\arcsin u = \dfrac{1}{\sqrt{1-u^2}}\dfrac{\mathrm{d}u}{\mathrm{d}x}$

(127) $\dfrac{d}{dx}\arccos u = \dfrac{-1}{\sqrt{1-u^2}}\dfrac{du}{dx}$

(128) $\dfrac{d}{dx}\arctan u = \dfrac{1}{1+u^2}\dfrac{du}{dx}$

(129) $\dfrac{d}{dx}\text{arccot} u = \dfrac{-1}{1+u^2}\dfrac{du}{dx}$

(130) $\dfrac{d}{dx}\text{arcsec} u = \dfrac{1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}$

(131) $\dfrac{d}{dx}\text{arccsc} u = \dfrac{-1}{|u|\sqrt{u^2-1}}\dfrac{du}{dx}$

(e) **Funciones Hiperbólicas**

Definiciones e Identidades Hiperbólicas Básicas

(132) $\sinh x = \dfrac{e^x - e^{-x}}{2}$

(133) $\cosh x = \dfrac{e^x + e^{-x}}{2}$

(134) $\tanh x = \dfrac{\sinh x}{\cosh x}$

(135) $\coth x = \dfrac{\cosh x}{\sinh x}$

(136) $\text{sech} x = \dfrac{1}{\cosh x}$

(137) $\text{csch} x = \dfrac{1}{\sinh x}$

(138) $\cosh^2 x - \sinh^2 x = 1$

(139) $\text{sech}^2 x + \tanh^2 x = 1$

(140) $\coth^2 - \text{csch}^2 x = 1$

(141) $\sinh 2x = 2\sinh x\cosh x$

(142) $\cosh 2x = \cosh^2 x + \sinh^2 x$

(143) $\tanh 2x = \dfrac{2\tanh x}{1+\tanh^2 x}$

(144) $\dfrac{d}{dx}\sinh u = \cosh u\dfrac{du}{dx}$

(145) $\dfrac{d}{dx}\cosh u = \sinh u\dfrac{du}{dx}$

(146) $\dfrac{d}{dx}\tanh u = \text{sech}^2 u\dfrac{du}{dx}$

(147) $\dfrac{d}{dx}\coth u = -\text{csch}^2 u\dfrac{du}{dx}$

(148) $\dfrac{d}{dx}\text{sech} u = -\text{sech} u\tanh u\dfrac{du}{dx}$

(149) $\dfrac{d}{dx}\text{csch} u = -\text{csch} u\coth u\dfrac{du}{dx}$

(f) **Funciones Hiperbólicas Inversas**

(150) $\dfrac{d}{dx}\sinh^{-1} u = \dfrac{1}{\sqrt{u^2+1}}\dfrac{du}{dx}$

(151) $\dfrac{d}{dx}\cosh^{-1} u = \dfrac{\pm 1}{\sqrt{u^2-1}}\dfrac{du}{dx}$

(152) $\dfrac{d}{dx}\tanh^{-1} u = \dfrac{1}{1-u^2}\dfrac{du}{dx}$

(153) $\dfrac{d}{dx}\coth^{-1} u = \dfrac{1}{1-u^2}\dfrac{du}{dx}$

(154) $\dfrac{d}{dx}\text{sech}^{-1} u = \dfrac{\mp 1}{u\sqrt{1-u^2}}\dfrac{du}{dx}$

(155) $\dfrac{d}{dx}\text{csch}^{-1} u = \dfrac{-1}{|u|\sqrt{1+u^2}}\dfrac{du}{dx}$

7. Propiedades de la Integral

Definición de Integral Definida

$$\boxed{\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x}$$

Sean $a, b, c \in \mathbb{R}$.

(156) $\displaystyle\int_a^b f(x)dx = F(b) - F(a)$

(157) $\displaystyle\int_a^b f(x)dx = -\int_b^a f(x)dx$

(158) $\displaystyle\int_a^a f(x)dx = 0$

(159) $\displaystyle\int_a^b c dx = c(b-a)$

(160) $\displaystyle\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

(161) $\displaystyle\int_a^b cf(x)dx = c\int_a^b f(x)dx$

(162) $\displaystyle\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

(163) $\displaystyle\int_a^\infty f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$

(164) $\displaystyle\int \sum_{i=1}^{n} f_i(x)dx = \sum_{i=1}^{n}\int f_i(x)dx$

8. Fórmulas de Integración

(se omite la constante de integración)

(a) **Formas Básicas**

(165) $\displaystyle\int k dx = kx;\ k\in\mathbb{R}$

(166) $\displaystyle\int u^n du = \frac{u^{n+1}}{n+1};\ n \neq -1$

(167) $\displaystyle\int \frac{du}{u} = \ln|u|$

(168) $\displaystyle\int e^u du = e^u$

(169) $\displaystyle\int a^u du = \frac{1}{\ln a}a^u$

(170) $\displaystyle\int \sin u du = -\cos u$

(171) $\displaystyle\int \cos u du = \sin u$

(172) $\displaystyle\int \sec^2 u du = \tan u$

(173) $\displaystyle\int \csc^2 u du = -\cot u$

(174) $\displaystyle\int \sec u\tan u du = \sec u$

(175) $\displaystyle\int \csc u\cot u du = -\csc u$

(176) $\displaystyle\int \tan u du = \ln|\sec u|$

(177) $\displaystyle\int \cot u du = \ln|\sin u|$

(178) $\int \sec u du = \ln|\sec u + \tan u|$

(179) $\int \csc u du = \ln|\csc u - \cot u|$

(180) $\int \frac{1}{\sqrt{a^2-u^2}}du = \sin^{-1}\frac{u}{a}$

(181) $\int \frac{1}{a^2+u^2}du = \frac{1}{a}\tan^{-1}\frac{u}{a}$

(182) $\int \frac{1}{u\sqrt{u^2-a^2}}du = \frac{1}{a}\sec^{-1}\frac{u}{a}$

(183) $\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right|$

(184) $\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right|$

(b) **Formas Trigonométricas**

(185) $\int \sin^2 u du = \frac{1}{2} - \frac{1}{4}\sin(2u)$

(186) $\int \cos^2 u du = \frac{1}{2}u + \frac{1}{4}\sin(2u)$

(187) $\int \tan^2 u du = \tan u - u$

(188) $\int \cot^2 u du = -\cot u - u$

(189) $\int \sin^3 u du = -\frac{1}{3}(2+\sin^2 u)\cos u$

(190) $\int \cos^3 u du = \frac{1}{3}(2+\cos^2 u)\sin u$

(191) $\int \tan^3 u du = \frac{1}{2}\tan^2 u + \ln|\cos u|$

(192) $\int \cot^3 u du = -\frac{1}{2}\cot^2 u - \ln|\sin u|$

(193) $\int \sec^3 u du = \frac{1}{2}\sec u\tan u + \frac{1}{2}\ln|\sec u + \tan u|$

(194) $\int \csc^3 u du = -\frac{1}{2}\csc u\cot u + \frac{1}{2}\ln|\csc u - \cot u|$

(c) **Formas Trigonométricas Inversas**

(195) $\int \sin^{-1} u du = u\sin^{-1} u + \sqrt{1-u^2}$

(196) $\int \cos^{-1} u du = u\cos^{-1} u - \sqrt{1-u^2}$

(197) $\int \tan^{-1} u du = u\tan^{-1} u - \frac{1}{2}\ln(1+u^2)$

(198) $\int u\sin^{-1} u du = \frac{2u^2-1}{4}\sin^{-1} u + \frac{u\sqrt{1-u^2}}{4}$

(199) $\int u\cos^{-1} u du = \frac{2u^2-1}{4}\cos^{-1} u - \frac{u\sqrt{1-u^2}}{4}$

(200) $\int u\tan^{-1} u du = \frac{u^2+1}{2}\tan^{-1} u - \frac{u}{2}$

(d) **Formas Exponenciales y Logarítmicas**

(201) $\int ue^{au}du = \frac{1}{a^2}(au-1)e^{au}$

(202) $\int e^{au}\sin(bu)du = \frac{e^{au}}{a^2+b^2}[a\sin(bu) - b\cos(bu)]$

(203) $\int e^{au}\cos(bu)du = \frac{e^{au}}{a^2+b^2}[a\cos(bu) + b\sin(bu)]$

(204) $\int \ln u du = u\ln u - u$

(205) $\int \frac{du}{u\ln u} = \ln|\ln u|$

(e) **Formas Hiperbólicas**

(206) $\int \sinh u du = \cosh u$

(207) $\int \cosh u du = \sinh u$

(208) $\int \tanh u du = \ln(\cosh u)$

(209) $\int \coth u du = \ln|\sinh u|$

(210) $\int \mathrm{sech} u du = \tan^{-1}|\sinh u|$

(211) $\int \mathrm{csch} u du = \ln\left|\tanh\left(\frac{u}{2}\right)\right|$

(212) $\int \mathrm{sech}^2 u du = \tanh u$

(213) $\int \mathrm{csch}^2 u du = -\coth u$

(214) $\int \mathrm{sech} u\tanh u du = -\mathrm{sech} u$

(215) $\int \mathrm{csch} u\coth u du = -\mathrm{csch} u$

9. Series de Taylor

(216) $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n;\ |x|<1$

(217) $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n;\ |x|<1$

(218) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!};\ |x|<\infty$

(219) $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2m+1}}{(2n+1)!};\ |x|<\infty$

(220) $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!};\ |x|<\infty$

(221) $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n};\ -1<x\le 1$

(222) $\ln\frac{1+x}{1-x} = 2\tanh^{-1} x = 2\sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1};\ |x|<1$

(223) $\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1};\ |x|\le 1$

(224) $(1+x)^m = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k;\ |x|<1$

10. Métodos de Integración

(225) **Sustitución Algebraica**

$$\int f(g(x))g'(x)dx = \int f(u)du$$

(226) **Integración por Partes**

$$\int udv = uv - \int vdu$$

(227) **Fracciones Parciales**

Supóngase que $\dfrac{f(x)}{g(x)}$ es una fracción propia.

i. Sea $x-r$ factor de $g(x)$. Si $(x-r)^m$ es la potencia más grande de $x-r$ que divide a $g(x)$ entonces para cada factor lineal distinto de $g(x)$ se asigna la suma de m fracciones parciales:

$$\frac{A_1}{x-r}+\frac{A_2}{(x-r)^2}+\ldots+\frac{A_m}{(x-r)^m}$$

ii. Sea x^2+px+q un factor cuadrático irreducible de $g(x)$. Si $(x^2+px+q)^n$ es la potencia más grande de este factor que divide a $g(x)$ entonces para cada factor cuadrático distinto de $g(x)$ se asigna la suma de n fracciones parciales:

$$\frac{B_1x+C_1}{x^2+px+q}+\frac{B_2x+C_2}{(x^2+px+q)^2}+\ldots+\frac{B_nx+C_n}{(x^2+px+q)^n}$$

(228) **Integrales Trigonométricas**

$$\int \sin^m x\cos^n x\mathrm{d}x$$

Caso 1 Si m es impar entonces $m=2k+1$; usar $\sin^2 x = 1-\cos^2 x$ y entonces

$$\begin{aligned}\sin^m x = \sin^{2k+1} x &= (\sin^2 x)^k \sin x\\ &= (1-\cos^2 x)^k \sin k\end{aligned}$$

Caso 2 Si m es par y n impar entonces $n=2k+1$; usar $\cos^2 x = 1-\sin^2 x$ y entonces

$$\begin{aligned}\cos^n x = \cos^{2k+1} x &= (\cos^2 x)^k \cos x\\ &= (1-\sin^2 x)^k \cos x\end{aligned}$$

Caso 3 Si m y n son pares, usar las fórmulas (81) y (82).

(229) **Sustituciones Trigonométricas**

i. $\sqrt{a^2-x^2} \longrightarrow x = a\sin\theta \quad \theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

ii. $\sqrt{a^2+x^2} \longrightarrow x = a\tan\theta \quad \theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

iii. $\sqrt{x^2-a^2} \longrightarrow x = a\ \mathrm{sec}\theta \quad \theta\in[0,\frac{\pi}{2})\cap(\frac{\pi}{2},\pi]$

iv. $z=\tan\dfrac{x}{2}$ luego

$$\cos x = \frac{1-z^2}{1+z^2}$$
$$\sin x = \frac{2z}{1+z^2}$$
$$\mathrm{d}x = \frac{2\mathrm{d}z}{1+z^2}$$