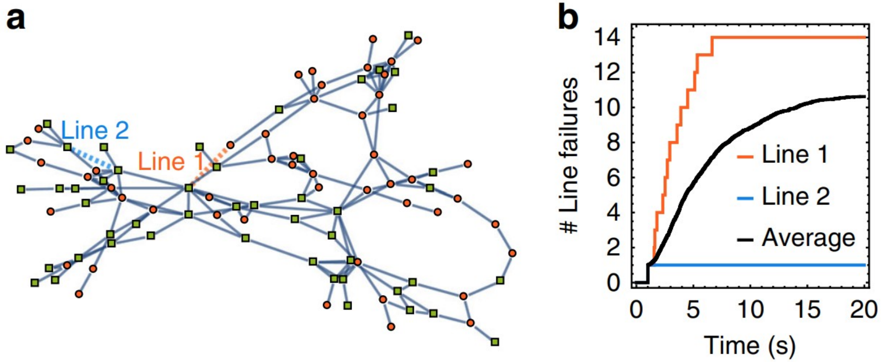
Power Grid Analysis with Applications to Power grid Security

Why Do we care?

A failure in a line causes stress on the power grid.

When looking at the transient after failing a line, we can see other power line reach a flow higher than the lines capacity.

These additional failure can lead to cascading failure.

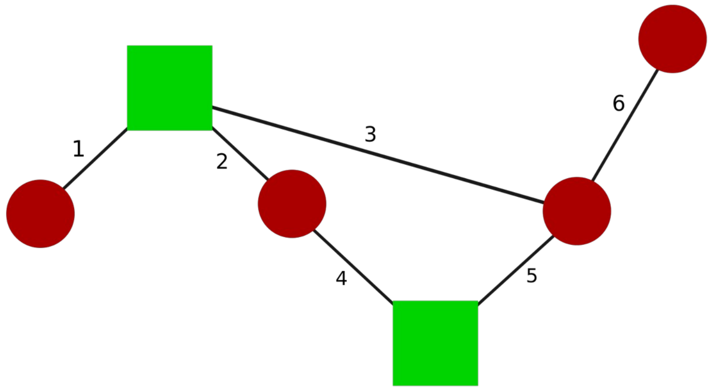


One way to analyze these

Swing Equation:

$$\ddot{\theta}_I = P_i - \gamma_i \dot{\theta} + \Sigma_j K_{ij} \sin(\theta_j - \theta_i)$$

$$power = K_{ij} \sin(\theta_j - \theta_i)$$



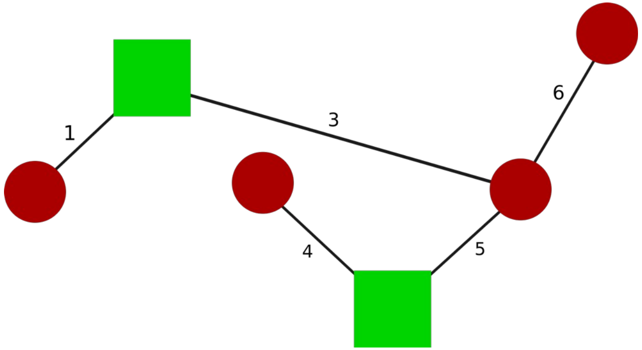
A line will fail if the power reach the threshold flow.

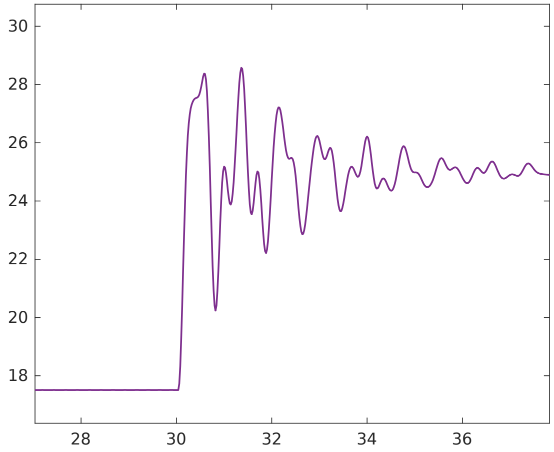
$$\alpha \in [0,1]$$
$$\alpha K_{ij} < K_{ij} \sin(\theta_j - \theta_i)$$

Example of Line Failure

An example showing the transient after a line failure.

We can see how power flow jump much larger than the steady state.



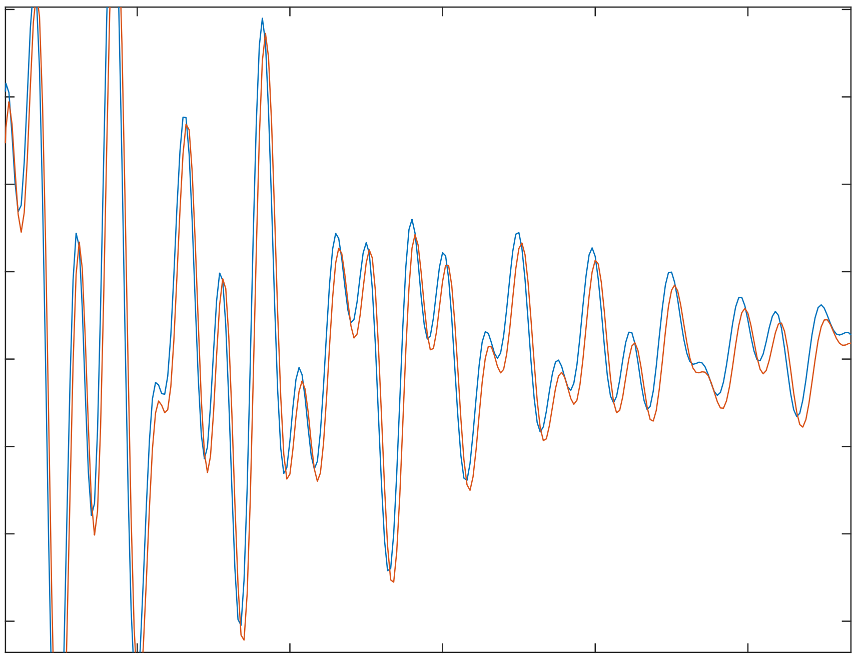


Linear Swing equation

Now we look at a linear swing equation:

$$\ddot{\theta}_I = P_i - \gamma_i \dot{\theta} + \Sigma_j K_{ij} (\theta_j - \theta_i)$$

The results of the transient is similar to the nonlinear swing equitation in the networks we are viewing.



Linearizing The Swing Equation

We now look at the linear swing equation.

$$\ddot{\theta}(t)=P-\gamma\dot{\theta}(t)+L\,\theta(t)$$

$$Where \quad L \quad is \quad the \quad Laplacian \quad \text{matrix}$$

We now diagonalize L. Where V is the eigenvector and lambda are the eigenvalues .

$$L=V\,\Lambda\,V^{-1}$$
$$we \quad multiply \quad the \quad \text{left} \quad side \quad by \quad V^{-1}$$

We now set

$$\eta(t)=V^{-1}\,\theta(t)$$
$$Q=V^{-1}\,P$$

And get

$$\ddot{\eta}(t)=Q-\gamma\,\dot{\eta}(t)-\Lambda\,\eta(t)$$

We can break up this equation

$$\ddot{\eta}_i(t)=Q_i-\gamma\,\dot{\eta}_i(t)-\lambda_i\,\eta_i(t)$$

Linearize Swing Equation

We now have decoupled the systems. We can now analyze eta independently of other etas

To get back theta we use:

$$\theta_i(t) = \Sigma_J V_{ij}\, \eta_j(t)$$